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Heat and Mass Transfer of an MHD Casson Fluid with Variable Properties and Non-Linear Thermal Radiation Over an Elastic Sheet

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Abstract
The study of heat and mass transfer is germane due to its numerous scientific and technological relevance. This work investigates heat and mass transmission of an MHD Casson fluid flow over an elastic sheet under the influence of nonlinear thermal radiation, chemical reaction, and varying thermal conductivity and diffusion coefficient. The flow models that describe the fluid behaviour are reduced to a non-dimensional form with suitable similarity variables. The resulting dimensionless models are admissible to the shooting method technique to obtain the model parametric analysis. The responses of different emerging parameters are shown on the fluid profiles through graphs. It is observed that the augmentation of magnetic field parameters boosts the boundary layer thickness thereby hindering the fluid's velocity while the direct relationship is recorded with the fluid's temperature. Findings also reveal that thermal radiation and temperature-dependent thermal conductivity parameters augment the temperature of the fluid.

Keywords: Casson Fluid, Thermal Radiation, Variable Thermal Conductivity, Chemical Reaction.

Introduction

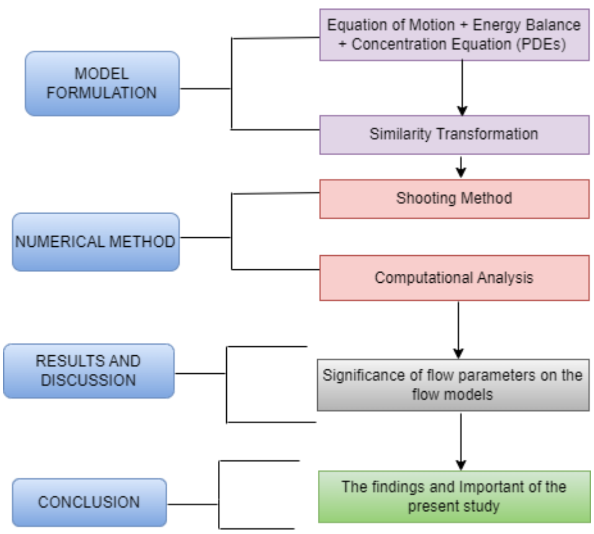
Non-Newtonian fluids are widely used in the manufacturing industries which include heat exchangers, lubricants, coolants, and in the production of drugs. Their broad industrial applications are appealing to scientists and engineers. A theory known as the Navier-Stoke equation is insufficient to predict the behaviour of such categorized fluids because no one model can describe the features of every fluid. As a result, a variety of non-Newtonian fluid models are proposed to describe the properties. These include Jeffery fluid, Maxwell fluid, Reinner-Philippoff, and Casson fluid models to mention but a few. Casson (1959) developed a model to estimate the flow behaviour of pigment oil suspensions employed in the printing of inks. Recent years have seen a variety of complex scenarios added to the study of heat transfer processes and fluid dynamics, including non-Newtonian fluids, magnetohydrodynamics (MHD), and the incorporation of variable fluid characteristics. Among them, the Casson fluid model has garnered the most attention since it explains non-Newtonian behaviours observed in paints, blood, and inks, to mention but a few. Meanwhile, the relationship between thermal radiation and elasticity in heat transfer processes has developed as an important area of research with ramifications ranging from materials processing to biomedical engineering. Understanding the characteristics of heat and mass movement in such intricate systems advances technology and contributes significantly to our fundamental knowledge of science.

The study of MHD examines how electrically conducting fluids react when subjected to magnetic fields, which may be observed in a range of man-made and natural situations. The interaction of magnetic field fluid motion frequently results in significant changes in flow layout and heat transmission rates. The incorporation of MHD effects in the study of Casson fluids raises intriguing questions since complicated flow behaviours may occur from the combination of electromagnetic forces with non-Newtonian rheology. The combination of MHD with Casson fluid dynamics has applications in metallurgy, aerospace, and environmental engineering, among other technical domains. The research into fluid dynamics and heat transport mechanisms has extended recently, and a variety of complex scenarios have been included, including magnetohydrodynamics (MHD), non-Newtonian fluid, and the usage of changeable fluid.

Several studies have employed this phenomenon to comprehend the thermal and solutal dynamics of the fluid. Salleh et al. (2010) investigated the thermal characteristics of boundary layer flow on a smooth circuitous cylinder and their findings showed that surface heat transmission has a direct relation to the local surface temperature. Sehra et al. (2021) addressed the issue of convective heat transport with MHD flow across an upright plate incorporating a chemical reaction. The authors investigated the influence of molecular diffusion and observed that a boost in fluid reaction augments the fluid's diffusivity. Nayak et al.(2016) illustrated the influences of slip velocity, radiation and magnetohydrodynamic consequence taking into consideration the stagnation point flow over an elastic sheet. Makinde et al. (2005) investigated the influence of time-dependent magnetized thermal radiation along a spongy channel. Gebhart and Para(1971) investigated how mass transfer affects buoyancy-induced flow on an upright isothermal plate. The simultaneous responses of heat emission and chemical reaction on buoyancy-induced spongy fluid dynamics were investigated by Deka and Neog (2009). Muhammed and Aishah (2017) probed a time-varying hydromagnetic convective flow through a porous media. Reddy et al. (2014) investigated the thermal flow attributes of a magnetized permeable sheet with heat dissipation and the reaction property. This study addresses the coupling responses of thermal radiation, chemical reactions, and varying thermal properties on the MHD Casson fluid flow over an elastic vertical plate. This has various scientific and technological applications such as in the production of inks, paints, and other manufacturing outputs.

The Structure of the Part

The order of the paper is shown in Figure 1. Section 3encompasses an analysis of mathematical model formulation resulting in dimensional governing equations of flow and similarity transformation technique. In section 4, we discuss the numerical procedure of the resulting non-dimensional governing equations. The discussion of the results and findings is depicted in section 5.



Formulation of the Model

Consider a steady, two-dimensional, incompressible, MHD Casson fluid flow over a vertically stretching sheet. A uniform magnetic field of strength B_0 acts along *the* $y-$direction. The responses of temperature-dependent thermal conductivity, varying diffusion coefficients, chemical reactions, and nonlinear thermal radiation are taken into account. Given these assumptions, the resulting conservation equations of Casson fluid model are given as (see [Asogwa & Ibe, 2020; Lawal et al., 2022; Wael et al., 2021; Kasali et al., 2023)

$$B_1 \frac{\partial B_1}{\partial x} + B_2 \frac{\partial B_1}{\partial y} = 0 \quad (1)$$

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$$B_1\frac{\partial B_1}{\partial x}+\frac{\partial B_1}{\partial y}=v(1+\frac{1}{\beta})\frac{\partial^2 B_1}{\partial y^2}+g\beta B_3(B_3-B_3\infty)+g\beta B_4-B_4\infty-\frac{\sigma B^2{}_0}{\rho}(B_1-B_1\infty), \quad (2)$$

$$B_1\frac{\partial B_3}{\partial x}+B_2\frac{\partial B_3}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k(B_3)\frac{\partial B_3}{\partial y}\right)+\frac{v}{pc_p}(1+\frac{1}{\beta})(\frac{\partial B_1}{\partial y})^2+\frac{\sigma B^2{}_0}{\rho}(B_1-B_1\infty)^2-\frac{1}{\rho c_p}\frac{\partial q_0}{\partial y} \quad (3)$$

$$B_1\frac{\partial B^3}{\partial x}+B_2\frac{\partial B^4}{\partial y}=\frac{\partial}{\partial y}\left(D(B_4)\frac{\partial B^4}{\partial y}\right)-K'r(B_4-B_4\infty) \quad (4)$$

where $\left\{q_0=\frac{4\sigma\prime}{3k}\frac{\partial B^{43}}{\partial z}, k(B_3)=k\infty[1+\chi_1(B_3-B_3\infty)], D(B_4)=D\infty[1+\chi_2(B_4-B_4\infty)]\right.$ (5)

The associated boundary conditions are given as (Asogwa & Ibe, 2020);

$$B_1=B_{1W},\ B_2=B_{2W},\ B_3=B_{3W},\ B_4=B_{4W}\quad at\ \ y=0, \quad (6)$$

$$B_1\rightarrow 0,\ B_2\rightarrow 0,\ B_3\rightarrow B_{3\infty},\ B_4\rightarrow B_{4\infty}\quad at\ \ y\rightarrow\infty. \quad (7)$$

Where B_1, B_2, B_3, B_4, v, B_{3W}, $B_{3\infty}$, B_{4W}, $B_{4\infty}$, B_0, $B_{1\infty}$, $k(B_3)$, ρ, c_p, σ, $D(B_4)$ depict flow velocity along (x, y)-axes, fluid temperature, fluid concentration, kinematic viscosity, temperature of the fluid at the wall, far-field fluid temperature, wall fluid concentration, far-field fluid concentration, constant magnetic field, free stream velocity, temperature-dependent thermal conductivity, dynamic viscosity, specific heat capacity, electric conductivity, concentration-dependent molecular diffusivity.

Adopting the similarity variables employed by (Asogwa & Ibe, 2020), we have

$$\eta=\frac{\sqrt{\xi}}{\sqrt{\nu}}y,\ \psi=\sqrt{\xi\nu x}\,f(\eta),\quad \theta=\frac{B_3-B_{3\infty}}{B_{3w}-B_{3\infty}},\ \varphi=\frac{B_4-B_{4\infty}}{B_{4w}-B_{4\infty}} \quad (8)$$

Equations (1) – (7) after using equation (8) become

$$(1+\frac{1}{\beta})f'''+ff''-M(f'-1)-f'^2+Gr\theta+Gc\phi=0, \quad (9)$$

$$(1+\gamma\theta)\theta''+\gamma\theta'^2-\frac{4R}{3}(2\theta''-3(1+(T[a]-1)\theta)\theta''-3(T[a]-1)\theta'^2)+Prf\theta'$$

$$+(1+\frac{1}{\beta})EcPr\,f''^2+MEcPr(f'-1)^2=0 \quad (10)$$

$$(1+\delta\phi)\phi''+\delta\phi'^2+Scf\phi'-Kr\phi=0 \quad (11)$$

The corresponding non-dimensional boundary conditions are given as

$$f(0)=f_w,\ f'(0)=1, \theta(0)=1,\ \phi(0)=1,\ f'(\infty)\rightarrow 0, \theta(\infty)\rightarrow 0, \phi(\infty)\rightarrow 0 \quad (12)$$

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The relevant non-dimensional parameters are;

$M = \frac{\sigma B^2{}_0}{\rho \xi}$, $Pr = \frac{(\mu cp)}{k_\infty}$, $R = \frac{4\sigma' B_{3\infty}}{k' k_\infty}$, $E_C = \frac{B^2{}_{1w}}{cp(B_{3w} - B_{3\infty})}$, $\gamma = \epsilon_a (B_{3w} - B_{3\infty})$, $T[a] = \frac{B_{3w}}{B_{3\infty}}$,

$Kr = \frac{K\prime r}{v}$, $\delta = \epsilon_b(B_{4w} - B_{4\infty})$, $G_r = \frac{g\beta_{B_3} (B_{3w} - B_{3\infty})}{\xi B_{1w}}$, $G_c = \frac{g\beta_{B_4} (B_{4w} - B_{4\infty})}{\xi B_{1w}}$, $S_c = \frac{v}{D\infty}$.

Where M, Pr, R, Ec, γ, T[a], Kr, δ, Gr, Gc, Sc represent magnetic field parameter, Prandtl number, thermal radiation parameter, Eckert number, temperature-dependent thermal conductivity parameter, temperature ratio, chemical reaction parameter, diffusion-coefficient parameter, thermal Grashof number, solutal Grashof number, Schmidt number.

Numerical Procedures

Dueto the non-linearity of the non-dimensional model (9)-(11) with the boundary conditions (12), the closed-form solutions are not easy to come by. To overcome this, we proposed the shooting method numerical technique. This technique solves boundary value problems by converting them into systems of first-order initial value problems (Khalil-Ur-Rehman& Malik, 2017; Irshad et al., 2024). The equations (9)-(11) are reduced to the first order system equations by assigning

$$\zeta_1 = f,\ \zeta_2 = f',\ \zeta_3 = f'',\ \zeta_4 = \theta,\ \zeta_5 = \theta',\ \zeta_6 = \phi,\ \zeta_7 = \phi'. \qquad (13)$$

The corresponding systems of equations (9) – (11) after implementing the assigned variables are given as;

$$\zeta'_1 = \zeta_2$$

$$\zeta'_2 = \zeta_3$$

$$\zeta'_3 = \frac{M(\zeta_2 - 1) + \zeta^2{}_2 - Gr\zeta_4 - Gc\zeta_6 - \zeta_1\zeta_3}{(1+\frac{1}{\beta})}$$

$$\zeta'_4 = \zeta_5$$

$$\zeta'_5 = \frac{-\gamma\zeta_5 - 4R(T[a]-1)\zeta^2{}_5 - Pr\zeta_1\zeta_5 - \left(1+\frac{1}{\beta}\right)EcPr\zeta^2{}_3 - MEcPr(\zeta_2-1)^2}{1+\gamma\zeta_4 - \frac{8R}{3} + 4R(1+(T[a]-1)\zeta_4)} \qquad (14)$$

$$\zeta'_6 = \zeta_7$$

$$\zeta'_7 = \frac{-\delta\zeta^2{}_7 - Sc\zeta_1\zeta_7 + Kr\zeta_6}{1+\delta\zeta_6}$$

The corresponding boundary conditions (12) are provided as

$\zeta_1(0) = f_w$, $\zeta_2(0) = 1$, $\zeta_4(0) = 1$, $\zeta_6(0) = 1$,

(4.3)

$\zeta_2(\infty) \to 0$, $\zeta_4(\infty) \to 0$, $\zeta_6(\infty) \to = 1$.

The above system of equations is solved using the MAPLE computing platform.

Results

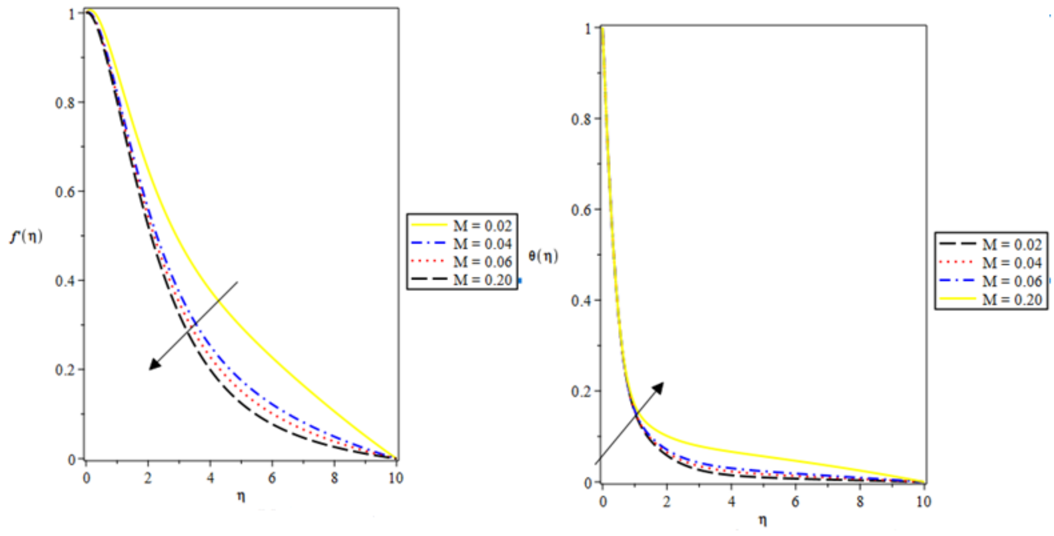

Figure 2: Responses of M on f'

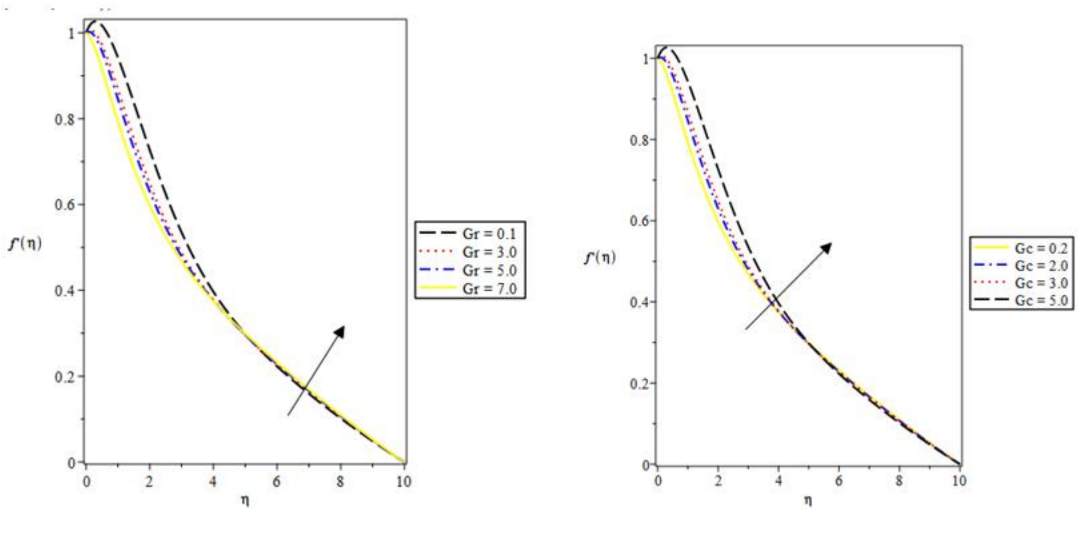

Figure 3: Responses of Gr and Gc on f'

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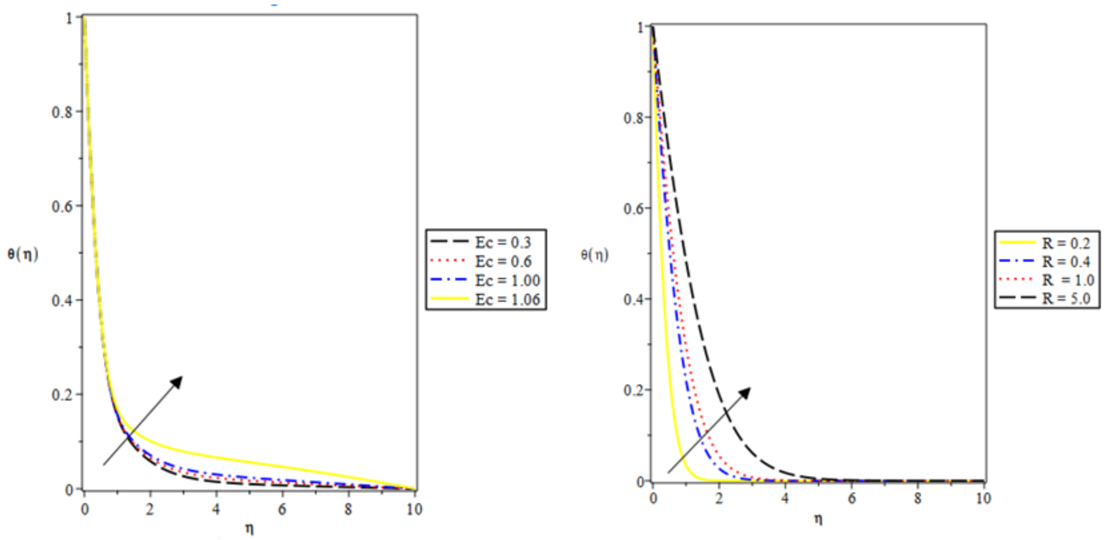

Figure 4: Responses of $Ec$and$R$ on θ

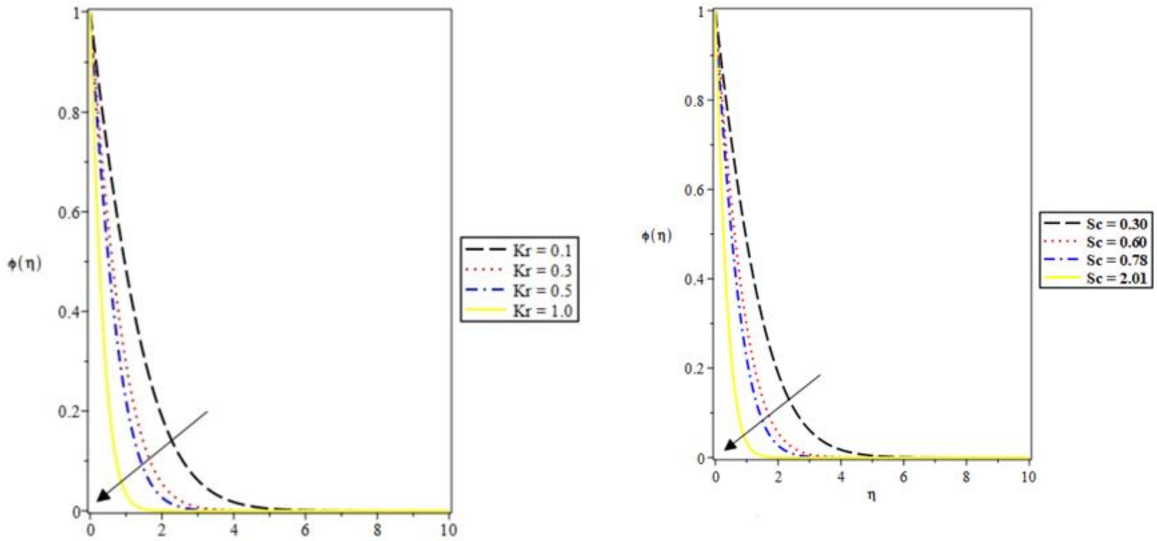

Figure 5: Responses of Kr and Sc on ϕ

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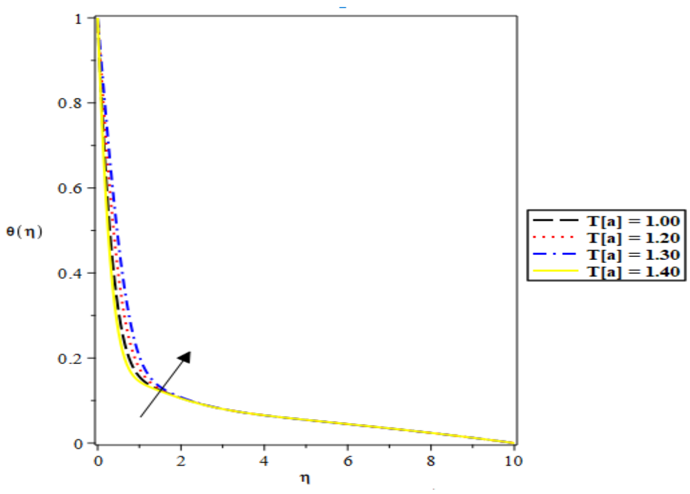

Figure 6: Response of $T[a]$ on θ

Discussion

The various effects of pertinent fluid parameters are thoroughly elucidated through graphs in this section. Figure 2displays the significance of magnetic field parameters on fluid profiles, showing that an increase in M slows down fluid velocity. This is attributed to the retarding force known as Lorentz force on the fluid velocity. Conversely, fluid temperature increases linearly with the magnetic field parameter, reflecting the relationship between heat transmission and the magnetic field. Figure 3 depicts the impacts of Grashof numbers on the fluid motion. Augmenting the thermal Grashof number leads to the dominance of buoyancy force over viscous force, resulting in enhanced fluid flow. The same effect is observed for the solutal Grashof number. The consequence of the presence of thermal radiation and heat dissipation parameters on the temperature of the fluid are displayed in Figure 4. The Eckert number, representing the relation of kinetic energy to enthalpy energy, shows that a rise in the Eckert number parameter boosts heat generation due to the drag force between fluid particles, thereby aiding the fluid temperature. Figure 5 illustrates the importance of chemical reaction parameters, Kr and Schmidt number, *Sc* on fluid concentration profiles. An increase in the chemical reaction parameter retards the fluid concentration profiles with a boost in the rate of reaction, consuming the reactant species and thereby downsizing fluid specie. This observation aligns with the findings of Lawal et al. (2022). Similarly, a growth in the *Sc* reduces the concentration profiles. The Schmidt number, representing the relationship between momentum and mass diffusivities, indicates that higher *Sc* lower species diffusivity compared to momentum diffusivity. Consequently, concentration variations diffuse more slowly, resulting in slower diffusion of concentration as depicted in Figure 5. The contribution of the temperature ratio T[a] on the thermal profile is observed in Figure 6. A rise in the temperature ratio T[a] augments the thermal attitude of the fluid. This observation is not surprising, being a temperature-associated parameter.

Conclusion

In this work, the impacts of some pertinent thermophysical parameters on an MHD Casson fluid flow over an elastics sheet have been carried out. The mathematical model that described the fluid flow was formulated with the aid of the conservation laws. The resulting non-dimensional coupled systems of ordinary differential equations were solved using the shooting technique. Parametric analysis was conducted to showcase the influence of the emergence parameters through graphs. The increment in the viscous dissipation parameter (Ec) and thermal radiation parameter (R) boost the thermal boundary layer of the fluid flow thereby give rise to the enhancement of the fluid temperature.By enhancing the magnetic parameter (M), the fluid velocity slows down while the temperature of the fluid experiences a surge. A reduction in concentration profiles is observed with the rise in the chemical reaction parameter and the Schmidt number.There is a linear relationship between the fluid velocity and the thermal and solutal Grashof numbers.

Recommendation

The authors recommend that for heat and mass transfer occurring at very high temperatures such as hot roller, nuclear reactor, etc and to account for the heat loss, thermal radiation and thermal conductivity are expressed as function of temperature.

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