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Symmetry in Rhotrices and Rhomtree Applications for Hybrid Orbitals in Chemical Compounds

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Abstract
This work explores the application of the robust multiplication method of even dimensional rhotrices in determining the symmetric lines of some organic chemical compounds and graphical representation of energy distribution of some these compounds using rhomtrees. The co-minors of R_2 and M_2 rhotrices that produced a given symmetric pattern that analyzed the unique properties of even dimensional rhotrices were of great help. By partitioning the co-minors of these systems the underlying rhotrices were used to obtain the rhomtrees corresponding to the chemical compounds investigated, showing distribution pathways of energy bond using these abstract structures.

Keywords: Symmetric, Co-Minors, Transpose, Heart-Oriented, Heartless Rhotrix

Introduction
Ajibade (2003) introduced a mathematical array which are, in some ways, between 2×2 and 3×3 dimensional matrices as an extension of ideas on matrix-tertions and matrix- noitrets proposed by Atanassov and Shannon (1998). Thus, a rhotix is defined by Ajibade as:

$$\left\langle \begin{matrix} & a & \\ b & h(R) & d \\ & e & \end{matrix} \right\rangle \quad (1)$$

Cayley (1889) introduced a theorem on trees. Mohammed (2011) introduced the construction of rhomtrees as graphical representation of rhotrices. The definition above of a rhotrix is called a heart-oriented form of rhotrix. Where $h(R)$ is the heart of the rhotrix. Verma (2023) propose the introduction of a novel visualization concept for rhotrices, referred to as rhomtrees. Rhomtrees are graphical trees, denoted as $T(m)$, where m represents the order of the tree. The total number of nodes in the tree is determined by $m=\frac{1}{2}(n^2+1)$, and the edges given by $k=\frac{1}{2}(n^2-1)$, where n is an integer that satisfies the condition $n\in 2Z^+ +1$. The root of the rhomtree is connected to four vertices or four components of binary branches. Consequently, equation (1) can be effectively represented and interpreted through this visual representation as;

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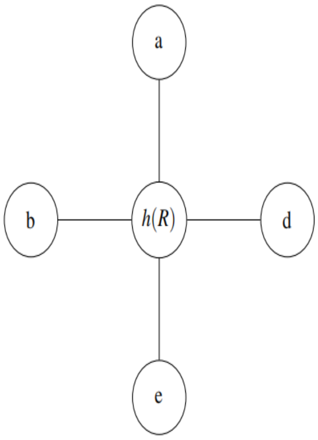

Figure 1: Rhomtree of order 5 $[T(5)]$

The above rhomtree $[T(5)]$ can represent the chain of a chemical compound called methane, as illustrated in Figure 2 below. Methane is one of the simplest saturated Alkane hydrocarbons, which have the general formula C_nH_{2n+2}. When considering isomers of saturated hydrocarbons with a specific number of carbon atoms (n), the rhomtree can be used to enumerate these isomers.

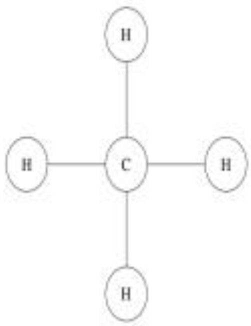

Figure 2: Methane corresponding to rhomtree T(5).

Moving on, Isere (2018) introduced another type of rhotrix called the heartless (or even dimensional rhotrix) defined as:

$$\left\langle \begin{array}{ccc} & a & \\ b & & d \\ & e & \end{array} \right\rangle \qquad (2)$$

This can also be represented graphically using an Even rhomtree as;

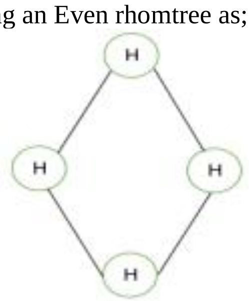

Figure 3. Butane corressponding to Rhomtree of T_4

The T_4 contains four hidden carbon atoms at each end which are attached to H atoms respectively. A Rhomtree representation for R_4 becomes

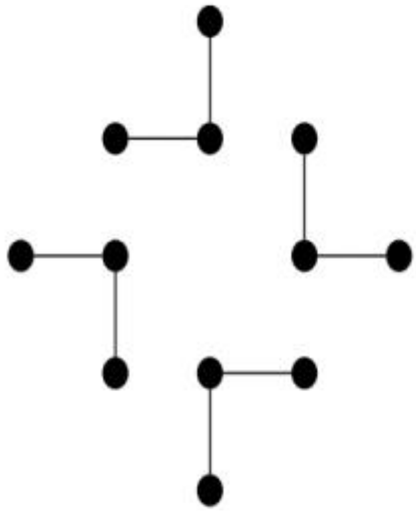

Figure 4. Cyclobuthane Rhomtree of $T(12)$

The above rhomtree $T(12)$ can represent the chain of a chemical compound called Cyclobuthane, as illustrated in Figure 4, $T(12)$ contains four carbon atoms at each end which are attached to H_2 atoms respectively. Where the total number of nodes can be determined by $m = \frac{1}{2}(n^2 + 2n)$and the total number of edges, $k = \frac{1}{2}(n^2 - 2n)$, for all $n \in 2Z^+$. Ajibade (2003) in the concluding section of his work was challenged by further development regarding how a rhotrix can be converted to a matrix and vice versa for its mathematical enrichment. In quest to solve this challenge. Sani (2004) proposed the first alternative rhotrix multiplication method using the row-column based method for rhotrix multiplication. This procedure gives an interesting and important result which gave room for more literatures in rhotrix algebra. Similarly, the concepts of even dimensional rhotrices by Isere (2018) and the New Multiplication Method for even dimensional rhotrices. Utoyo et al. (2023) will be used to show the proportionate and balanced similarity that is found in two halves of an object in this work. In computational algebra, we mostly encounter matrices with real entries, recently, we now encounter rhotrices with real entries too. However, rhotrices are capable of operating on more than one linear system when dealing with real-life problems. So, this work will be exploring real cases of symmetries in even dimensional rhotrices. Symmetries in even-dimensional rhotrices are all about exploring the patterns and transformations that these rhotrices exhibits. A new multiplication approach for even dimensional rhotrices has been introduced by Utoyo et al. (2023). In even dimensions, we find interesting symmetries that govern the behaviour of rhotrices using this new multiplication method. These symmetries involve rotations, reflections and scaling of the rhotrix elements which can help us to gain deeper understanding of the structure and properties of even dimensional rhotrix. Symmetries in even dimensional rhortices are essential because they help us to transform rhotrices in various ways.These transformations include rotations, reflections and scaling. By appling these transformations, we can discover fascinating patterns and unique properties of rhotrices.These insights are especially useful when representing transformation of objects in a virtual space like computer graphics. Rhotrices are like self-similar property where the rhotrix exhibits a repeating pattern at different scales determined by proportionate and balanced similarity that is found in two halves of the object if it is equal to the transpose of the object. One fascinating pattern in rhotrices is the concept of self-similarity. The transpose of a rhotrix is obtained from a given rhotrix R after interchanging its rows and columns. Transpose of R is denoted by R^T. If R is of order $m \times n$, then R^T is of the order $n \times m$. A rhotrix is said to be symmetric if the transpose of R is the rhotrix R itself i.e., $(R)^T = R$, (Verma, 2023).

The co-minors of the new multiplication approach for even dimensional rhotrices will help to determine whether, if given rhotrices R and S, the first column and the first row will have the same values or alphabets when operated on, which will introduce symmetries in even dimensional rhotrices.

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1. Preliminaries:

New mathematical rhomboid arrays and their classifications as abstract structures forms a unified framework for patterns analysis across systems.

Definition 1

Symmetric even dimensional rhotrices are square rhotrices that are equal to its transpose rhotrix.

The transpose rhotrix of any given rhotrix R can be given as R^t. Thus a rhotrix R is symmetric if $R = \left(R^t\right)^t$. It is like flipping the rhotrix horizontally and vertically. This allows us to study its properties and symmetries in a different way.

Definition 2

A transposed even-dimensional rhotrix is a square rhotrix that is equal to its original rhotrix when you swap the rows and columns of the co-minors of that rhotrix.

Theorem 1

Let R be an hl-rhotrix. Then R is symmetric, if and only if the first column and row are of the same entries.

Proof.

If,

$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle$$

then, their corresponding co-minors are given as:

$$\left\langle \begin{array}{ccc} & a_{11} & \\ a_{31} & & a_{13} \\ & a_{33} & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & c_{11} & \\ c_{21} & & c_{12} \\ & c_{22} & \end{array} \right\rangle \text{ and } \begin{bmatrix} a_{21} & a_{12} \\ a_{32} & a_{23} \end{bmatrix}.$$

So that, R_4^T becomes

$$\left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{12} & c_{11} & a_{21} & \\ a_{13} & c_{12} & & c_{21} & a_{31} \\ & a_{23} & c_{22} & a_{32} & \\ & & a_{33} & & \end{array} \right\rangle$$

Therefore,

$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \text{ and } R_4^T = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle$$

If, $R_4 = R_4^T$, we say, is a symmetric rhotrix.

Addition Operation of Symmetric Even Dimensional Heartless Rhotrices

The sum of two symmetric even dimensional rhotrices gives the resultant as a Symmetric rhotrix. Suppose,

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$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \text{ and } S_4 = \left\langle \begin{array}{ccccc} & & b_{11} & & \\ & b_{21} & d_{11} & b_{12} & \\ b_{31} & d_{21} & & d_{12} & b_{13} \\ & b_{32} & d_{22} & b_{23} & \\ & & b_{33} & & \end{array} \right\rangle$$

Then, the sum of these two rhotrices is

$$R_4 + S_4 = \left\langle \begin{array}{ccccc} & & a_{11}+b_{11} & & \\ & a_{21}+b_{21} & c_{11}+d_{11} & a_{12}+b_{12} & \\ a_{31}+b_{31} & c_{21}+d_{21} & & c_{12}+d_{12} & a_{13}+b_{13} \\ & a_{32}+b_{32} & c_{22}+d_{22} & a_{23}+b_{23} & \\ & & a_{33}+b_{33} & & \end{array} \right\rangle$$

So that $R_4^T + S_4^T$

$$= \left\langle \begin{array}{ccccc} & & a_{11}+b_{11} & & \\ & a_{12}+b_{12} & c_{11}+d_{11} & a_{21}+b_{21} & \\ a_{13}+b_{13} & c_{12}+d_{12} & & c_{21}+d_{21} & a_{31}+b_{31} \\ & a_{23}+b_{23} & c_{22}+d_{22} & a_{32}+b_{32} & \\ & & a_{33}+b_{33} & & \end{array} \right\rangle$$

Therefore,

$$R_4 + S_4 = \left\langle \begin{array}{ccccc} & & a_{11}+b_{11} & & \\ & a_{21}+b_{21} & c_{11}+d_{11} & a_{12}+b_{12} & \\ a_{31}+b_{31} & c_{21}+d_{21} & & c_{12}+d_{12} & a_{13}+b_{13} \\ & a_{32}+b_{32} & c_{22}+d_{22} & a_{23}+b_{23} & \\ & & a_{33}+b_{33} & & \end{array} \right\rangle \text{and } R_4^T + S_4^T = \left\langle \begin{array}{ccccc} & & a_{11}+b_{11} & & \\ & a_{21}+b_{21} & c_{11}+d_{11} & a_{12}+b_{12} & \\ a_{31}+b_{31} & c_{21}+d_{21} & & c_{12}+d_{12} & a_{13}+b_{13} \\ & a_{32}+b_{32} & c_{22}+d_{22} & a_{23}+b_{23} & \\ & & a_{33}+b_{33} & & \end{array} \right\rangle$$

If, $R_4 + S_4 = R_4^T + S_4^T$, we say, $R_4 + S_4$ is a symmetric rhotrix.

Multiplication Operation of Symmetric Even Dimensional Heartless Rhotrices:

Multiplication operation of symmetric even dimensional rhotrices is not always true, but given a symmetric rhotrices R_4 and S_4, then, RS is symmetric if and only if R and S commutes. That is, if $RS = SR$. These condition can be true, if,

- S_4 is a zero rhotrix

Suppose S_4 is a zero rhotrix,

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$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \text{ and } S_4 = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 0 & 0 & 0 & \\ 0 & 0 & & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle \text{, then,}$$

$$R_4 \circ S_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \circ \left\langle \begin{array}{ccccc} & & 0 & & \\ & 0 & 0 & 0 & \\ 0 & 0 & & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle$$

Co-minors of $R_4 \circ S_4$ is

$$\left\langle \begin{array}{ccc} & a_{11} & \\ a_{31} & & a_{13} \\ & a_{33} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & 0 & \\ 0 & & 0 \\ & 0 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & c_{11} & \\ c_{21} & & c_{12} \\ & c_{22} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & 0 & \\ 0 & & 0 \\ & 0 & \end{array} \right\rangle \text{ and } \begin{bmatrix} a_{12} & a_{23} \\ a_{21} & a_{32} \end{bmatrix} \circ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Then, the multiplication operation $\circ$ of these two rhotrices is

$$R_4 \circ S_4 = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 0 & 0 & 0 & \\ 0 & 0 & & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle \text{ and } S_4 \circ R_4 = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 0 & 0 & 0 & \\ 0 & 0 & & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle$$

If, $R_4 \circ S_4 = S_4 \circ R_4$, hence S_4 is a zero rhotrix.

- S_4 is a unit rhotrix

Suppose S_4 is a unit rhotrix,

$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \text{ and } S_4 = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 1 & 1 & 1 & \\ 1 & 1 & & 1 & 1 \\ & 1 & 1 & 1 & \\ & & 1 & & \end{array} \right\rangle \text{, then,}$$

$$R_4 \circ S_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \circ \left\langle \begin{array}{ccccc} & & 1 & & \\ & 1 & 1 & 1 & \\ 1 & 1 & & 1 & 1 \\ & 1 & 1 & 1 & \\ & & 1 & & \end{array} \right\rangle$$

Co-minors of $R_4 \circ S_4$ is

$$\left\langle \begin{array}{ccc} & a_{11} & \\ a_{31} & & a_{13} \\ & a_{33} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & c_{11} & \\ c_{21} & & c_{12} \\ & c_{22} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 1 & \end{array} \right\rangle \text{ and } \begin{bmatrix} a_{21} & a_{12} \\ a_{32} & a_{23} \end{bmatrix} \circ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Then, the multiplication operation $\circ$ of these two rhotrices is

$$R_4 \circ S_4 = \left\langle \begin{array}{ccccc} & & a_{11}+a_{13} & & \\ & a_{21}+a_{12} & c_{11}+c_{12} & a_{21}+a_{12} & \\ a_{31}+a_{33} & c_{21}+c_{22} & & c_{11}+c_{12} & a_{11}+a_{13} \\ & a_{32}+a_{23} & c_{21}+c_{22} & a_{32}+a_{23} & \\ & & a_{31}+a_{33} & & \end{array} \right\rangle \text{ and } S_4 \circ R_4 = \left\langle \begin{array}{ccccc} & & a_{13}+a_{11} & & \\ & a_{12}+a_{21} & c_{12}+c_{11} & a_{12}+a_{21} & \\ a_{33}+a_{31} & c_{22}+c_{21} & & c_{12}+c_{11} & a_{13}+a_{11} \\ & a_{23}+a_{32} & c_{22}+c_{21} & a_{23}+a_{32} & \\ & & a_{33}+a_{31} & & \end{array} \right\rangle$$

If, $R_4 \circ S_4 = S_4 \circ R_4$, hence S_4 is a unit rhotrix.

- S_4 is an identity rhotrix

Suppose S_4 is a identity rhotrix,

$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \text{ and } S_4 = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 1 & 1 & 0 & \\ 0 & 0 & & 0 & 0 \\ & 0 & 1 & 1 & \\ & & 1 & & \end{array} \right\rangle, \text{ then,}$$

$$R_4 \circ S_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \circ \left\langle \begin{array}{ccccc} & & 1 & & \\ & 1 & 1 & 0 & \\ 0 & 0 & & 0 & 0 \\ & 0 & 1 & 1 & \\ & & 1 & & \end{array} \right\rangle$$

Co-minors of $R_4 \circ S_4$ is

$$\left\langle \begin{array}{ccc} & a_{11} & \\ a_{31} & & a_{13} \\ & a_{33} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & 1 & \\ 0 & & 0 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & c_{11} & \\ c_{21} & & c_{12} \\ & c_{22} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & 1 & \\ 0 & & 0 \\ & 1 & \end{array} \right\rangle \text{ and } \begin{bmatrix} a_{21} & a_{12} \\ a_{32} & a_{23} \end{bmatrix} \circ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then, the multiplication operation $\circ$ of these two rhotrices is

$$R_4 \circ S_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \text{ and } S_4 \circ R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle$$

If, $R_4 \circ S_4 = S_4 \circ R_4$, hence S_4 is an identity rhotrix.
Since the multiplication operation of symmetries in even dimensional rhotrices satisfies the three conditions above, we say that multiplication is symmetric in these abstract structures.

R_n Operation for Symmetric Even Dimensional Heartless Rhotrices

Theorem 2. For any integer n, R_n is symmetric if R is symmetric.
Proof.
Suppose,

$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle$$

When, $n = 2$, R_4 is

$$R_4^2 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle \times \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle$$

$$\left\langle \begin{array}{ccc} & a_{11} & \\ a_{31} & & a_{13} \\ & a_{33} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & a_{11} & \\ a_{31} & & a_{13} \\ & a_{33} & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & c_{11} & \\ c_{21} & & c_{12} \\ & c_{22} & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & c_{11} & \\ c_{21} & & c_{12} \\ & c_{22} & \end{array} \right\rangle \text{ and } \begin{bmatrix} a_{21} & a_{12} \\ a_{32} & a_{23} \end{bmatrix} \circ \begin{bmatrix} a_{21} & a_{12} \\ a_{32} & a_{23} \end{bmatrix}.$$

$$\left\langle \begin{array}{ccc} & a_{11}^2 + a_{13}a_{31} & \\ a_{31}(a_{11}+a_{33}) & & a_{13}(a_{11}+a_{33}) \\ & a_{31}a_{13} + a_{33}^2 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & c_{11}^2 + c_{12}c_{21} & \\ c_{21}(c_{11}+c_{22}) & & c_{12}(c_{11}+c_{22}) \\ & c_{21}c_{12} + c_{22}^2 & \end{array} \right\rangle$$

$$\text{and } \begin{bmatrix} a_{21}^2 + a_{12}a_{32} & a_{12}(a_{21}+a_{23}) \\ a_{32}(a_{21}+a_{23}) & a_{32}a_{12} + a_{23}^2 \end{bmatrix}.$$

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Then, R_4^2 will be

$$\left\langle \begin{array}{ccccc} & & a_{11}^2+a_{13}a_{31} & & \\ & a_{21}^2+a_{12}a_{32} & c_{11}^2+c_{12}c_{21} & a_{12}(a_{21}+a_{23}) & \\ a_{31}(a_{11}+a_{33}) & c_{21}(c_{11}+c_{22}) & & c_{12}(c_{11}+c_{22}) & a_{13}(a_{11}+a_{33}) \\ & a_{32}(a_{21}+a_{23}) & c_{21}c_{12}+c_{22}^2 & a_{32}a_{12}+a_{23}^2 & \\ & & a_{31}a_{13}+a_{33}^2 & & \end{array} \right\rangle$$

The transpose of R_4^2 is

$\left(R_4^2\right)^T =$

$$\left\langle \begin{array}{ccccc} & & a_{11}^2+a_{13}a_{31} & & \\ & a_{21}^2+a_{12}a_{32} & c_{11}^2+c_{12}c_{21} & a_{32}(a_{21}+a_{23}) & \\ a_{13}(a_{11}+a_{33}) & c_{12}(c_{11}+c_{22}) & & c_{21}(c_{11}+c_{22}) & a_{31}(a_{11}+a_{33}) \\ & a_{12}(a_{21}+a_{23}) & c_{21}c_{12}+c_{22}^2 & a_{32}a_{12}+a_{23}^2 & \\ & & a_{31}a_{13}+a_{33}^2 & & \end{array} \right\rangle$$

Since, $R_4^2 = \left(R_4^2\right)^T$, where $a_{12}(a_{21}+b_{23}) = a_{32}(a_{21}+b_{23}),\ a_{13}=a_{31}$ and $c_{12}=c_{21}$; R_4^2 is symmetric if R_4 is symmetric.

Inverse Operation for Symmetric Even Dimensional Heartless Rhotrices

If the inverse R^{-1} of an even dimensional rhotrix exist, it is symmetric if and only if R is symmetric.

For

$$R_4 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{array} \right\rangle, \textbf{ the inverse of } R_4 \textbf{ will yield}$$

$$\left\langle \begin{array}{ccc} & a_{11} & \\ a_{31} & & a_{13} \\ & a_{33} & \end{array} \right\rangle = a_{11}a_{33}-a_{31}a_{13},\quad R_2^{-1} = \frac{1}{a_{11}a_{33}-a_{31}a_{13}} \left\langle \begin{array}{ccc} & a_{33} & \\ -a_{31} & & -a_{13} \\ & a_{11} & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & \frac{a_{33}}{A} & \\ -\frac{a_{31}}{A} & & -\frac{a_{13}}{A} \\ & \frac{a_{11}}{A} & \end{array} \right\rangle$$

$$\left\langle \begin{array}{ccc} & c_{11} & \\ c_{21} & & c_{12} \\ & c_{22} & \end{array} \right\rangle = c_{11}c_{22} - c_{21}c_{12},\ R_2^{-1} = \frac{1}{c_{11}c_{22}-c_{21}c_{12}} \left\langle \begin{array}{ccc} & c_{22} & \\ -c_{12} & & -c_{21} \\ & c_{11} & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & \frac{c_{22}}{B} & \\ -\frac{c_{12}}{B} & & -\frac{c_{21}}{B} \\ & \frac{c_{11}}{B} & \end{array} \right\rangle$$

$$\text{and} \begin{bmatrix} a_{21} & a_{12} \\ a_{32} & a_{23} \end{bmatrix} = a_{21}a_{23} - a_{12}a_{32},\ R_2^{-1} = \frac{1}{a_{21}a_{23}-a_{12}a_{32}} \begin{bmatrix} a_{21} & -a_{12} \\ -a_{32} & a_{23} \end{bmatrix} = \begin{bmatrix} \frac{a_{23}}{C} & -\frac{a_{12}}{C} \\ -\frac{a_{32}}{C} & \frac{a_{21}}{C} \end{bmatrix}$$

So that R_4 will become

$$R_4^{-1} = \left\langle \begin{array}{ccccc} & & \frac{a_{33}}{A} & & \\ & \frac{a_{23}}{C} & \frac{c_{22}}{B} & -\frac{a_{12}}{C} & \\ -\frac{a_{31}}{A} & -\frac{c_{21}}{B} & & -\frac{c_{12}}{B} & -\frac{a_{13}}{A} \\ & -\frac{a_{32}}{C} & \frac{c_{11}}{B} & \frac{a_{21}}{C} & \\ & & \frac{a_{11}}{A} & & \end{array} \right\rangle$$

The transpose of R_4^{-1} will be

$$\left(R_4^{-1}\right)^T = \left\langle \begin{array}{ccccc} & & \frac{a_{33}}{A} & & \\ & \frac{a_{23}}{C} & \frac{c_{22}}{B} & \frac{a_{32}}{C} & \\ \frac{a_{13}}{A} & \frac{c_{12}}{B} & & \frac{c_{21}}{B} & \frac{a_{31}}{A} \\ & \frac{a_{12}}{C} & \frac{c_{11}}{B} & \frac{a_{21}}{C} & \\ & & \frac{a_{11}}{A} & & \end{array} \right\rangle$$

Therefore, $R_4^{-1} = \left(R_4^{-1}\right)^T$, where $a_{13} = a_{31},\ a_{12} = a_{32}$ and $c_{12} = c_{21}$; so R_4^{-1} is symmetric if R_4 is symmetric.

Results

Rhotrices are used to visualize various objects. this research helps us to visualize the symmetric structures of some chemical compounds and how their arrangement affect their molecular arrangement.

Cyclobutane:

This chemical compound has C_4 atoms and H_8 atoms making it a total of 4 nodes and 8 branches.

If the structural arrangement of cyclobutane is:

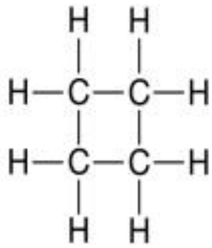

Figure 5. Cyclobutane

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R_4 structure of a cyclobutane will be

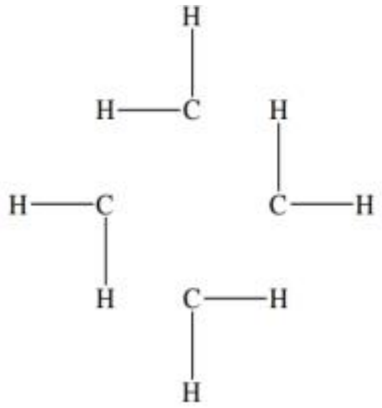

Figure 6: Structure of a Cyclobutane (R4)

Therefore the Line graph cyclic chain rhomtree will be

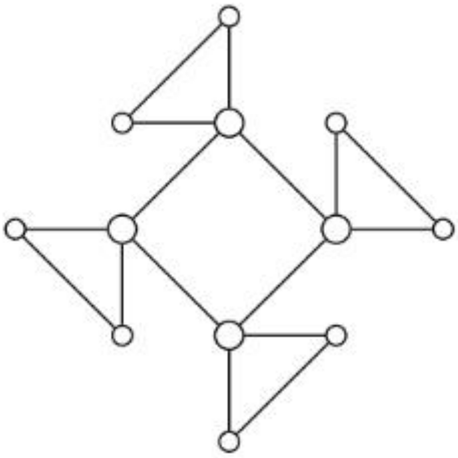

Figure 7: Rhomtree of R_4

Cyclooctane:

This chemical compound has C_8 atoms and H_{16} atoms making it a total of 8 nodes and 24 branches.
If the structural arrangement of cyclooctane is:

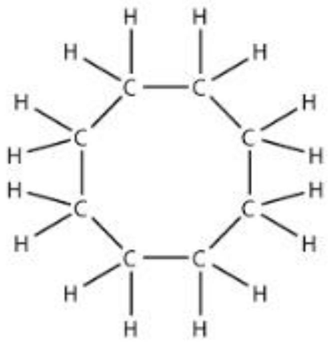

Figure 8: Cyclooctane

R_6 structure of a cyclooctane will be

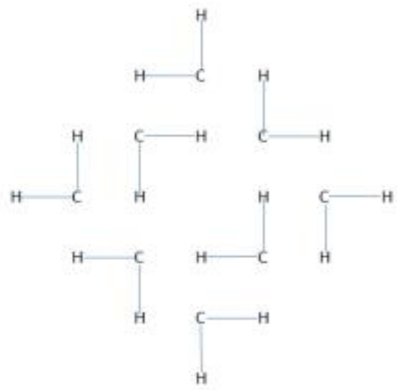

Figure 9: Structure of Cyclooctane Rhomtree of $T(24)$

Therefore the cyclic chain orbital rhomtree will be

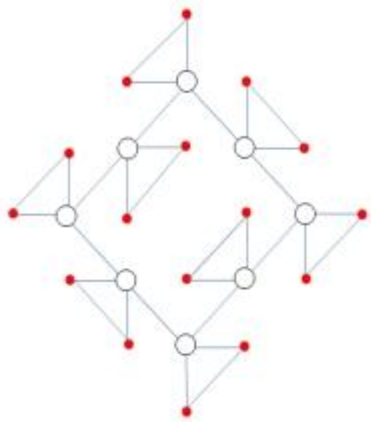

Figure 10: Rhomtree of R_6

Dodecahedrane:

This chemical compound has C_{20} atoms and H_{20} atoms making it a total of 8 nodes and 24 branches. If the structural arrangement of cyclooctane is:

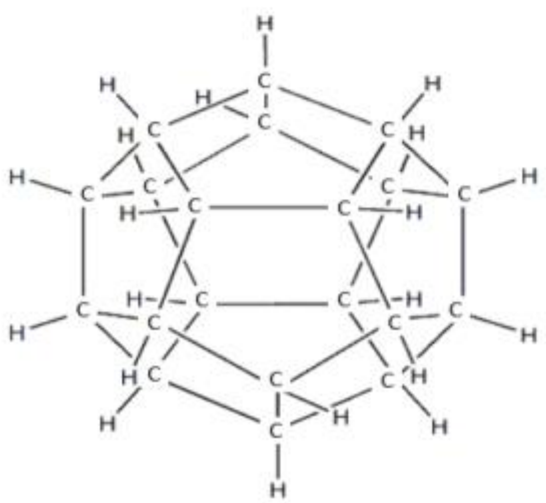

Figure 11: dodecahehedrane

The structure of a dodecahehedrane will be

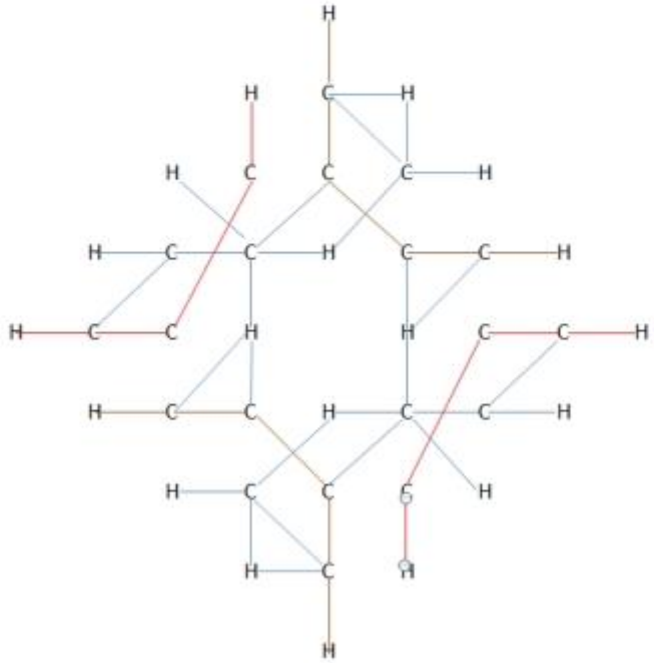

Figure 12: Structure of dodecahehedrane Rhomtree of $T(40)$

Therefore the line graph of the dodecahedrane will be

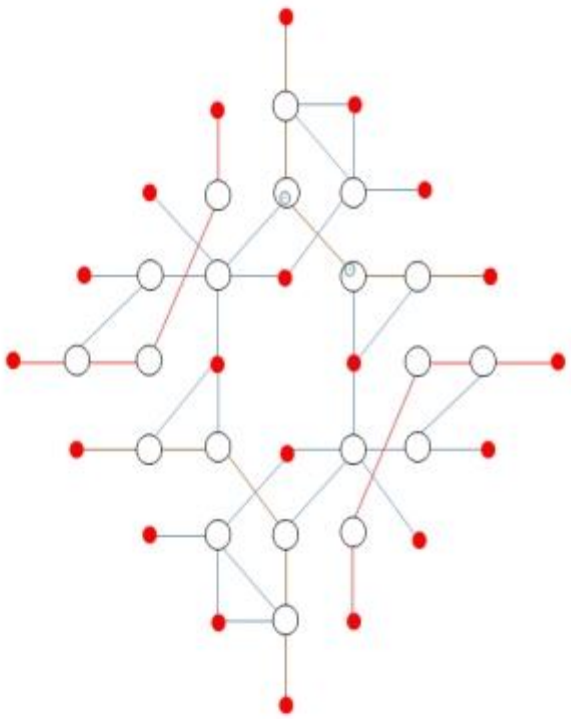

Figure 13: Rhomtree of R_8

The dodecahedrane is a three dimensional geometric shape with twelve faces, each being a regular pentagon with twenty carbon atoms and 20 hydrogen atoms. Their nodes are aesthetically linked with their destination. The dodecahedrane are used in architecture and design to create aesthetically pleasing structure. Cayley (1874) presented that, in Chemistry, dodecahedrane can be used as building blocks for complex molecules or as templates for the synthesis of new material. Dodecahedrane are use for recreational games like the rubrik's cube.

The Linear Combination of Atomic Orbitals (LCAO) approximation

This is a method used in molecular orbital theory to describe the electronic structure of molecules (Eschrig, 1989). It is based on the idea that the molecular orbitals of a molecule can be constructed by combining (or linearly combining) the atomic orbitals of its constituent atoms.

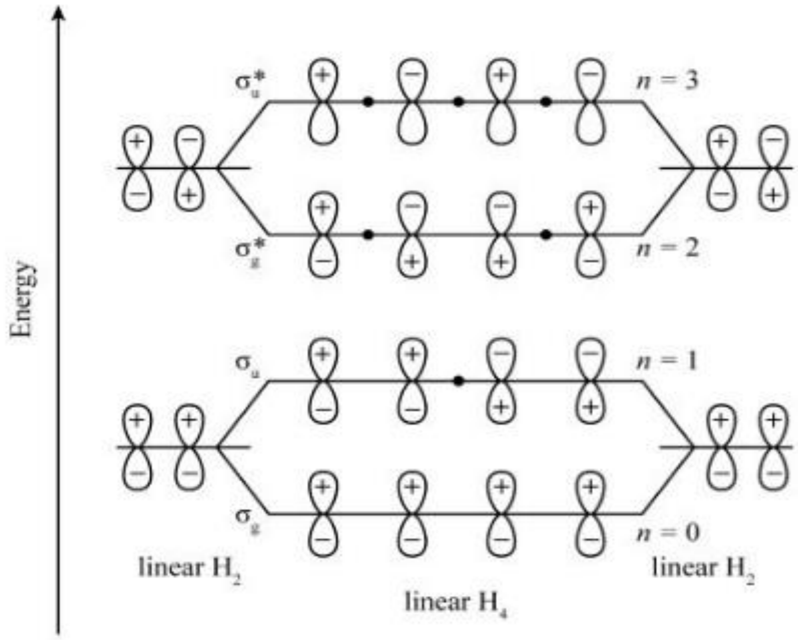

Figure 14: Linear *H*4 is a hypothetical molecule used to illustrate concepts in Group Theory

if α is the energy interactions between atomic orbitals and β is the overlap Rhotrix which accounts for overlap between orbitals, then

$$R_4 = \left\langle \begin{array}{ccccc} & & \alpha_{11} & & \\ & \alpha_{21} & \beta_{11} & \alpha_{12} & \\ \alpha_{31} & \beta_{21} & & \beta_{12} & \alpha_{13} \\ & \alpha_{32} & \beta_{22} & \alpha_{23} & \\ & & \alpha_{33} & & \end{array} \right\rangle$$

and the structure of Methylcyclopropane is

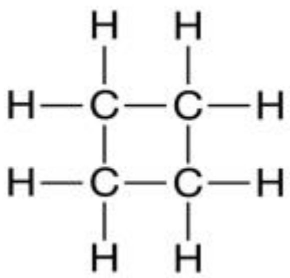

Figure 15: Methylcyclopropane Rhomtree of $T(12)$

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the rhotrix R_6 is

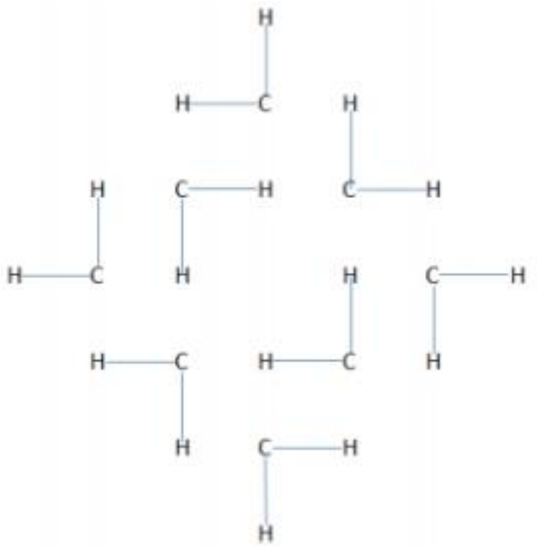

Figure 16: R_6 Structure of Methylcyclopropane

Therefore the cyclic chain orbital rhomtree will be

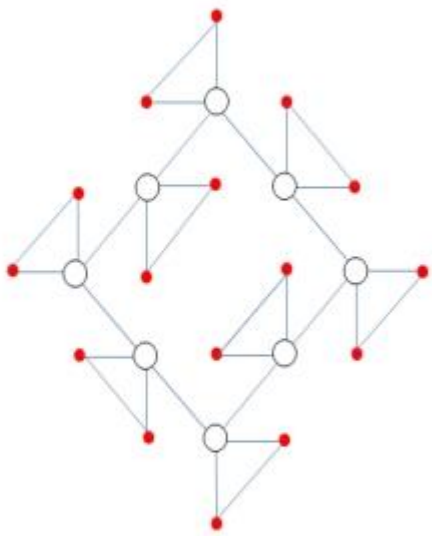

Figure 17: R_6 Rhomtree of Methylcyclopropane

These Rhomtrees illustrates linear combination of hybrid orbitals. The carbon atoms are hybridized (the circles), and the molecular orbitals (red nodes) are formed from the linear combination of hybrid orbitals. The bonding is composed of σ bonds formed through the overlap of sp3 hybridized atomic orbitals. Each carbon atom undergoes sp3 hybridization, resulting in four sp3 hybrid orbitals that form sp3 bonds with adjacent carbon and hydrogen atoms. this configuration leads to a stable, saturated hydrocarbon structure.

Conclusion

This paper presents a symmetric even-dimensional rhotrices and some properties. These rhotrices are equal to their transpose. These holds when their rhotrices dimension are equal and symmetric . These rhotrices where able to divide the object into two halves having the same properties. The new multiplication approach of even dimensional rhotrices was of great help in this work. The study contributes to the field by providing a novel approach to calculating the symmetry of objects, nature and organic chemical compounds using even dimensional heartless rhotrices. The concept of rhomtrees as graphical method of representing rhotrices of energy pathways for electrons bonding were presented. Finally, this work provide a valuable tool for researchers and practitioners working with these specialized abstract structure.

Recommendation

The distribution pathways of these abstract structures is an essential study in further research work.

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