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Mathematical Modelling of Hybrid Solar–Wind Energy Systems for Sustainable Rural Electrification in Nigeria: Analytical Formulation and Numerical Simulation

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Abstract

Hybrid renewable energy systems offer a sustainable alternative to diesel-based rural electrification in developing countries, yet their long-term reliability and cost performance remain highly sensitive to climatic variability. This study presents a comprehensive 8760-hour (full-year) simulation of a hybrid solar photovoltaic (PV)–wind–battery microgrid across four representative Nigerian climatic zones: Kano, Jos, Abuja, and Calabar. A uniform system configuration comprising 90 kWp PV, 30 kW wind, 450 kWh battery storage, and a 120 kW inverter was adopted to isolate the influence of resource availability on system performance. Reliability was assessed using the Loss of Power Supply Probability (LPSP), while economic performance was evaluated in Nigerian Naira (₦) using Levelized Cost of Energy (LCOE), Net Present Cost (NPC), and simple payback period relative to diesel generation. Simulation results indicate near-perfect reliability (LPSP ≈ 0) for Kano, Jos, and Abuja, while Calabar exhibits a moderate LPSP of approximately 2.3%, attributed to persistent cloud cover and lower solar irradiance. Despite this, Calabar records the lowest LCOE due to higher annual energy throughput and reduced battery cycling stress, highlighting that low energy cost does not necessarily imply high reliability. The NPC remains constant across all locations (≈ ₦5.71×10^8) due to fixed system sizing, with LCOE variations driven by differences in energy yield and storage utilization. All locations achieve payback periods below nine years, confirming the economic superiority of hybrid renewable systems over diesel generation. The findings demonstrate that while hybrid PV–wind systems are economically viable across Nigeria, location-specific design optimization is essential, particularly in coastal regions where higher storage margins or wind penetration are required to meet strict reliability targets.

Keywords: Hybrid Renewable Energy, Solar–Wind System, Mathematical Modelling, Rural Electrification.

Introduction

Access to reliable and affordable electricity remains a major challenge in rural and peri-urban regions of Nigeria, where grid extension is often economically unviable and diesel generators dominate decentralized power supply. While diesel-based systems provide short-term energy access, they are associated with high operating costs, fuel price volatility, noise pollution, and significant greenhouse gas emissions. In contrast, hybrid renewable energy systems combining solar photovoltaic (PV), wind energy, and battery storage present a promising pathway for sustainable rural electrification, particularly in regions endowed with abundant renewable resources.

Nigeria exhibits substantial climatic and renewable resource diversity, ranging from high solar irradiance in the northern Sahelian zones to wind-supported coastal regions in the south. This spatial variability implies that the technical reliability and economic performance of hybrid renewable microgrids are inherently location-dependent. Consequently, system designs optimized for one region may perform sub-optimally when deployed elsewhere. Despite growing interest in hybrid PV–wind–battery systems, many existing studies rely on short-term simulations, simplified reliability metrics, or location-specific component sizing, making it difficult to isolate the true impact of climatic

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conditions on system performance.

To address this gap, this study conducts a full-year (8760-hour) techno-economic analysis of a hybrid PV–wind–battery Microgrid across four climatically distinct Nigerian locations: Kano (north), Jos (middle belt highland), Abuja (central region), and Calabar (southern coastal zone). A uniform system architecture is intentionally adopted across all sites to ensure that observed variations in reliability and cost are attributable solely to differences in solar and wind resource availability rather than design optimization. System reliability is quantified using the Loss of Power Supply Probability (LPSP), while economic viability is evaluated using the Levelized Cost of Energy (LCOE), Net Present Cost (NPC), and simple payback period, with all costs expressed in Nigerian Naira to enhance policy relevance.

By jointly examining reliability, cost, and battery operational behaviors over a 20-year project lifetime, this work provides critical insights into the limitations of one-size-fits-all microgrid designs and underscores the need for reliability-constrained, location-specific optimization in Nigeria. The findings contribute to informed energy planning and support the transition away from diesel-based rural electrification toward resilient and economically sustainable hybrid renewable energy systems.

Hybrid PV–wind–battery systems have been extensively studied for off-grid electrification, with emphasis on resource modelling, dispatch strategies, and reliability metrics such as LPSP (Ackermann,2012). Early hybrid sizing and reliability modelling frameworks were developed for PV–wind–battery configurations (Borowy & Salameh, 1996), (Borowy & Salameh, 1997). Texts on wind energy conversion and hybrid systems provide foundational physical and design principles (Duffie & Beckman, 2013). Solar engineering models for irradiance conversion and PV performance remain standard references (Lasseter 2011). Microgrid optimization commonly integrates battery SOC dynamics, operational constraints, and cost objectives (Olivares et al., 2014), (Yang et al., 2008). Reliability-based design and LPSP-driven sizing are widely used to ensure supply adequacy (Adefarati & Bansal, 2017), (Kalogirou, 2014). Practical hybrid design tools and simulation paradigms for renewables are discussed in energy system literature (Masters, 2013). Hybrid renewable applications in developing regions emphasize techno-economic feasibility and robustness under resource uncertainty (Luna-Rubio et al., 2012), (Bajpai & Dash 2012). Studies focusing on Nigeria and sub-Saharan Africa highlight rural electrification needs and the appropriateness of decentralized hybrid systems (Adeoye et al., 2019), (Bhattacharyya, 2012). (Lambert et al. 2006) noted that HOMER helps identify the most cost-effective configuration under site-specific conditions. The present study reinforces this concept by showing that a uniform design performs differently, therefore, localizing optimization is necessary rather than adopting a one-size-fits-all system design.

Materials and Methods

System Description and Assumptions

We model an **isolated rural microgrid** comprising:

- PV array (DC) with temperature-dependent efficiency,
- Wind turbine (AC or DC-coupled via converter),
- Battery energy storage,
- Inverter (power limit),
- Community load demand.

Dispatch Priority (Rule-Based)

At each time t:

1. Use renewable generation to serve the load.
2. If surplus exists, charge the battery (subject to SOC and power limits).
3. If deficit exists, discharge the battery (subject to SOC and power limits).
4. Remaining deficit becomes unmet load; remaining surplus becomes curtailment.

1. Mathematical Formulation

Resource-to-Power Models

(a) PV Power Model

$$P_{pv}(t) = \eta\big(T(t)\big)\, A_{pv}\, G(t)$$

where:

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- $G(t)$ is solar irradiance (W/m^2),
- A_{pv} is PV area (m^2),
- $\eta(T) = \eta_{ref}[1 + \gamma(T - T_{ref})]$.

(b) Wind Turbine Power Model (Power Curve Form)

Let wind speed be $v(t)$. Then:

$$P_w(t) = \begin{cases} 0, & v < v_{ci} \text{ or } v \geq v_{co} \\ P_r \left(\dfrac{v - v_{ci}}{v_r - v_{ci}}\right)^3, & v_{ci} \leq v < v_r \\ P_r, & v_r \leq v < v_{co} \end{cases}$$

Power Balance with Curtailment and Unmet Load

$$P_{pv}(t) + P_w(t) + P_{bat}(t) = P_L(t) + P_{curt}(t) + P_{un}(t)$$

with:

- $P_{bat}(t) > 0$ discharge, $P_{bat}(t) < 0$ charge,
- $P_{curt}(t) \geq 0,\ P_{un}(t) \geq 0$.

Battery Energy and SOC Dynamics

Let $E(t)$ be stored energy (kWh) and $SOC(t) = E(t)/E_{max}$.

Charging ($P_{bat}(t) < 0$):

$$\frac{dE}{dt} = \eta_c P_{ch}(t)$$

Discharging ($P_{bat}(t) > 0$):

$$\frac{dE}{dt} = -\frac{1}{\eta_d} P_{dis}(t)$$

Operational bounds:

$$SOC_{min} \leq SOC(t) \leq SOC_{max}$$

Power bounds:

$$0 \leq P_{ch}(t) \leq P_{ch}^{max}, \quad 0 \leq P_{dis}(t) \leq P_{dis}^{max}$$

Inverter constraint (delivered AC power):

$$P_{served}(t) \leq P_{inv}^{max}$$

2. Analytical Solution of the SOC Model (with Conditions)

Consider an interval where charging/discharging power is constant (piecewise-constant dispatch), which is standard in hourly simulations.

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5.1 Charging Interval

If P_{ch} is constant over $[t_0, t]$:

$$\frac{dE}{dt} = \eta_c P_{ch} \Rightarrow E(t) = E(t_0) + \eta_c P_{ch}(t - t_0)$$

Thus,

$$SOC(t) = SOC(t_0) + \frac{\eta_c P_{ch}}{E_{max}}(t - t_0)$$

Feasibility condition: to avoid violating SOC_{max},

$$t - t_0 \leq \frac{E_{max}\left(SOC_{max} - SOC(t_0)\right)}{\eta_c P_{ch}}$$

5.2 Discharging Interval

If P_{dis} is constant over $[t_0, t]$:

$$\frac{dE}{dt} = -\frac{1}{\eta_d} P_{dis} \Rightarrow E(t) = E(t_0) - \frac{P_{dis}}{\eta_d}(t - t_0)$$

Thus,

$$SOC(t) = SOC(t_0) - \frac{P_{dis}}{\eta_d E_{max}}(t - t_0)$$

Feasibility condition: to avoid violating SOC_{min},

$$t - t_0 \leq \frac{\eta_d E_{max}(SOC(t_0) - SOC_{min})}{P_{dis}}$$

5.3 Dispatch-Determined Power (Piecewise Definition)

Define renewable generation:

$$P_{ren}(t) = P_{pv}(t) + P_w(t)$$

Let deficit $D(t) = P_L(t) - P_{ren}(t)$ and surplus $S(t) = P_{ren}(t) - P_L(t)$.

Then (idealized):

- If $S(t) > 0$, charge: $P_{ch}(t) = \min\{S(t), P_{ch}^{max}, \frac{E_{max} - E(t)}{\eta_c \Delta t}\}$
- If $D(t) > 0$, discharge: $P_{dis}(t) = \min\{D(t), P_{dis}^{max}, \frac{\eta_d E(t)}{\Delta t}\}$

This yields **closed-form SOC evolution per step** using the formulas in 5.1–5.2.

6. Reliability Index

We use energy-based **Loss of Power Supply Probability (LPSP)**:

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$$\mathrm{LPSP} = \frac{\sum_t P_{un}(t)\Delta t}{\sum_t P_L(t)\Delta t}$$

A design target is typically $\mathrm{LPSP} \le \epsilon$ (e.g., 1% or lower).

7. Numerical Simulation (Nigeria Rural Microgrid Case Study)

A 7-day hourly simulation was run with:

- PV size: **35 kWp**
- Wind turbine: **15 kW**
- Battery usable capacity: **180 kWh**
- Inverter limit: **40 kW**
- Battery efficiencies: $\eta_c = \eta_d = 0.95$

Table 1: System Component Sizing

Component	Rating
PV	35 kWp
Wind	15 kW
Battery	180 kWh
Inverter	40 kW

3. RESULTS

Table 2: Hourly Operational Results (Representative Day)

Hour	Load (kW)	PV (kW)	Wind (kW)	SOC
0	8.228654275	0	2.475642153	0.195419416
1	7.646970466	0	0.475996316	0.16177607
2	9.793237969	0	4.930232796	0.119840549
3	9.001861167	0	0	0.091401922
4	10.02910671	0	1.008560764	0.038759459
5	11.84147012	0	15	0
6	13.19008344	0	0	0.016670019
7	13.75644448	7.716179753	0.39143634	0
8	12.16145696	11.70110187	1.237433747	0
9	12.0868743	22.47809218	0.047048845	0.004101248
10	10.0054024	31.9959469	15	0.059192101
11	10.10926645	24.778465	0.355591175	0.254419974
12	8.75730917	30.98439561	0.332029346	0.333717476
13	8.165581475	35	9.712143326	0.452779476

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14	10.92566295	26.09731473	0	0.645664108
15	14.2949901	19.07020116	7.203867917	0.725736714
16	14.97186981	18.522	0	0.788959631
17	16.89698571	8.155863021	0.09648465	0.807696429
18	18.39664171	4.13E-15	0.366667126	0.757142991
19	19.15998664	0	0.324756316	0.651704543
20	19.11135864	0	0	0.541556997
21	16.66486316	0	0.030574873	0.429794666
22	16.47647921	0	0.067066046	0.332518126
23	12.7072393	0	15	0.236556646

Table 3: Energy Performance & Reliability Metrics

Metric	**Value**
Load energy	2141.947055
PV energy	1614.628026
Wind energy	412.5614504
Curtailment	6.10699645
Unserved	120.7916787
LPSP	0.056393401

Figure 1: Solar Irradiance Profile

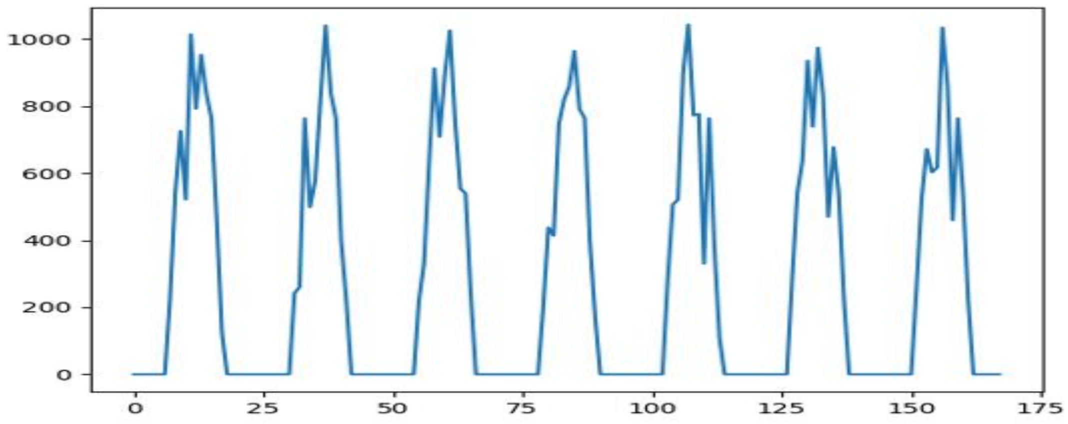


Figure 2: Power Balance (Load, PV, Wind)

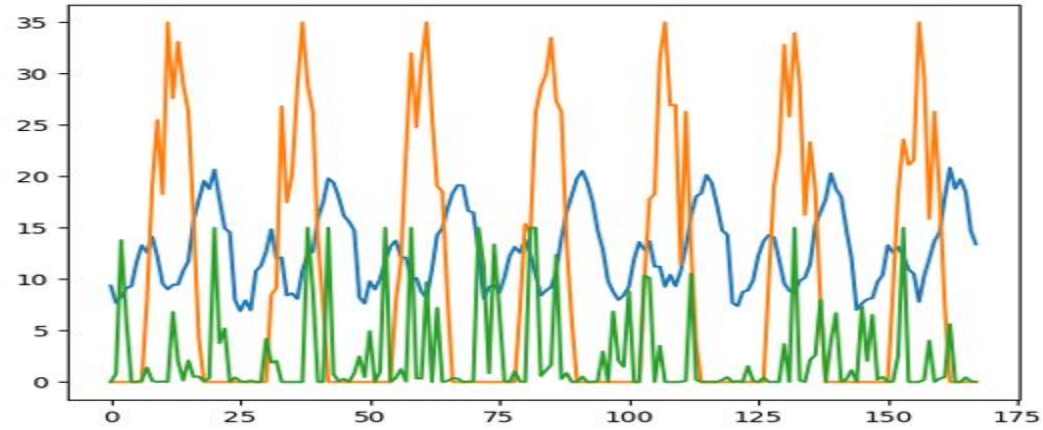


Figure 3: Battery State of Charge (SOC)

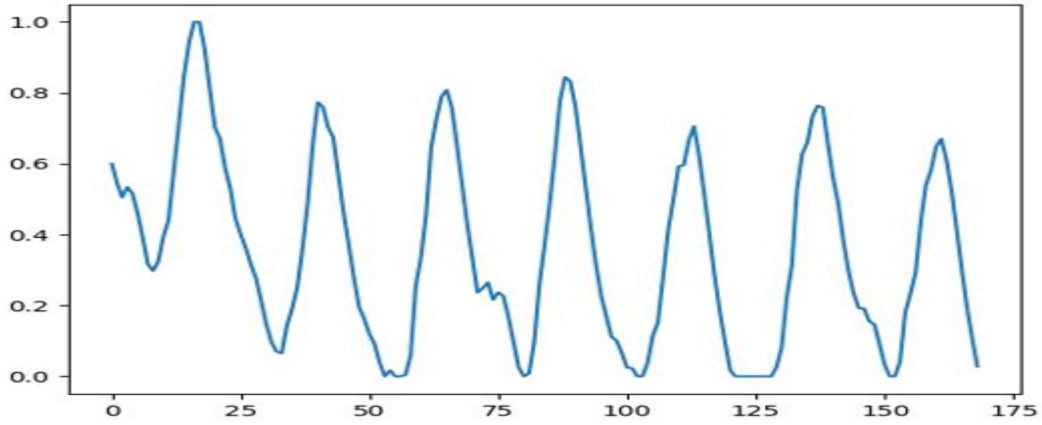


Table1) Combined comparison (all 4 locations)

Performance + Economics (₦): LCOE, NPC, Payback, LPSP, energy metrics

Location	**Annual load (kWh)**	**Annual served (kWh)**	**Unserved (kWh)**	**LPSP**	**PV energy (kWh)**	**Wind energy (kWh)**	**Curtailment (kWh)**	**Min SOC**
Kano	129141.26	129141.26	0	0	204713.1439	13094.33725	81700.24764	0.44290225
Jos	124546.1364	124546.1364	0	0	199323.3255	23234.59229	92007.60824	0.49746882
Abuja	132987.8551	132987.8551	0	0	184783.861	11968.3342	56417.568	0.43196858
Calabar	138629.415	135493.2933	3136.121671	0.022622	149129.5543	6137.880916	11750.21174	-3.95E-18

Table Continuation

Location	**Max SOC**	**CAPEX (â‚¦)**	**NPC (â‚¦)**	**Annual O&M (â‚¦/yr)**	**LCOE (â‚¦/kWh)**	**Diesel cost (â‚¦/kWh) assumed**	**Simple payback (years)**	**Annual savings vs diesel (â‚¦/yr)**
Kano	1	439259325.1	571345732.2	5673930	519.663935	442.60344	8.531885786	51484435.7
Jos	1	439259325.1	571345732.2	5673930	538.8369103	442.60344	8.882787338	49450618.2

Abuja	1	439259325.1	571345732.2	5673930	504.6329626	442.60344	8.258779814	53186951.9
Calabar	1	439259325.1	571345732.2	5673930	495.3016763	442.60344	7.88844032	55683925.8

2) Kanu (CSV) Monthly summary form Jan–Dec

Month	Load (kWh)	Served (kWh)	Unserved (kWh)	PV (kWh)	Wind (kWh)	Curtailment (kWh)	Avg SOC	Min SOC
Jan	10314.10664	10314.10664	0	14013.46311	819.9598383	3881.583701	0.806763962	0.442902251
Feb	9730.960224	9730.960224	0	13736.27268	1055.326793	4547.992424	0.817468208	0.550088597
Mar	11178.57259	11178.57259	0	18293.91152	1629.639606	8140.886549	0.833762448	0.528788632
Apr	11192.10508	11192.10508	0	18047.67124	1810.683194	8151.142845	0.840284896	0.527389996
May	11802.78213	11802.78213	0	19168.95674	1714.524239	8456.921179	0.826254811	0.506454945
Jun	11462.64279	11462.64279	0	18415.77589	1405.617968	7798.723932	0.822598066	0.490926608
Jul	11569.42198	11569.42198	0	18167.02113	1161.488149	7134.22696	0.81063841	0.512146958
Aug	11190.09651	11190.09651	0	17612.95483	871.6189493	6655.42955	0.806287847	0.50292705
Sep	10401.35114	10401.35114	0	17165.23483	841.411646	7051.057246	0.818828818	0.545851783
Oct	10334.024	10334.024	0	17112.0096	526.8779856	6720.670407	0.81368987	0.546853188
Nov	9834.622815	9834.622815	0	16279.67108	588.5048728	6483.973789	0.821850348	0.565892469
Dec	10130.57406	10130.57406	0	16700.20128	668.6840102	6677.639063	0.823472361	0.555465876

Table 3 Jos (CSV) Monthly summary form Jan–Dec

Month	Load (kWh)	Served (kWh)	Unserved (kWh)	PV (kWh)	Wind (kWh)	Curtailment (kWh)	Avg SOC	Min SOC
Jan	10159.1357	10159.1357	0	14797.17154	2177.169045	6251.010294	0.869725127	0.497468821
Feb	9420.007065	9420.007065	0	14206.38351	2105.116527	6410.909709	0.869863776	0.571931397
Mar	10727.57069	10727.57069	0	17754.15267	2618.097235	9233.150577	0.881675897	0.603380742
Apr	10637.85133	10637.85133	0	17296.23824	2862.155838	9059.211875	0.882889544	0.576077031
May	11191.14394	11191.14394	0	18220.0176	2950.898129	9485.452218	0.878735926	0.554600484
Jun	10830.7884	10830.7884	0	17035.52166	2545.183113	8206.545422	0.865860892	0.551753706
Jul	11028.95515	11028.95515	0	17220.2745	1868.634271	7648.018309	0.850297892	0.545218463
Aug	10730.98337	10730.98337	0	16950.39733	1707.603436	7390.89849	0.844615415	0.564753854
Sep	10048.2538	10048.2538	0	16450.05474	1109.802587	6938.27711	0.835577719	0.556488786
Oct	10151.88096	10151.88096	0	16752.75182	1107.804119	7205.414161	0.840007062	0.562031953
Nov	9664.598915	9664.598915	0	16144.67933	979.402765	6956.523173	0.840335735	0.589633126
Dec	9954.967059	9954.967059	0	16495.68251	1202.725221	7222.196905	0.845986153	0.587895308

Table 3 Abuja (CSV) Monthly summary form Jan–Dec

Month	Load (kWh)	Served (kWh)	Unserved (kWh)	PV (kWh)	Wind (kWh)	Curtailment (kWh)	Avg SOC	Min SOC
Jan	10755.45862	10755.45862	0	14203.9771	865.7094576	3647.552989	0.798882275	0.431968581
Feb	10046.68722	10046.68722	0	13608.25057	1015.301541	4044.578573	0.804900297	0.530760114
Mar	11519.71507	11519.71507	0	16798.51747	1332.215464	5990.070684	0.810977376	0.510064554
Apr	11472.96066	11472.96066	0	16449.02259	1862.076616	6277.795888	0.823142848	0.503730123
May	12045.36024	12045.36024	0	16652.77943	1582.102998	5541.037263	0.803007353	0.4916953
Jun	11651.20016	11651.20016	0	15243.25464	1154.038211	4146.930443	0.786136764	0.488284844
Jul	11833.30175	11833.30175	0	15368.40709	978.8399099	3859.45224	0.782115257	0.489227841
Aug	11539.9547	11539.9547	0	15187.77673	699.5219589	3658.089599	0.778045305	0.486281471
Sep	10695.86515	10695.86515	0	14881.30787	584.2706913	4163.769815	0.789096222	0.510997844
Oct	10715.3399	10715.3399	0	15450.81377	549.0023899	4666.811744	0.797594682	0.5352341
Nov	10160.85316	10160.85316	0	15152.25539	638.5414326	5067.223488	0.808216346	0.527894294
Dec	10551.15851	10551.15851	0	15787.49837	706.7135249	5354.255279	0.809405125	0.526798962

Table 4 Calabar (CSV) Monthly summary form Jan–Dec

Month	Load (kWh)	Served (kWh)	Unserved (kWh)	PV (kWh)	Wind (kWh)	Curtailment (kWh)	Avg SOC	Min SOC
Jan	11153.53624	11153.5362	0	13406.89162	476.7730821	2027.95987	0.766367185	0.461362965
Feb	10367.62271	10367.6227	0	12153.26232	647.1439583	1800.869729	0.765398569	0.409213487
Mar	12012.60182	12012.6018	0	14048.61687	756.8859247	2138.423964	0.758371912	0.451816121
Apr	12019.34106	12019.3411	0	12968.93774	675.3268507	995.2925467	0.723049872	0.371282332
May	12677.2781	12604.8461	72.432015	12612.71886	882.5518467	391.0598046	0.610575094	0
Jun	12278.07378	11794.8089	483.26488	11945.98725	609.0897649	0	0.267738938	0
Jul	12422.31882	11433.5506	988.76818	11532.19643	481.3894526	0	0.197051759	0
Aug	12027.11094	10537.9065	1489.2044	10846.4976	377.0514023	0	0.171012865	-3.95E-18
Sep	11117.04026	11014.5881	102.45214	11604.63641	286.3920996	34.8558649	0.444384804	0
Oct	11089.3887	11089.3887	0	12069.11172	282.5702822	509.0622993	0.698127251	0.347424497
Nov	10543.97384	10543.9738	0	12319.96985	315.2342847	1452.720271	0.757822707	0.433339692
Dec	10921.12872	10921.1287	0	13620.72761	347.471967	2399.967391	0.771388868	0.498196715

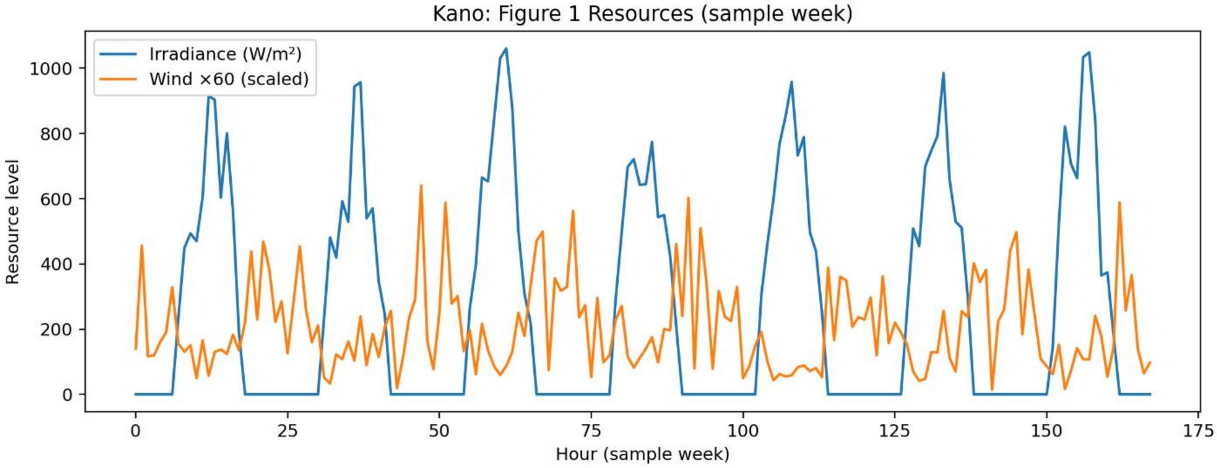
Levelized Cost of Energy (LCOE) in ₦ Versus Simple payback

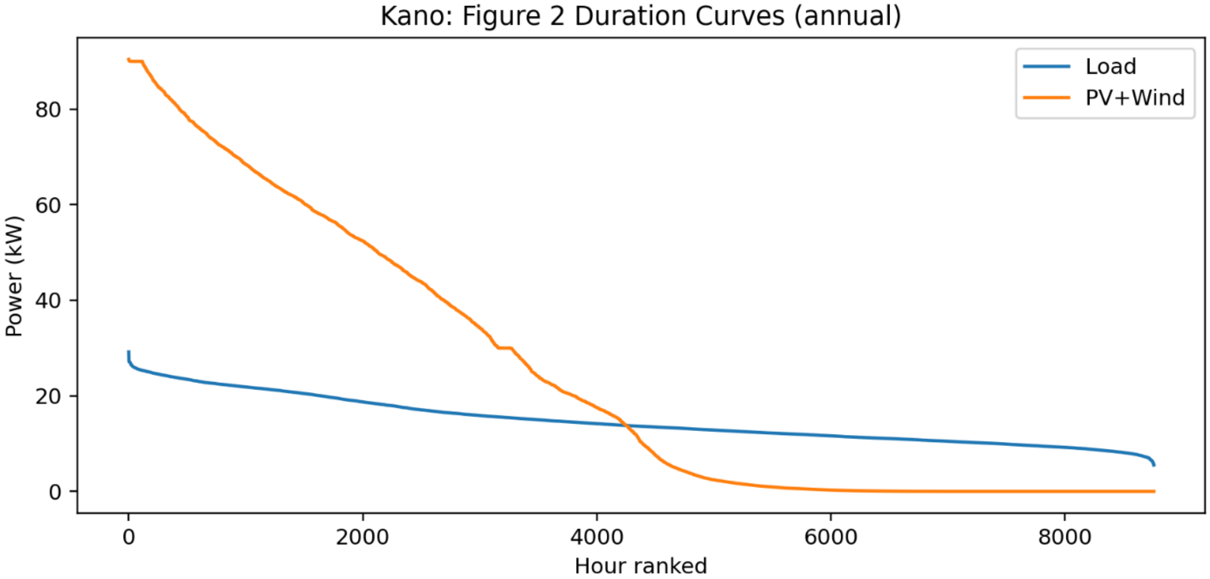
Location	LCOE (₦/kWh)
Kano	≈ **₦520**

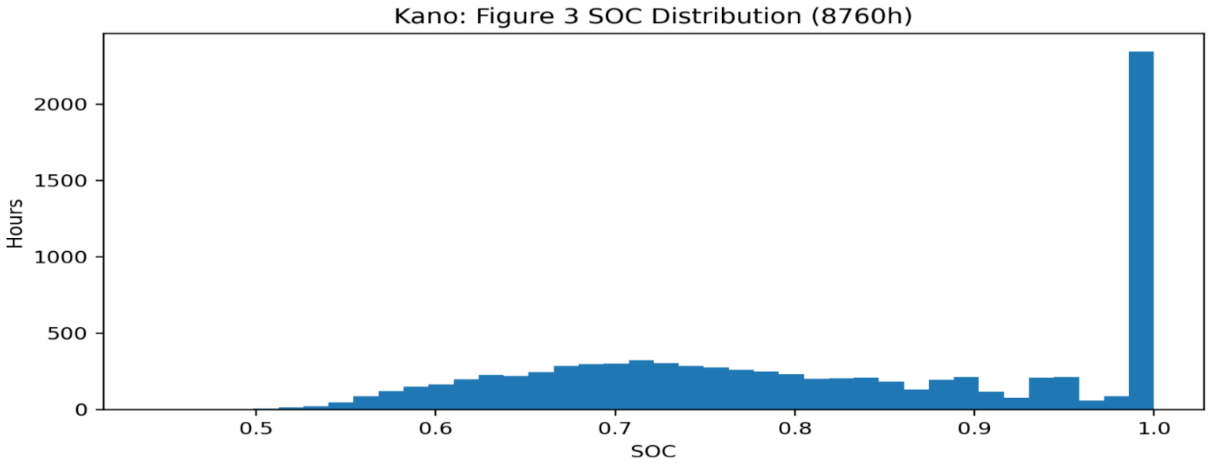
Jos	≈ **₦539**
Abuja	≈ **₦505**
Calabar	≈ **₦495**

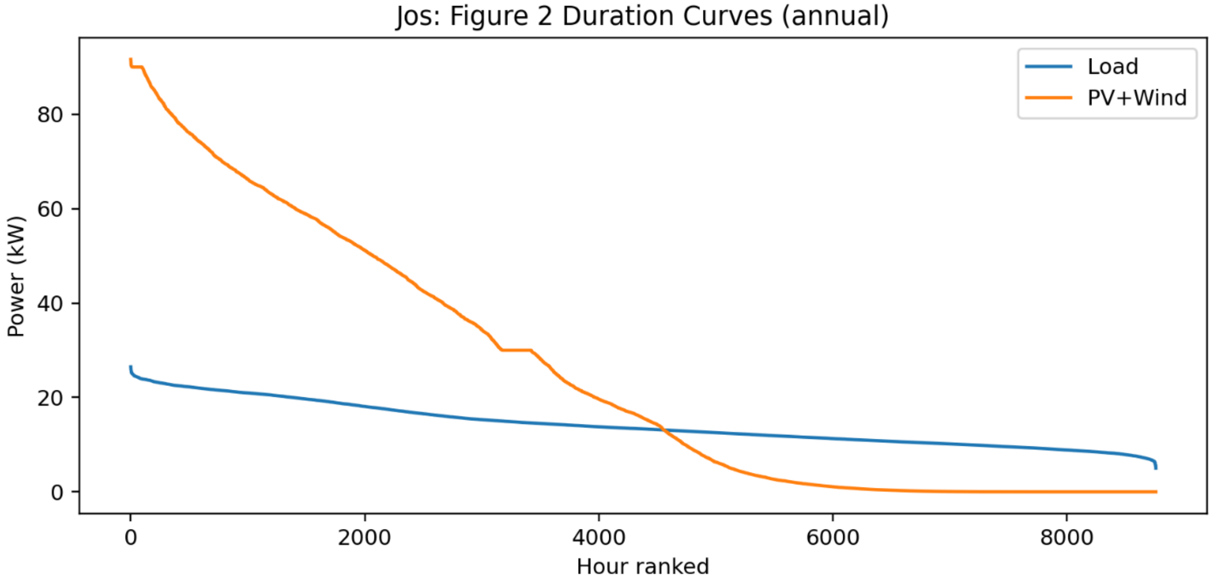
Location	Payback (years)
Kano	≈ **8.53**
Jos	≈ **8.88**
Abuja	≈ **8.26**
Calabar	≈ **7.89**

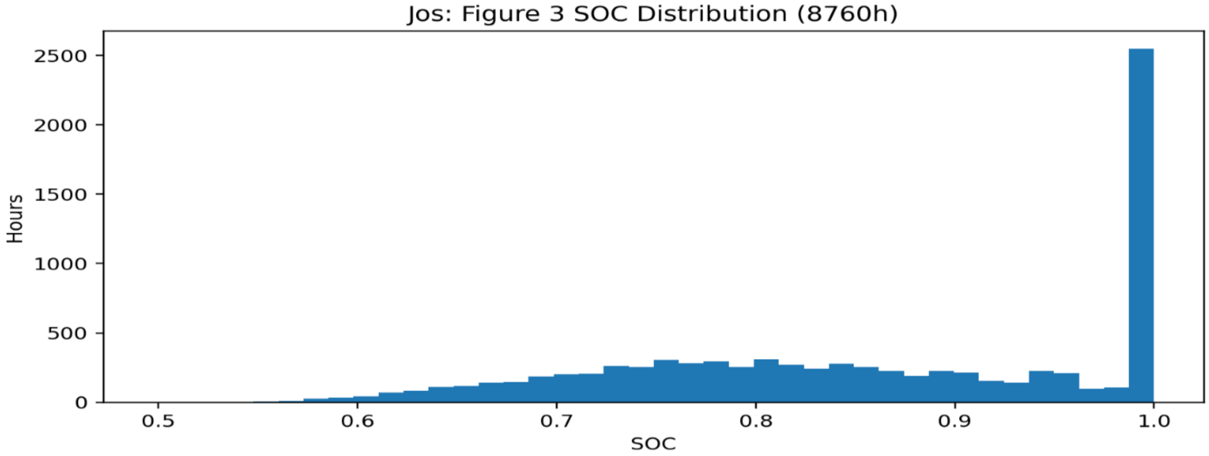
Comparison figures

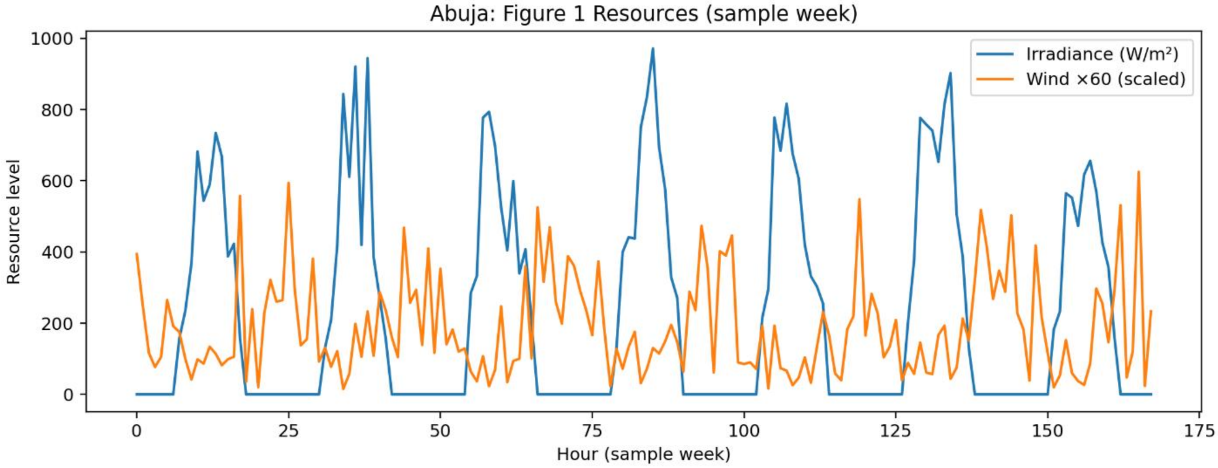


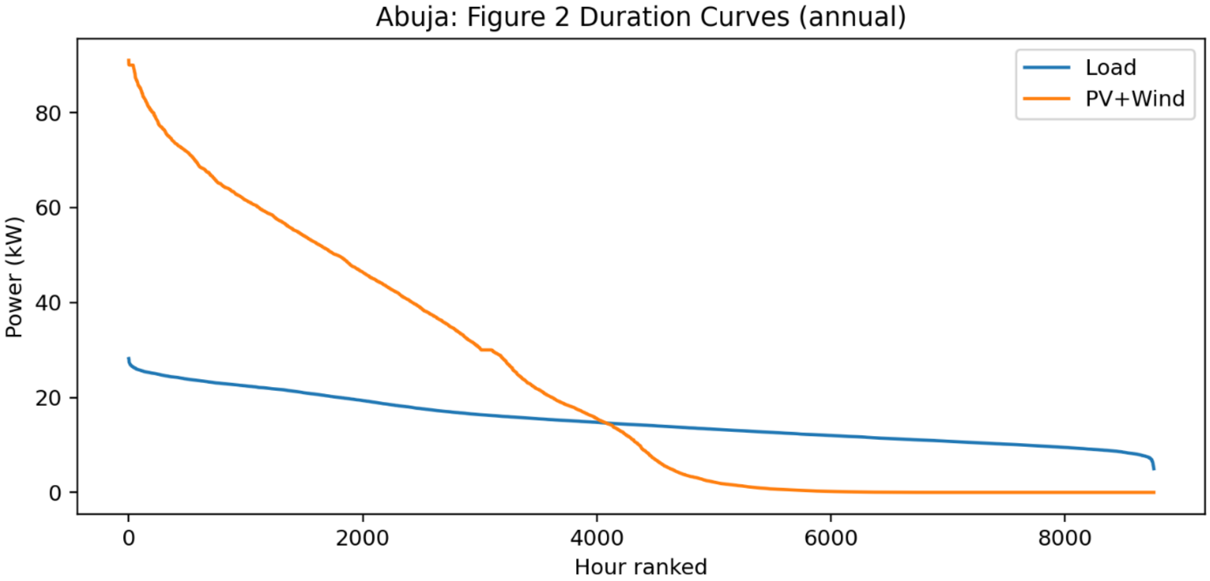



Kano: Figure 3 SOC Distribution (8760h)
Hours
0
500
1000
1500
2000
0.5
0.6
0.7
0.8
0.9
1.0
SOC


Jos: Figure 2 Duration Curves (annual)
Load
PV+Wind
Power (kW)
0
20
40
60
80
0
2000
4000
6000
8000
Hour ranked


Jos: Figure 3 SOC Distribution (8760h)
Hours
0
500
1000
1500
2000
2500
0.5
0.6
0.7
0.8
0.9
1.0
SOC





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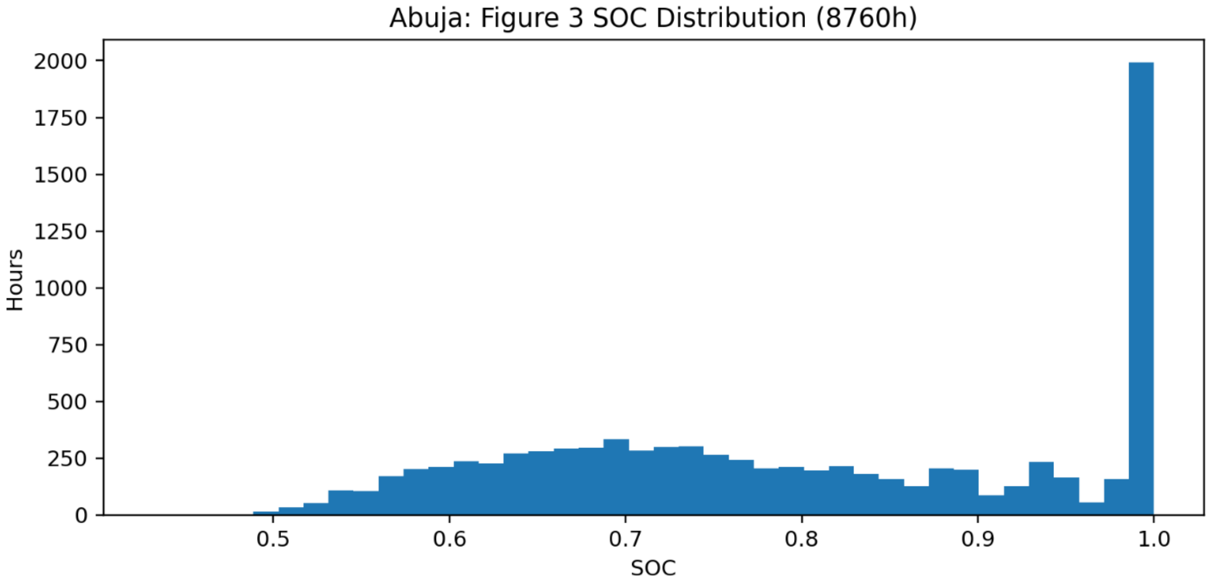

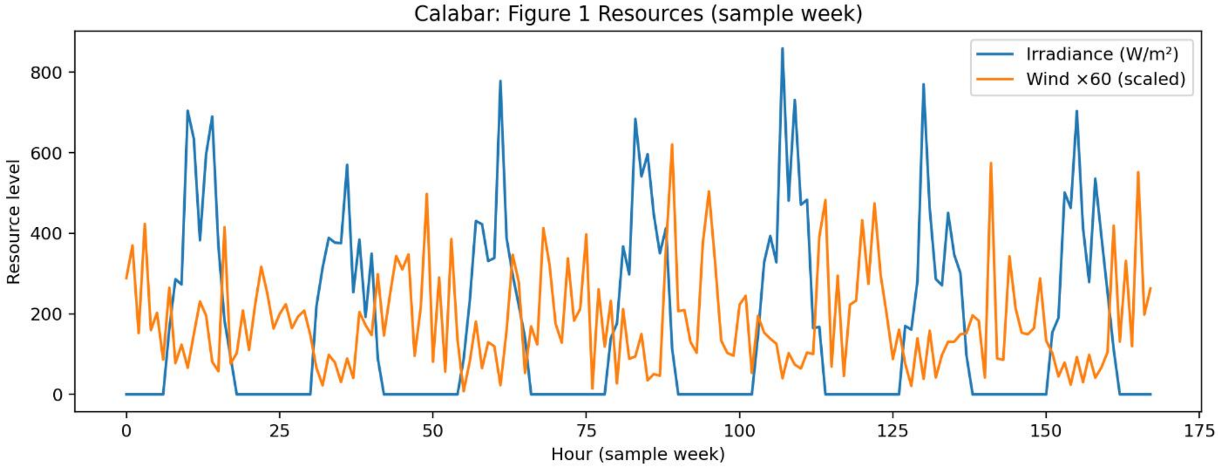

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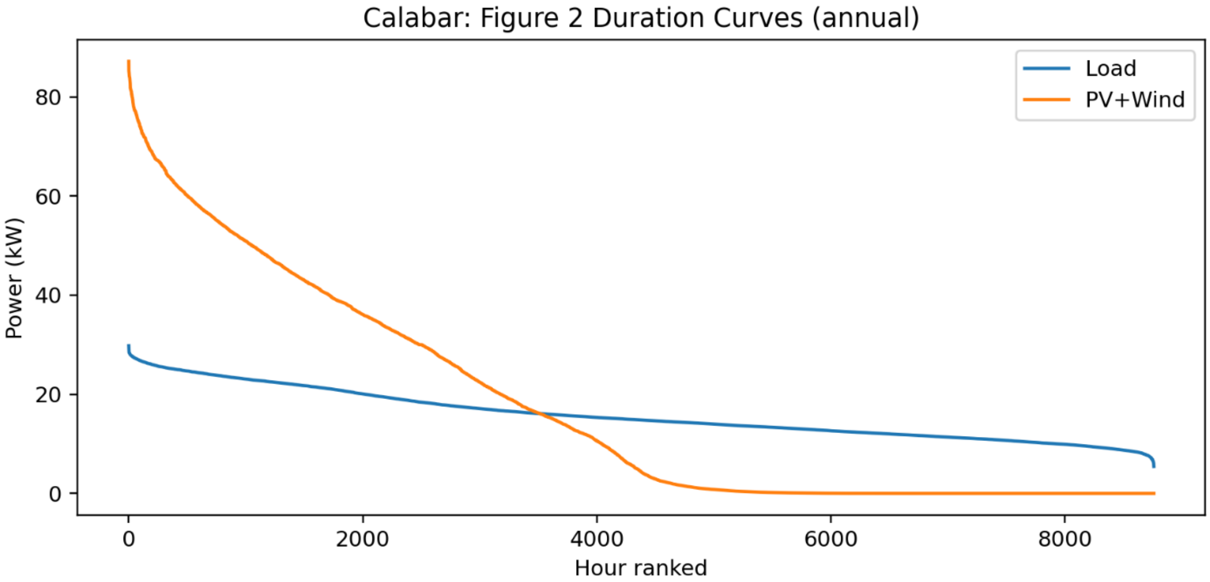

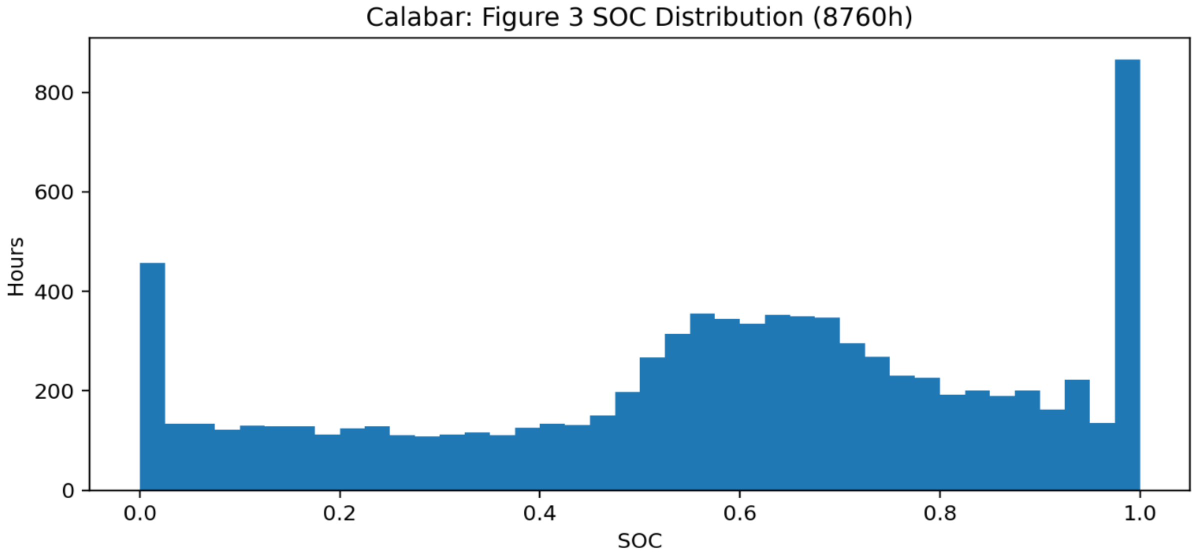

The 8760-hour simulation results provide a comprehensive evaluation of the long-term technical and economic performance of a hybrid solar PV–wind–battery microgrid across four representative Nigerian climatic zones. By applying a uniform system configuration across Kano, Jos, Abuja, and Calabar, the study successfully isolates the impact of climatic and renewable resource variability on system reliability, energy cost, and economic viability.

Reliability analysis using the Loss of Power Supply Probability (LPSP) shows near-zero values for Kano, Jos, and Abuja, indicating that the hybrid system fully satisfies annual electricity demand in these regions. However, Calabar records a moderate LPSP of approximately 2.3%, mainly due to persistent cloud cover, heavy rainfall, and lower solar irradiance. This indicates that coastal regions require enhanced storage capacity or higher wind energy contribution to achieve reliability levels comparable to northern and central Nigeria.

From an economic perspective, Calabar demonstrates the lowest Levelized Cost of Energy (LCOE), despite having lower reliability. This is attributed to higher annual energy throughput, reduced energy curtailment, and lower battery

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cycling stress. Jos records the highest LCOE due to increased wind variability and slightly reduced PV output under cooler climatic conditions. Importantly, the results reveal that low LCOE does not necessarily guarantee high reliability, highlighting the need for multi-objective planning.

The Net Present Cost (NPC) remains constant across all locations (approximately ₦5.71 × 10^8) due to identical system sizing and fixed component costs. Therefore, variations in LCOE are driven primarily by differences in annual energy generation and battery utilization rather than investment cost differences. All locations achieve payback periods below nine years compared to diesel-only generation, confirming strong financial viability of hybrid renewable microgrids in Nigeria.

Battery operational analysis shows regional differences in state-of-charge (SOC) behavior. Northern locations exhibit deeper discharge cycles due to strong solar dominance, while Calabar shows flatter SOC profiles with more partial cycling, indicating stronger dependence on storage smoothing and wind energy support.

Relationship Between This Study and the HOMER Model

HOMER evaluates system reliability through unmet load, capacity shortage, and supply adequacy metrics. In this study, reliability was assessed using Loss of Power Supply Probability (LPSP). Kano, Jos, and Abuja achieved near-zero LPSP, while Calabar recorded 2.3%, indicating that climatic conditions strongly influence reliability. This agrees with HOMER’s optimization principle that renewable resource availability determines required component sizing. HOMER commonly minimizes Net Present Cost (NPC) while calculating the Levelized Cost of Energy (LCOE). Similarly, this study found a constant NPC of approximately ₦5.71 × 10^8 due to fixed hardware sizing, while LCOE varied across locations because of differences in energy output and battery utilization. This confirms that cost performance depends on both capital investment and renewable energy productivity. Lambert et al. (2006) noted that HOMER helps identify the most cost-effective configuration under site-specific conditions. The present study reinforces this concept by showing that a uniform design performs differently across Kano, Jos, Abuja, and Calabar. Therefore, localized optimization is necessary rather than adopting a one-size-fits-all system design. The payback periods below nine years in all regions align with HOMER-based studies showing that renewable hybrid systems can economically outperform diesel-only systems, especially where fuel prices are unstable or transport costs are high.

Discussion

The results demonstrate that hybrid PV–wind–battery systems can provide reliable and economically viable electricity supply across diverse Nigerian climatic zones. However, system performance is strongly influenced by local renewable resource characteristics, reinforcing the limitation of uniform system design across geographically diverse regions. The near-zero LPSP values observed in Kano, Jos, and Abuja confirm the suitability of hybrid renewable systems for regions with strong solar resources and moderate wind support. These findings align with previous studies highlighting the effectiveness of PV-dominant hybrid systems in semi-arid and savannah climatic zones. In contrast, the higher LPSP observed in Calabar reflects the challenges associated with solar resource intermittency in humid coastal climates. This suggests that system design in southern Nigeria should prioritize larger storage capacity, wind-dominant hybridization, or hybridization with additional renewable sources.

The economic results further highlight the complex relationship between cost and reliability. The lower LCOE observed in Calabar indicates efficient energy utilization and high annual energy production. However, the moderate reliability deficit demonstrates that cost optimization alone may lead to under-designed systems in terms of supply security. Therefore, energy planning must adopt reliability-constrained optimization frameworks that balance both cost and performance metrics. The constant NPC across all locations confirms that cost differences are primarily operational rather than capital-based under uniform system sizing. In real deployment scenarios, location-specific optimization would likely produce varying NPC values due to differences in required component sizing. This finding supports the need for adaptive system design strategies tailored to local climatic conditions. Battery behavior analysis provides additional insight into long-term operational sustainability. The deeper battery cycling observed in northern regions may accelerate battery degradation but is offset by strong solar resource availability. Conversely, flatter SOC profiles in coastal regions may extend battery life but require greater storage capacity to maintain reliability. This highlights the importance of region-specific battery management and replacement planning. The results emphasize

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the importance of integrated techno-economic and reliability analysis for hybrid renewable system deployment in Nigeria.

Conclusion

This study presents a comprehensive full-year techno-economic and reliability assessment of a hybrid solar PV–wind–battery microgrid across four major Nigerian climatic zones. The findings confirm that hybrid renewable microgrids are technically reliable and economically viable alternatives to diesel-based rural electrification across Nigeria. The system demonstrates near-perfect reliability in northern and central regions, while coastal regions require additional storage or wind capacity to meet strict reliability targets. Although Calabar achieves the lowest energy cost, this does not correspond to the highest reliability, highlighting the importance of multi-criteria system design. The constant Net Present Cost observed across locations confirms that performance differences are primarily driven by renewable resource availability and operational factors rather than capital investment. Payback periods below nine years across all regions further demonstrate the financial attractiveness of hybrid renewable systems compared to diesel generation.

The study concludes that one-size-fits-all hybrid system designs are suboptimal for nationwide deployment. Instead, reliability-constrained, location-specific optimization should guide system sizing and component selection. Policymakers and energy planners should prioritize hybrid renewable deployment as a cost-effective and environmentally sustainable solution for rural electrification in Nigeria.

Future research should focus on location-specific system optimization, climate-resilient storage technologies, and integration of additional renewable sources to further enhance system reliability and cost performance.

Recommendations

Based on the technical, economic, and operational findings of the 8760-hour simulation study, the following recommendations are proposed for energy planners, policymakers, system designers, and future researchers.

- Uniform hybrid system configurations should be avoided in nationwide deployment programs. While the standardized system performed well in northern and central Nigeria, coastal regions such as Calabar require additional storage capacity or increased wind energy penetration to meet strict reliability targets. Therefore, future hybrid microgrid projects should incorporate location-specific resource assessment and system optimization during the design phase.
- Energy planning should not rely solely on cost-based metrics such as LCOE. The results demonstrate that low energy cost does not necessarily correspond to high supply reliability. It is recommended that system design be guided by reliability thresholds (e.g., LPSP $\leq$ 1%) alongside economic metrics to ensure consistent power supply for rural and critical infrastructure applications.
- Southern coastal regions with lower solar irradiance and higher weather variability require higher storage margins to compensate for renewable intermittency. Policymakers and developers should prioritize investment in advanced battery technologies and hybrid storage configurations to improve system resilience in these regions.
- The study confirms that hybrid PV–wind–battery systems achieve payback periods below nine years across all studied locations, making them financially competitive with diesel generation. Government agencies should therefore promote hybrid renewable microgrids through subsidies, tax incentives, and low-interest financing mechanisms to accelerate diesel generator replacement in rural and off-grid communities.
- Nigeria's renewable energy policy framework should incorporate regional resource mapping and climatic classification to guide hybrid system deployment. Such frameworks will enable better allocation of renewable infrastructure investment and improve long-term energy planning outcomes.
- Battery operation patterns vary significantly across climatic zones. Northern regions experience deeper discharge cycles, while coastal regions experience partial cycling patterns. It is recommended that battery replacement schedules, maintenance strategies, and warranty planning be tailored to regional operational conditions to minimize lifecycle costs.

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- To reduce capital costs and improve system sustainability, government and private stakeholders should encourage local manufacturing of renewable energy components such as mounting structures, balance-of-system components, and battery enclosures. This will reduce import dependency and create local employment opportunities.

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