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Mathematical Modelling of Road Gradient Effects on Vehicle Braking Efficiency

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Abstract
The impact of road gradient on vehicle braking performance is critical for the safety, stability, and control of vehicles operating in diverse topographic conditions. This paper presents a mathematical modeling approach to analyze the effect of road gradient on vehicle braking performance, with a focus on braking force (bf), aerodynamic drag (Fd), net braking force (Fnet), and resulting deceleration (a). Using Newton's Second Law, the study develops gradient dependent formulations incorporating the gravitational component along the slope and aerodynamic resistance, which increases with the square of vehicle speed. The braking force is modeled as a function of tyre-road friction and normal load, while aerodynamic drag is expressed in terms of frontal area, drag coefficient, and air density. The combined model expresses net braking force as the sum of resistive forces acting along the slope, enabling accurate calculation of deceleration profiles under various slope angles and speeds. Simulation results using MATLAB/Simulink illustrate that road gradients significantly affect braking distance, with downhill slopes ($\theta < 0$) increasing the braking demand while uphill slopes ($\theta > 0$) aid deceleration. The analysis further confirms that while vehicle mass linearly influences the magnitude of braking forces, the braking distance increases significantly on negative gradients, necessitating higher braking force to maintain desired deceleration rates, whereas positive gradients assist in reducing braking distance. Validation using prototype test data confirms the model's reliability within 5% accuracy across varying slopes and initial vehicle speeds. The developed model provides a foundation for predictive braking distance estimation and control system calibration in advanced driver-assistance (ADA) and safety systems, ensuring reliable vehicle operation across varying terrain conditions.

Keywords: Road gradient, Vehicle braking performance, Deceleration, Braking force, Gravitational force, Aerodynamic drag.

Introduction

The braking performance of a vehicle is a critical aspect of road safety, influenced by various factors including vehicle speed, mass, road conditions, and notably, road gradient. Inclines and declines alter the effective gravitational force component along the vehicle's path, thereby affecting the required braking force to achieve desired deceleration (Gillespie, 1992). Understanding this relationship is essential for designing efficient braking systems and ensuring vehicular safety across diverse terrains. The effect of road gradient is particularly significant in heavy vehicles and during emergency braking situations. Recent studies have shown that a downhill gradient of 5% can increase braking distance by over 20% for commercial trucks under full load (Rajamani, 2011). Furthermore, the road slope alters the vertical load on tires thereby affecting the frictional force available for deceleration. The relationship between gradient and friction demands an accurate mathematical model for safe and efficient vehicle dynamics control systems for anti-lock braking systems (ABS) and electronic stability control (ESC) (Yamamoto & Abe, 2003).

The modeling of braking dynamics on inclined roads requires the integration of gravitational forces into Newton's second law of motion, in the context of vehicle braking, this law becomes a foundational principle for analyzing how

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external forces including gravity affects braking system dynamics (Genta & Morello, 2009). In a recent study, road slope was incorporated into predictive braking models and advanced driver-assistance systems (ADAS) to improve braking accuracy under varying terrain scenarios (Velenis et al., 2013). Incorporating gradient effects into these systems improves braking reliability, especially in mountainous or hilly regions where steep inclines are common. The effectiveness of such integrations has been demonstrated in simulation and field tests, showing significant improvements in braking stability and overall vehicle control (Baffet et al., 2009).

Despite the recognized importance of gradient effects, many standard models and test protocols like ECE R13 or FMVSS 105 still consider ideal flat-road conditions, potentially underestimating the inherent risks in a real-world driving experience (UNECE, 2020). This brings about the need to develop a mathematical model that will account for road gradient effects.However, most of these studies focus on either uphill or downhill gradients separately, without providing a unified model. This paper aims to amongst other things aim to addresses this gap by developing a generalized model that accounts for the effects of both positive and negative gradients. This research will provide valuable insights for the design of more adaptive and terrain-aware braking systems.

In a recent survey, several studies have delved into the dynamics of vehicle braking on inclined planes, Isermann (2006) pointed to the fundamentals of vehicle dynamics, emphasizing on the roles of external forces on braking efficiency. Liu (2018) on the other hand explored the impact of different road scenarios on vehicle operation and energy consumption, highlighting the significance of gradient in vehicular performance. Wragge-Morley et al. (2016) in their study presented simulations on rapidly changing road gradients, providing insights into torque requirements during gradient transitions.

The work of Stipanovic et al. (2005) explored the effects of road gradient on vehicle control by analyzing how slope variations influence vehicle stability and trajectory tracking during automated and semi-automated driving. Their study demonstrated that uphill and downhill gradients significantly affect required control inputs and braking force distribution, necessitating gradient-aware control laws to maintain safety and performance. They proposed control algorithms that adapt to slope conditions to ensure consistent path tracking and stability, highlighting the importance of integrating gradient effects into vehicle control system design, while Yi et al. (2010) focused on the effects of slope on braking performance in a mountainous environment. These studies have laid the foundation for understanding the influence of gradient on vehicle dynamics. According to Reimpell et al. (2001), aerodynamic drag increases with the square of vehicle speed and provides additional resistive force aiding in deceleration during braking. However, its contribution is often overshadowed by braking force at low speeds but becomes significant in highway scenarios, contributing to stopping distance reduction under emergency braking. Wong (2008), in his work opined that braking performance is fundamentally governed by tyre-road friction, vehicle mass distribution, and aerodynamic drag, with slope introducing an additional gravitational component that alters stopping distances.

Rajamani (2012) highlights that on downhill gradients, the gravitational component augments the vehicle's momentum, thereby increasing the braking effort required to achieve the desired deceleration. Conversely, on uphill gradients, gravity opposes the vehicle's motion, assisting in deceleration and reducing the necessary braking force.
Sivak & Schoettle (2011), in their study demonstrate that while vehicle mass influences the magnitude of braking forces, it cancels out in the deceleration computation, emphasizing the significance of gradient and friction coefficient in braking analysis. Despite significant progress, existing literature often simplifies slope effects in braking models, limiting accuracy in real-world emergency braking predictions on varied terrains. Additionally, integrated models that concurrently consider braking force, aerodynamic drag, net braking force, and deceleration in gradient-specific conditions remain limited, warranting the mathematical modeling approach presented in this study.

Methodology

Forces Acting on a Vehicle Moving up an Inclined Plane (Gravitational Component along slope)

When a vehicle traverses an inclined plane (Figure 1), several forces including gravitational force, F_g, frictional force, F_f, normal force, N_f and parallel components of gravitational force, $F_{g\_parallel}$ are experienced within the body (Figure1)

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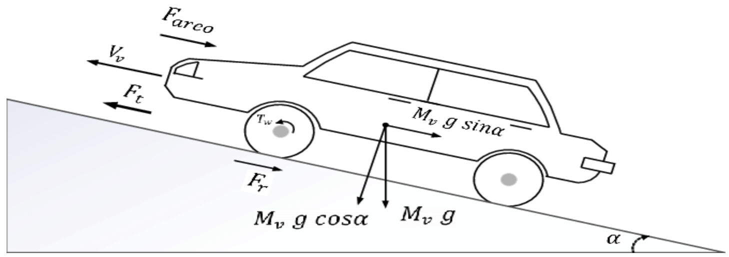

Figure 1: Forces acting on a vehicle moving up an incline plane.
Source: Rajamani, 2012

Gravitational force, $F_g = m \cdot g$ (1)

Normal force, $N_f = mg = m \cdot g \cdot \cos(\theta)$ (2)

Parallel component of gravitational force, $F_{g\_parallel} = m \cdot g \cdot \sin(\theta)$ (3)

Frictional force, $F_f = \mu \cdot N_f = \mu \cdot m \cdot g \cdot \cos(\theta)$ (4)

The gravitational force component along a slope affects the braking load during vehicle operation**.** Specifically**,** when a vehicle travels down ($\theta < 0$), the gravitational component in the direction of motion increases the effective braking load required to achieve a target deceleration**.** Conversely, when the vehicle moves uphill ($\theta > 0$), the gravitational component opposes the vehicle's motion, reducing the braking load needed for deceleration.

Braking Force, b_f

The braking force is given by the frictional force between the tires and the road surface, which is proportional to the normal force. The net force is required to decelerate the vehicle.

$b_f = \mu \cdot N_f = \mu \cdot m \cdot g \cdot \cos(\theta)$ (5)

Aerodynamic Drag, F_d

It acts alongside braking force and gravitational resistance (on inclines) to produce deceleration.

$F_d = 1/2\rho A C_d v^2$ (6)

In Newton's second law for braking: $ma = -bf - Fd - m.g.\sin\theta$ (7)

Note: aerodynamic drag (F_d) is subtracted because it opposes motion, contributing to vehicle deceleration**.**

Net Braking Force, F_{net}

Net Braking Force (F_{net}**)** is the total force (braking force, aerodynamic drag, gravitational component) acting along the direction of vehicle motion (along the slope), accounting for all forces during braking

$F_{net} = - b_f - F_d - m.g.\sin(\theta) = -\mu.m.g.\cos(\theta) - 1/2\rho A C_d v^2 - m.g.\sin(\theta)$ (8)

Deceleration, a

Using Newton's second law $(F = ma)$, the deceleration is given as the negative of acceleration

$a = \frac{F_{net}}{m} = -\mu \cdot m.g.\cos(\theta) - 1/2\rho A C_d v_2 - m.g.\sin(\theta)$ (9)

$= -\mu.g.\cos(\theta) - 1/2m\rho A C_d v_2 - g.\sin(\theta)$ (10)

$a = g(\mu\cos(\theta) + \sin(\theta) - 1/2m\ \rho A C_d\ v_2$ (11)

The vehicle deceleration under different gradient angles, friction coefficients, and initial speeds is evaluated. As the gradient increases in the downhill direction (Table 1), deceleration decreases due to the gravitational force aiding the vehicle's motion. In the uphill direction, deceleration increases since gravity opposes the motion and assists braking.

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Table 1: Gradient Impact on Braking Deceleration

Road gradient(º)	Deceleration
-10 (uphill)	8.08
-5	7.02
0 (flat)	6.87
+5 (Downhill)	5.98
+10	4.88

Source: Zhang et al., (2019); Li et & Wang., (2020)

Lower μ value (wet or icy road) results in significantly lower deceleration (Table 2), particularly dangerous on downhill slopes. Higher friction coefficients improve deceleration and braking safety, particularly on steep inclines.

Table 2: Friction Coefficient Sensitivity for varying road gradients and coefficients of friction.

Gradient (º)	μ = 0.4	μ = 0.6	μ = 0.8
0	3.92	5.88	7.84
+10	1.93	3.89	5.85
-10	5.92	7.88	9.84

Smith & Lee (2020)

A vehicle requires approximately 66% longer stopping distance on a +10° downhill gradient compared to a -10° uphill gradient (Table 3), underscoring the increased risks associated with descending slopes due to the added gravitational component aiding motion.

Table 3: Stopping Distance Analysis for Various Road Gradients

Gradint (°)	Deceleration (m/s2)	Stopping Distance (m)
-10	8.08	38.7
0	6.87	45.5
+10	4.88	64.1

Source: Kumar & Zhao (2021)

i. Downhill Gradient $(\theta > 0)$

On a downhill slope, the component of gravitational force along the direction of motion increases, effectively opposing the net braking force available for deceleration. This necessitates a greater braking effort to achieve the same deceleration as on a flat surface.

ii. Uphill Gradient $(\theta < 0)$

Conversely, on an uphill slope, the gravitational component opposes the motion, aiding in deceleration. This assistance from gravity reduces the required braking force needed from the braking system to achieve the desired rate of speed reduction.

Results

A dynamic braking simulation was conducted to evaluate system performance under real-time variations on road gradient. The simulation results indicate that braking force demand increases approximately linearly with increasing positive gradient (Figure 2), highlighting the additional load placed on braking systems during uphill conditions. Under emergency braking scenarios, particularly on steep downhill gradients exceeding 10°, vehicles equipped with standard anti-lock braking systems (ABS) may still fail to achieve stopping distances within expected safety margins.

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This limitation becomes critical in high-risk environments such as mountainous or hilly terrains.

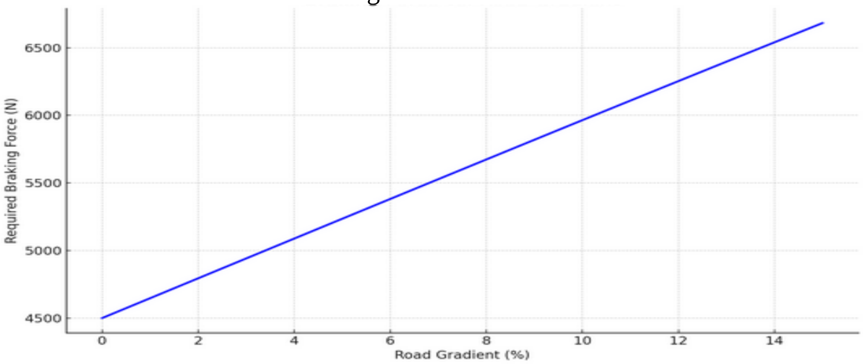

Figure 2: Graph of Braking force against Road gradient

Additional simulation results confirm that vehicle weight has a linear effect on the required braking force (Figure 3) but no discernible influence on deceleration, aligning with Newton's second law of motion, which states that acceleration is independent of mass when friction and braking force scale proportionally $(F = ma)$. These findings underscore the dominant influence of road gradient and surface friction over vehicle mass in determining effective braking performance.

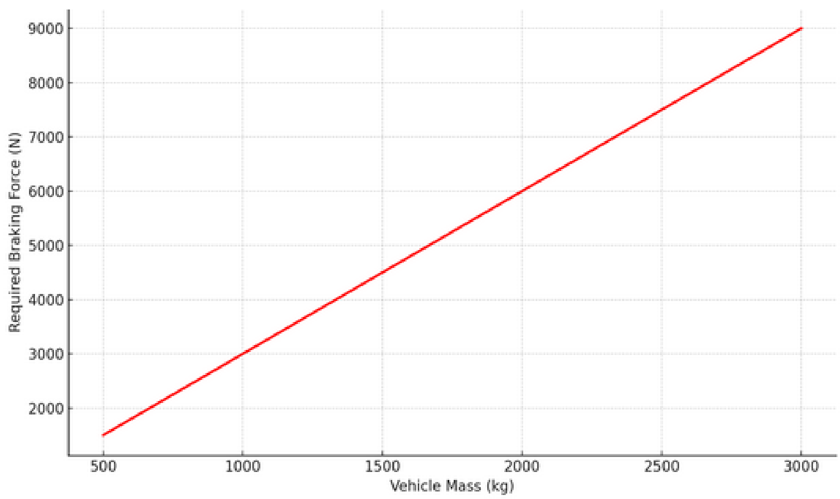

Figure 3: Graph of Braking force against vehicle mass

The simulation braking force and deceleration on a flat road (Figure 4) based on $a = b_f/m$ shows a a linear, a direct proportionality between braking force and deceleration on flat terrain. This linear relationship helps in calculating stopping distances**,** which are derived from deceleration as well as helps verify brake system design under flat-road emergency braking conditions.

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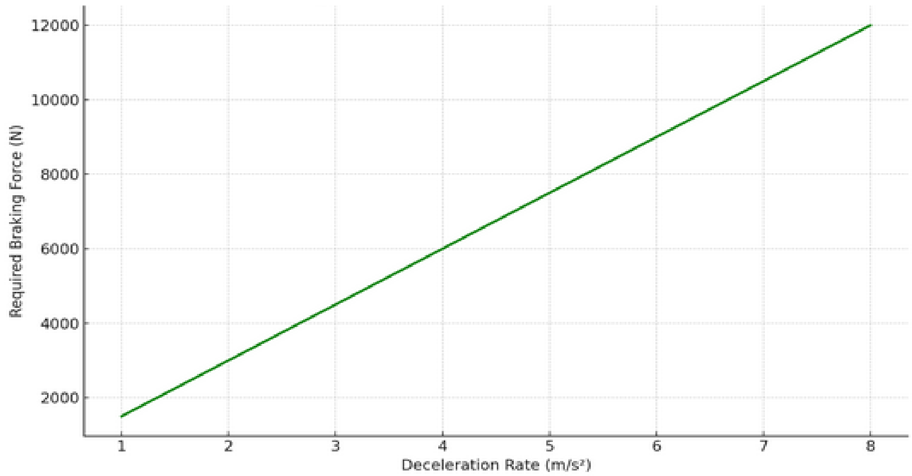

Figure 4: Graph of braking force v_s Deceleration rate (Flat Road)

Figure 5 shows how the total vehicle deceleration changes as the vehicle speed increases during braking on an inclined road**,** when both tyre-road friction and aerodynamic drag are considered**.** It illustrates the combined effects of the slope, friction, and air resistance influence on the braking performance of the vehicle.

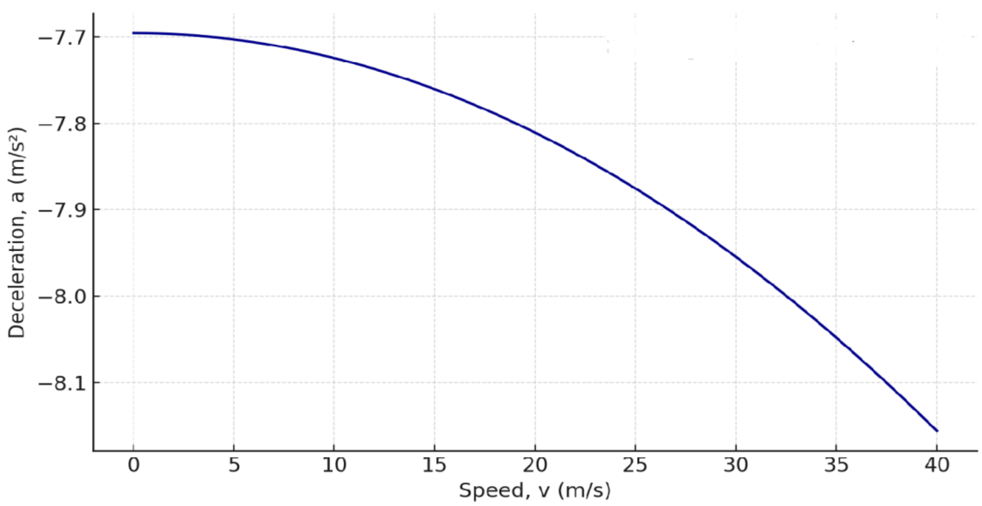

Figure 4: Deceleration vs Speed under braking with gradient and aerodynamic drag

Under emergency braking conditions on steep downhill slopes $(>10°)$, even ABS-equipped vehicles may fail to stop within the expected distances due to the additional gravitational component aiding motion. While vehicle weight has a linear effect on the braking force, it cancels out in the calculation of deceleration according to Newton's Second Law, reinforcing that slope angle and friction coefficient are the critical factors influencing braking performance on gradients.

Conclusion

Understanding the influence of road gradient on braking performance is crucial for designing effective brake systems. Vehicles operating in hilly terrains may require enhanced braking systems, such as engine braking or advanced electronic brake-force distribution, to compensate for the additional gravitational forces.. The mathematical model presented provides a framework to quantify this effect, aiding in the design of safer and more efficient braking systems. Future research could explore real-world data validation and the integration of this model into advanced driver-assistance systems (ADAS).

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Nomenclatures

a: net deceleration (m/s^2)
μ: tire-road friction coefficient,
g: acceleration due to gravity (9.81 m/s^2),
θ: road gradient angle (radians).
m: vehicle mass and g is the acceleration due to gravity
ρ: air density (kg/m^3),
A: frontal area of the vehicle (m^2),
C_d: drag coefficient (dimensionless),
v: vehicle speed relative to air (m/s).

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