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On The Theoretical Development And Applications Of The Sine Type Ii Generalized Topp-Leone Exponential Distribution.

***[1]Faweya, O., [2]Olumi ,T. T., & [3]Akinyemi, O.**
[1, 3]Department of Statistics, Ekiti State University, Ado-Ekiti
[2]Department of Statistics, Federal Polytechnic Orogun, Delta State.

***Corresponding author email**: olumitoba@yahoo.com.

Abstract
This study introduces and investigates the Sine Type II Generalized Topp-Leone Exponential (STIIGTLE) distribution, a novel extension constructed by integrating the Sine Type II generator with the generalized Topp-Leone exponential baseline model. We derived the fundamental statistical properties of the STIIGTLE distribution, including its probability density function, cumulative distribution function, quantile functions and moments. Parameter estimation is addressed using the maximum likelihood estimation (MLE) and maximum product of spacing (MPS) techniques, and the performance of the estimators is evaluated through simulation studies to access consistency and efficiency under different sample sizes. To demonstrate its practical relevance, the STIIGTLE model is applied to real-life from biomedical field. Comparative analyses with existing distributions reveal that the proposed model provides superior goodness-of-fit, as evidenced by standard selection criteria such as AIC, BIC, and log-likelihood values. The results confirm that the STIIGTLE distribution is a robust and flexible tool for modeling skewed and heavy-tailed data

Keywords: Probability, Exponential, Sine, Topp-Leone, Parameter.

Introduction

The continuous growth of applied statistics has led to an increasing demand for probability model that can adequately describe complex real world phenomena. In many practical situations such as reliability engineering, medical survival studies, and financial risk analysis, data often exhibit features that cannot be captured by classical distributions. Although the exponential remains one of the most widely used lifetime models due to its simplicity and mathematical convenience, its assumption of a constant hazard rate significantly limits its applicability in modeling real-life processes (Gupta and Kundu, 2001; Nadarajah and Kotz, 2006).

To overcome such limitations, statisticians have focused on developing generalized families of distributions by introducing additional parameters or by embedding existing models within flexible generating frameworks. One of the early contributions in this direction is the Topp-Leone distribution proposed by Topp and Leone (1955), which has attracted attention because of its bounded support and usefulness in modeling lifetime data. Subsequent modifications and generalizations of this distribution have enhanced its applicability in different areas of statistical modeling.

Among these extensions, the Type II generalized Topp-Leone family has proven particularly useful due to its ability to accommodate various shapes of probability density and hazard rate functions. This flexibility allows it to model increasing, decreasing, and non-monotonic failure rates, which are commonly observed in engineering systems and biological processes (Hassan et al., 2019). As a result, it has become a suitable candidate for further generalization using modern generator techniques.

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In recent developments, generator-based approaches have played a significant role in constructing new distributions. These methods involve transforming a baseline distribution through a functional mechanism to obtain more flexible models. One notable example is the sine-generated family by Cordeiro et al. (2014) which has been applied to several distribution families, yielding models with better data-fitting capabilities.

The integration of sine-generated techniques with the type II generalized Topp-Leone framework represents a natural progression in the search for more versatile statistical models. When combined with the exponential distribution as a baseline, this approach results in a hybrid model capable of addressing the shortcomings of traditional lifetime distributions. In particular, the exponential component provides analytical simplicity, while the sine Type II generalized Topp-Leone structure introduces the flexibility required to capture complex data patterns (Al-Saiary and Bakoban, 2020; Moakofi et al., 2021).

Motivated by these advancements, the Sine Type II Generalized Topp-Leone Exponential distribution has been proposed as an improved model for analyzing lifetime data. This distribution is designed to offer greater flexibility in terms of skewness, kurtosis, and hazard rate behavior, thereby making it suitable for a wide range of applications. Its potential usefulness extends to areas such as reliability analysis, survival modeling, and risk assessment, where accurate representation of data behavior is essential for effective decision making.
In view of the above, studying the properties and applications of this new distributions is both relevant and necessary. It contributes to the expanding body of statistical literature by providing an alternative modeling tool that can outperform traditional distributions in capturing complex real world patterns.

Materials and Methods

- **Sine Type II Generalized Topp-Leone Exponential (STIIGTLE) Distribution**

The cdf and pdf of exponential distribution, which serves as our baseline distribution with parameter θ are given by:

$$H(x;\theta) = 1 - e^{-\theta x},\ x > 0, \theta > 0 \tag{1}$$

and

$$h(x;\theta) = \theta e^{-\theta x}, x > 0, \theta > 0 \tag{2}$$

The STIIGTLE has cdf and pdf as follows:

$$F(x;\ \alpha,\beta,\theta) = Sin\left[\frac{\pi}{2}\left[1 - \left[1 - \left[1 - e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]\right],\ x > 0, \alpha > 0, \beta > 0, \theta > 0 \tag{3}$$

and

$$f(x;\ \alpha,\beta,\theta) = \frac{\pi}{2} 2\alpha\beta\theta e^{-\theta x}\left[1 - e^{-\theta x}\right]^{2\beta-1}\left[1 - \left[1 - e^{-\theta x}\right]^{2\beta}\right]^{\alpha-1} Cos\left[\frac{\pi}{2}\left[1 - \left[1 - \left[1 - e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]\right] \tag{4}$$

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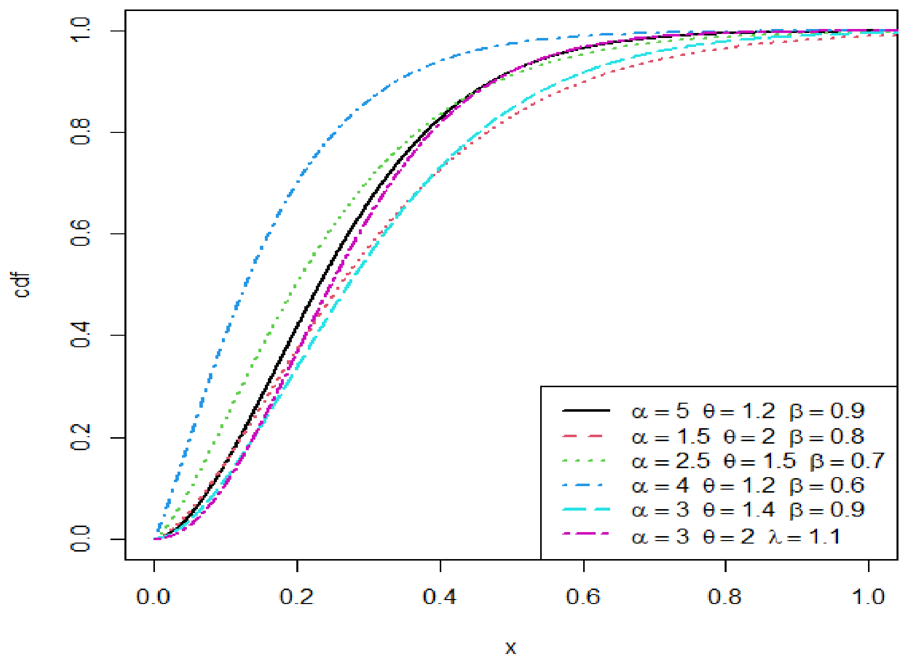

Figure 1: Plots of the cdf of STIIGTLE distribution with different parameter values

As seen from figure 1, we can deduced that the cdf has varying shapes. Some are steeper, while others are flatter and this indicates that the parameters significantly influence the overall shape of the distribution.

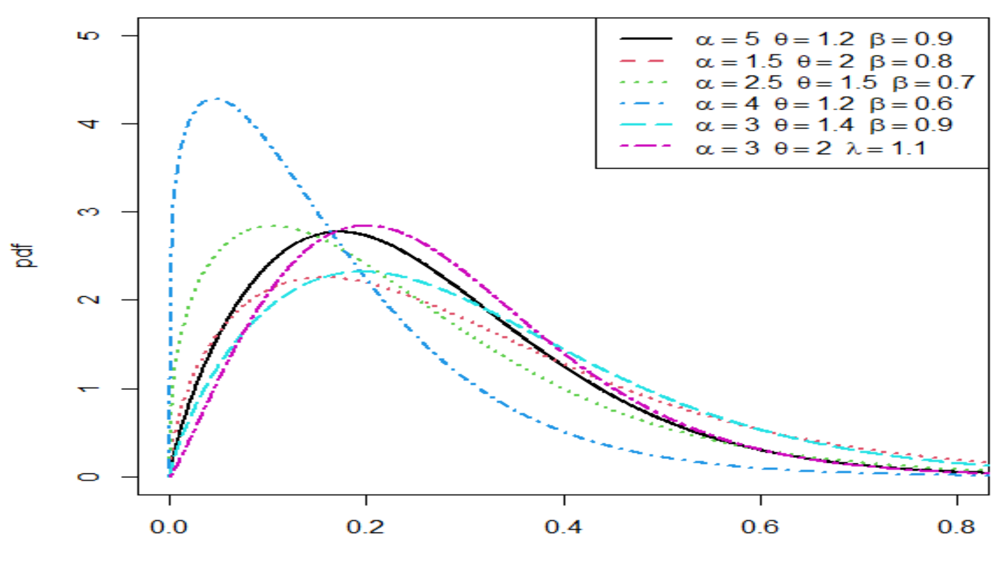

Figure 2: Plots of the pdf of STIIGTLE distribution under various parameter values are presented. The pdf plot shown in figure 2 explores how different parameter combination affect the shape of the distribution, and these parameters

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control the location, spread, and skewness of the distribution. We can see that the distribution has varying shapes. Some are more symmetric, while others are skewed. This indicates that the parameters have a substantial impact on the overall shape of the distribution.

Validity Check on Sine Type Ii Generalized Topp-Leone Exponential Distribution

It suffices to show that

$$\int_0^\infty f(x;\ \alpha,\beta,\theta)dx = 1 \tag{5}$$

$$\int_0^\infty \frac{\pi}{2} 2\alpha\beta\theta e^{-\theta x}\left[1-e^{-\theta x}\right]^{2\beta-1}\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha-1} Cos\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]\right] \mathrm{dx}$$

Let $y = \frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]$, when $x \Rightarrow 0, y \Rightarrow 0$; when$x \Rightarrow 0, y \Rightarrow \frac{\pi}{2}$;

Substituting into the original integral, we get

$$dx = \frac{dy}{\frac{\pi}{2} 2\alpha\beta\theta e^{-\theta x}[1-e^{-\theta x}]^{2\beta-1}[1-[1-e^{-\theta x}]^{2\beta}]^{\alpha-1}}$$

$$\int_0^{\frac{\pi}{2}} \frac{\pi}{2} 2\alpha\beta\theta e^{-\theta x}\left[1-e^{-\theta x}\right]^{2\beta-1}\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha-1} \frac{dy}{\frac{\pi}{2} 2\alpha\beta\theta e^{-\theta x}[1-e^{-\theta x}]^{2\beta-1}[1-[1-e^{-\theta x}]^{2\beta}]^{\alpha-1}}$$

$$\int_0^{\frac{\pi}{2}} Cos(y)dy = [Sin(y)]|_0^{\frac{\pi}{2}} = Sin\left(\frac{\pi}{2}\right) - Sin(0) = 1 - 0 = 1$$

Therefore,

$$f(x;\alpha,\beta,\theta) = \frac{\pi}{2} 2\alpha\beta\theta e^{-\theta x}\left[1-e^{-\theta x}\right]^{2\beta-1}\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha-1} Cos\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]\right]$$

is a legitimate pdf

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Suitable Expansion of Density for The Stiigtle Distribution

In this section, the expanded expressions of the pdf and cdf for the STIIGTLE distribution using standard mathematical techniques is derived. These include the Maclaurin series expansions for sine and cosine functions, along with the binomial expansion, expressed as follows:

$$Sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$Cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n!)} \qquad (6)$$

$$(1-y)^b = \sum_{k=0}^{\infty} (-1)^k \binom{b}{k} y^k \qquad (7)$$

The expanded form of the STIIGTLE distribution's pdf is derived through the application of Maclau expansion for the cosine function and the binomial expansion, as outlined in equations (7) and (8), to equat

$$\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\alpha},\boldsymbol{\beta},\theta) = \frac{\pi}{2} 2\alpha\beta\theta e^{-\theta x}\left[1-e^{-\theta x}\right]^{2\beta-1}\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha-1} Cos\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]\right] \qquad (8)$$

$$Cos\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]\right] = \sum_{a=0}^{\infty} \frac{(-1)^a \pi^{2a}}{(2a)!\,2^{2a}}\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]^{2a}$$

$$\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha(1+t)-1} = \sum_{p=0} (-1)^p \binom{\alpha(1+t)-1}{p}\left[1-e^{-\theta x}\right]^{2\beta p}$$

$$\left[1-e^{-\theta x}\right]^{2\beta(1+p)-1} = \sum_{l=0}^{\infty} (-1)^l \binom{2\beta(1+p)-1}{l}\left[e^{-\theta x}\right]^l$$

$$\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\alpha},\boldsymbol{\beta},\theta) = \sum_{\boldsymbol{a},\boldsymbol{t},\boldsymbol{p},l=0}^{\infty} \frac{(-1)^{a+t+p+l}\pi^{2a+1} 2\alpha\beta\theta}{(2a)!\,2^{2a+1}}\binom{2a}{t}\binom{\alpha(1+t)-1}{p}\binom{2\beta(1+p)-1}{l}\left[e^{-\theta x}\right]^l$$

The probability density function (pdf) above can be rewritten, as follows:

$$\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\alpha},\boldsymbol{\beta},\theta) = \sum_{\boldsymbol{a},\boldsymbol{t},\boldsymbol{p},\mathrm{l}=0}^{\infty} \boldsymbol{\Phi}\left[e^{-\theta x}\right]^{l+1} \qquad (9)$$

where

$$\boldsymbol{\Phi} = \sum_{\boldsymbol{a},\boldsymbol{t},\boldsymbol{p},l=0}^{\infty} \frac{(-1)^{a+t+p+\mathrm{l}}\pi^{2a+1} 2\alpha\beta\theta}{(2a)!\,2^{2a+1}}\binom{2a}{t}\binom{\alpha(1+t)-1}{p}\binom{2\beta(1+p)-1}{l}$$

Also, the cumulative distribution function (cdf) can be expressed in expanded form as follows:

$$\boldsymbol{F}(\boldsymbol{x};\boldsymbol{\alpha},\boldsymbol{\beta},\theta) = \boldsymbol{Sin}\left[\frac{\boldsymbol{\pi}}{\boldsymbol{2}}\left[\boldsymbol{1}-\left[\boldsymbol{1}-\left[\boldsymbol{1}-\boldsymbol{e}^{\theta x}\right]^{\boldsymbol{2\beta}}\right]^{\boldsymbol{\alpha}}\right]\right]$$

$$\boldsymbol{Sin}\left[\frac{\boldsymbol{\pi}}{\boldsymbol{2}}\left[\boldsymbol{1}-\left[\boldsymbol{1}-\left[\boldsymbol{1}-\boldsymbol{e}^{\theta x}\right]^{\boldsymbol{2\beta}}\right]^{\boldsymbol{\alpha}}\right]\right] = \sum_{\boldsymbol{g}=\boldsymbol{0}}^{\infty} \frac{(-\boldsymbol{1})^{\boldsymbol{g}}\boldsymbol{\pi}^{\boldsymbol{2g+1}}}{(\boldsymbol{2g}+\boldsymbol{1})!\,\boldsymbol{2}^{\boldsymbol{2c+1}}}\left[\boldsymbol{1}-\left[\boldsymbol{1}-\left[\boldsymbol{1}-\boldsymbol{e}^{-\theta x}\right]^{\boldsymbol{2\beta}}\right]^{\boldsymbol{\alpha}}\right]^{\boldsymbol{2g+1}}$$

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$$\left[\mathbf{1}-\left[\mathbf{1}-\left[\mathbf{1}-\boldsymbol{e}^{\theta x}\right]^{\mathbf{2}\boldsymbol{\beta}}\right]^{\boldsymbol{\alpha}}\right]^{\mathbf{2}\boldsymbol{g}+\mathbf{1}} = \sum_{\boldsymbol{w}=\mathbf{0}}^{\infty}(-\mathbf{1})^{\boldsymbol{w}}\binom{\mathbf{2}\boldsymbol{g}+\mathbf{1}}{\boldsymbol{w}}\left[\mathbf{1}-\left[\mathbf{1}-\boldsymbol{e}^{-\theta x}\right]^{\mathbf{2}\boldsymbol{\beta}}\right]^{\boldsymbol{\alpha}\boldsymbol{w}}$$

$$\left[\mathbf{1}-\left[\mathbf{1}-\boldsymbol{e}^{\theta x}\right]^{\mathbf{2}\boldsymbol{\beta}}\right]^{\boldsymbol{\alpha}\boldsymbol{w}} = \sum_{\boldsymbol{d}=\mathbf{0}}^{\infty}(-\mathbf{1})^{\boldsymbol{d}}\binom{\boldsymbol{\alpha}\boldsymbol{w}}{\boldsymbol{d}}\left[\mathbf{1}-\boldsymbol{e}^{-\theta x}\right]^{\mathbf{2}\boldsymbol{\beta}\boldsymbol{d}}$$

$$\left[\mathbf{1}-\boldsymbol{e}^{-\theta x}\right]^{\mathbf{2}\boldsymbol{\beta}\boldsymbol{d}} = \sum_{\boldsymbol{s}=\mathbf{0}}^{\infty}(-\mathbf{1})^{\boldsymbol{s}}\binom{\mathbf{2}\boldsymbol{\beta}\boldsymbol{d}}{\boldsymbol{S}}\left[\boldsymbol{e}^{-\theta x}\right]^{\boldsymbol{s}}$$

$$F(x;\ \alpha,\beta,\theta) = \sum_{g,w,d,s=0}^{\infty}\frac{(-1)^{g+w+d+s}\pi^{2g+1}}{(2g+1)!\,2^{2c+1}}\binom{2g+1}{w}\binom{\alpha w}{d}\binom{2\beta d}{S}\left[\boldsymbol{e}^{-\wp x}\right]^{\boldsymbol{s}}$$

The expression of the cdf above can be rewritten as follows:

$$F(x;\ \alpha,\beta,\theta) = \sum_{g,w,d,s=0}^{\infty}\Upsilon\left[\boldsymbol{e}^{-\theta x}\right]^{\boldsymbol{s}}$$

Where,

$$\Upsilon = \frac{(-1)^{g+w+d+s}\pi^{2g+1}}{(2g+1)!2^{2c+1}}\binom{2g+1}{w}\binom{\alpha w}{d}\binom{2\beta d}{S} \qquad (10)$$

Statistical Properties of the Stiigtle Distribution

In this section, various statistical properties of the NSTIIGTLE distribution were investigated.

Moments Of The Stiigtle Distribution

Given the significance of moments in statistical studies, particularly for real-world applications, we proceed to derive the r^{th} moments for STIIGTLE.

$$E(X^r) = \int_0^\infty x^r f(x)dx \qquad (11$$

The r^{th} moments of the STIIGTLE distribution are obtained by substituting equation (9) into equation (11), resulting in:

$$E(X^r) = \sum_{a,t,p,\wr=0}^{\infty}\Phi\int_0^\infty x^r\left[e^{-\theta x}\right]^{\wr+1}dx \qquad (12)$$

Simplify further, therefore

$$E(X^r) = \sum_{a,t,p,\wr=0}^{\infty}\frac{\Phi\Gamma(r+1)}{(\wr+1)^{r+1}\theta^r} \qquad (13)$$

Now

$$\boldsymbol{\Phi} = \frac{(-1)^{a+t+p+\wr}\pi^{2a+1}2\alpha\beta\theta}{(2a)!\,2^{2a+1}}\binom{2a}{t}\binom{\alpha(1+t)-1}{p}\binom{2\beta(1+p)-1}{\wr}$$

The equation (13) above represents the r^{th} moments of the STIIGTLE distribution, while the mean of the distribution can be calculated by setting r = 1 in (13).

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Moment Generating Function of the Stiigtle Distribution

The Moment Generating Function of x is given as

$$M_x(t) = \int_0^\infty e^{tx} f(x)dx \tag{14}$$

The moment generating function (MGF) of the STIIGTLE distribution is obtained by substituting equation (9) into equation (14), yields the following:

$$M_x(t) = \sum_{a,t,p,l=0}^{\infty} \Phi \int_0^\infty e^{tx}[e^{-\wp x}]^{l+1}dx \tag{15}$$

where the expansion of $e^{tx} = \sum_{z=0}^{\infty} \frac{t^z x^z}{z!}$ and using the moment-based approach described above, the mgf of the STIIGTLE distribution is given in equation (16) below.

$$M_x(t) = \sum_{a,t,p,l=0}^{\infty} \sum_{m=0}^{\infty} \frac{t^m \Gamma(m+1)}{(l+1)^{m+1}\theta^m m!} \tag{16}$$

Reliability Function of the Stiigtle Distribution

The reliability function represents the probability that a system or component continues to function without failure beyond a specified time. It measures survival over time and provides a fundamental description of longevity in reliability and lifetime modeling. It is defined as follows:

$$\boldsymbol{R}(\boldsymbol{x};\boldsymbol{\alpha},\boldsymbol{\beta},\theta) = \mathbf{1} - \boldsymbol{F}(\boldsymbol{x};\boldsymbol{\alpha},\boldsymbol{\beta},\theta)$$

$$\boldsymbol{R}(\boldsymbol{x};\boldsymbol{\alpha},\boldsymbol{\beta},\theta) = \mathbf{1} - \boldsymbol{Sin}\left[\frac{\boldsymbol{\pi}}{\mathbf{2}}\left[\mathbf{1} - \left[\mathbf{1} - \left[\mathbf{1} - \boldsymbol{e}^{-\theta x}\right]^{\mathbf{2}\boldsymbol{\beta}}\right]^{\boldsymbol{\alpha}}\right]\right] \tag{17}$$

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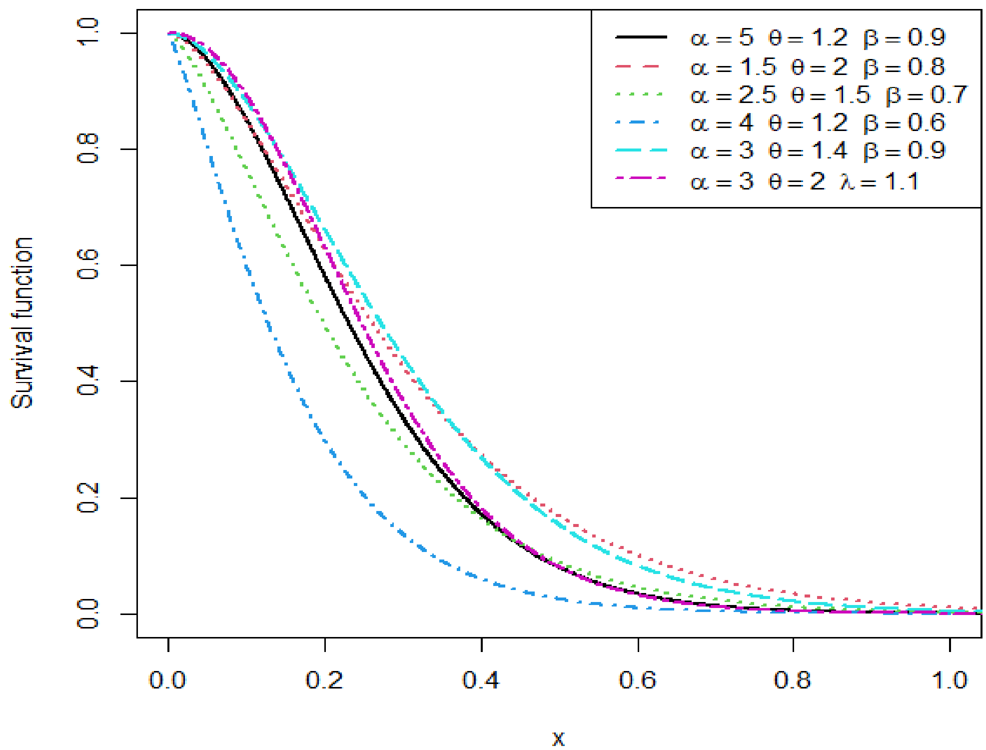

Figure 3: Plots of the survival function of STIIGTLE distribution with different parameter values

Hazard Function of the Stiigtle Distribution

The hazard function describes the instantaneous risk that an event occurs at a particular time, given that the subject has survived up to that time. It quantifies the rate at which failures happen over time and serves as a key tool for analyzing time-to-event data in reliability and survival studies, and it is given as:

$$T(x;\ \alpha,\beta,\theta)=\frac{f(x;\ \alpha,\beta,\theta)}{R(x;\ \alpha,\beta,\theta)} \tag{18}$$

$$T(x;\ \alpha,\beta,\theta)=\frac{\frac{\pi}{2}2\alpha\beta\theta e^{-\theta x}[1-e^{-\theta x}]^{2\beta-1}\left[1-[1-e^{-\theta x}]^{2\beta}\right]^{\alpha-1}Cos\left[\frac{\pi}{2}\left[1-\left[1-[1-e^{-\theta x}]^{2\beta}\right]^{\alpha}\right]\right]}{1-Sin\left[\frac{\pi}{2}\left[1-\left[1-[1-e^{-\theta x}]^{2\beta}\right]^{\alpha}\right]\right]} \tag{19}$$

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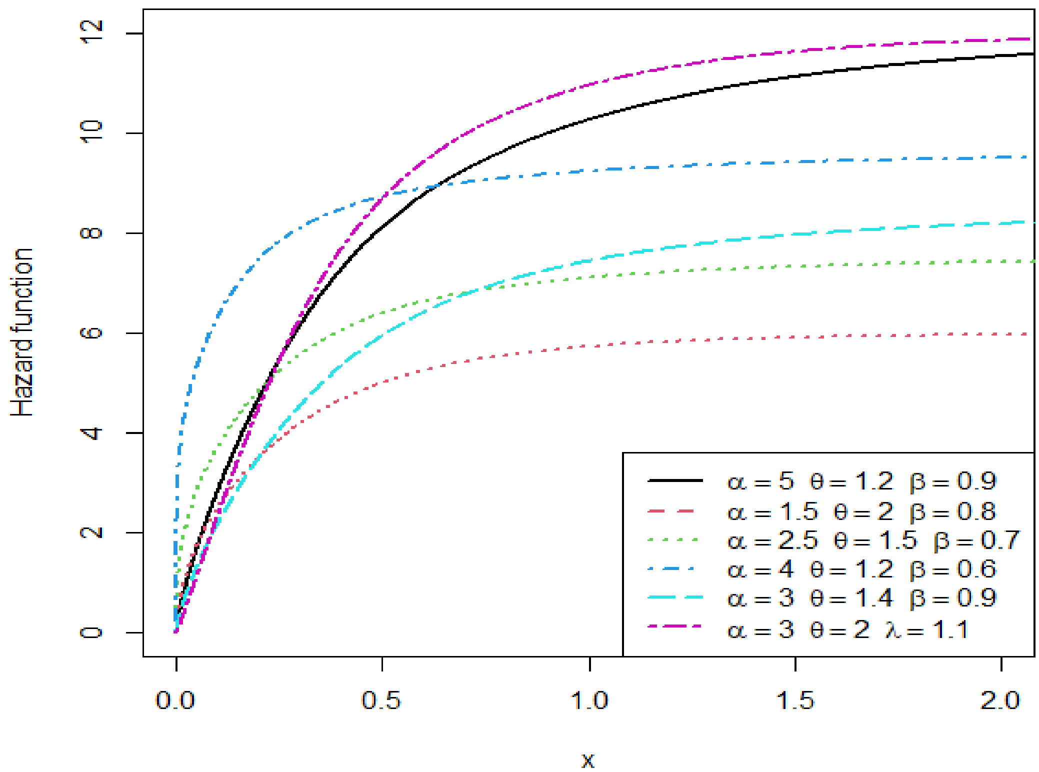

Figure 4: Plots of the hazard function of STIIGTLE distribution with different parameter values

Quantile Function of The Stiigtle Distribution

The quantile function, also referred to as the inverse cdf, of the STIIGTLE distribution is derived using the cdf given in equation (3).

$$F(x;\alpha,\beta,\theta) = Sin\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right]\right] = U$$

$$\left[1-\left[1-\left[1-e^{-\theta x}\right]^{2\beta}\right]^{\alpha}\right] = \frac{sin^{-1}(U)}{\frac{\pi}{2}}$$

$$\left[1-e^{-\theta x}\right]^{2\beta} = \left[1-\left[\frac{sin^{-1}(U)}{\frac{\pi}{2}}\right]\right]^{\frac{1}{\alpha}}$$

$$1-e^{-\theta x} = \left[1-\left[1-\left[\frac{sin^{-1}(U)}{\frac{\pi}{2}}\right]\right]^{\frac{1}{\alpha}}\right]^{\frac{1}{2\beta}}$$

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$$e^{-\theta x} = 1 - \left[1 - \left[1 - \left[\frac{sin^{-1}(U)}{\frac{\pi}{2}}\right]\right]^{\frac{1}{\alpha}}\right]^{\frac{1}{2\beta}}$$

$$-\theta x = log\left[1 - \left[1 - \left[\frac{sin^{-1}(U)}{\frac{\pi}{2}}\right]\right]^{\frac{1}{\alpha}}\right]^{\frac{1}{2\beta}}$$

$$x = \varrho(U) = \frac{1}{\theta}\left[-log\left[1 - \left[1 - \left[1 - \left[\frac{sin^{-1}(U)}{\frac{\pi}{2}}\right]\right]^{\frac{1}{\alpha}}\right]^{\frac{1}{2\beta}}\right]\right] \quad (20)$$

The median of the STIIGTLE distribution is obtained by setting u = 0:5 in equation (20), resulting in:

$$x = \varrho(0.5) = \frac{1}{\theta}\left[-log\left[1 - \left[1 - \left[1 - \left[\frac{sin^{-1}(0.5)}{\frac{\pi}{2}}\right]\right]^{\frac{1}{\alpha}}\right]^{\frac{1}{2\beta}}\right]\right] \quad (21)$$

Parameter Estimation of the STIIGTLE Distribution

In this section, we outline the methods for estimating the unknown parameters of the STIIGTLE distribution. Specifically, we explore two distinct approaches for parameter estimation.

Maximum Likelihood Estimation of the STIIGTLE Distribution

Let $x_1, x_2, ..., x_n$ represent a random sample of size n from the STIIGTLE distribution. The likelihood function corresponding to the vector of parameter $(a, b, q)^T$ can be expressed as:

$$logL = nlog\left(\frac{\pi}{2}\right) + nlog(2) + nlog(\alpha) + nlog(\beta) + nlog(\theta) - \theta\sum^{n} x_i + (2\beta - 1)\sum^{n} log\left[1 - e^{-\theta x_i}\right]$$
$$+(\alpha - 1)\sum_{i=1}^{n} log\left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right] + \sum_{i=1}^{n} log\left[Cos\left[\frac{\pi}{2}\left[1 - \left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right]^{\alpha}\right]\right]\right] \quad ((22)$$

By differentiating the log-likelihood (LL) with respect to α, β, θ and equating them to zero yields:

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} log\left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right] + \sum_{i=1}^{n}\frac{\pi}{2}\left[1 - \left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right]^{\alpha}\right]$$
$$log\left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right] Cos\left[\frac{\pi}{2}\left[1 - \left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right]^{\alpha}\right]\right] \quad (23)$$

$$\frac{\partial L}{\partial \beta} = \frac{n}{\beta} + 2\sum_{i=1}^{n} log\left[1 - e^{-\theta x_i}\right] - 2(\alpha - 1)\sum_{i=1}^{n}\frac{\left[1 - e^{-\theta x_i}\right]^{2\beta} log\left[1 - e^{-\theta x_i}\right]}{\left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right]}$$
$$-2\frac{\pi}{2}\alpha\sum_{i=1}^{n}\left[1 - \left[1 - e^{\theta x_i}\right]^{2\beta}\right]^{\alpha - 1}\left[1 - e^{\theta x_i}\right]^{2\beta} log\left[1 - e^{\theta x_i}\right] tan\left[\frac{\pi}{2}\left[1 - \left[1 - \left[1 - e^{-\theta x_i}\right]^{2\beta}\right]^{\alpha}\right]\right] \quad (24)$$

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$$\frac{\partial L}{\partial \theta}=\frac{n}{\theta}-\sum_{i=1}^{n}x_i+(2\beta-1)\sum_{i=1}^{n}\frac{e^{-\theta x_i}x_i}{[1-e^{-\theta x_i}]}-(\alpha-1)\sum_{i=1}^{n}\frac{2\beta\left[1-e^{-\theta x_i}\right]^{2\beta-1}e^{-\theta x_i}x_i}{[1-[1-e^{-\theta x_i}]^{2\beta}]}$$
$$-\sum_{i=1}^{n}\frac{\pi}{2}\alpha\left[1-\left[1-e^{\theta x_i}\right]^{2\beta}\right]^{\alpha-1}2\beta\left[1-e^{\theta x_i}\right]^{2\beta-1}e^{-\theta x_i}x_i tan\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x_i}\right]^{2\beta}\right]^{\alpha}\right]\right] \quad (25)$$

Equations (23), (24) and (25) above are non-linear and cannot be solved analytically, requiring the use of computational
methods obtain numerical solutions.

Maximum Product of Spacing Estimates of the Stiigtle Distribution

Suppose $x_1, x_2, \dots x_n$ represents a random samples from the STIIGTLE distribution have CDF $F(x;\alpha,\beta,\theta)$ and $x_1, x_2, \dots x_n$ be the corresponding ordered sample. Then the spacing:

$P_i = F(x_{(i)}) - F(x_{(i-1)}) \quad for \quad i = 1, 2, \dots n+1$

where

$$F(x_{(0)}) = 0 \quad and\ F(x_{(n+1)}) = 1 \quad (26)$$

Therefore

$$F(x_{(i)};\alpha,\beta,\theta) = Sin\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x_{(i)}}\right]^{2\beta}\right]^{\alpha}\right]\right] \quad (27)$$

$$F(x_{(i-1)};\alpha,\beta,\theta) = Sin\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x_{(i-1)}}\right]^{2\beta}\right]^{\alpha}\right]\right] \quad (28)$$

Substituting equations (27) and equation (28) into equation (26), we have:

$$P_i = \left[Sin\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x_{(i)}}\right]^{2\beta}\right]^{\alpha}\right]\right]\right] - \left[Sin\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x_{(i-1)}}\right]^{2\beta}\right]^{\alpha}\right]\right]\right] \quad (29)$$

The estimates of the parameters are obtained by maximizing equation (29)

$$T(x;\alpha,\beta,\theta) = \frac{1}{n+1}\sum_{i=1}^{n+1} log P_i \quad (30)$$

$$T(x;\alpha,\beta,\theta) = \frac{1}{n+1}\sum_{i=1}^{n+1} log\left\{\left[Sin\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x_{(i)}}\right]^{2\beta}\right]^{\alpha}\right]\right]\right] - \left[Sin\left[\frac{\pi}{2}\left[1-\left[1-\left[1-e^{-\theta x_{(i-1)}}\right]^{2\beta}\right]^{\alpha}\right]\right]\right]\right\} \quad (31)$$

Differentiating T with respect to each parameter provides the parameter estimates of $\hat{\alpha}_{MPS}, \hat{\beta}_{MPS}, \hat{\theta}_{MPS}$ and solving the resulting non-linear equations gives their values.

$$\frac{\partial T(x;\alpha,\beta,\theta)}{\partial \alpha} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{P_i}\left[D_{(1)}(x_{(i)};\alpha,\beta,\theta) - D_{(2)}(x_{(i-1)};\alpha,\beta,\theta)\right] \quad (32)$$

$$\frac{\partial T(x;\alpha,\beta,\theta)}{\partial \beta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{P_i}\left[R_1(x_{(i)};\alpha,\beta,\theta) - R_2(x_{(i-1)};\alpha,\beta,\theta)\right] \quad (33)$$

$$\frac{\partial T(x;\alpha,\beta,\theta)}{\partial \theta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{P_i}\left[V_1(x_{(i)};\alpha,\beta,\theta) - V_2(x_{(i-1)};\alpha,\beta,\theta)\right] \quad (34)$$

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The MPS are obtained by setting equation (32), (33) and (34) to zero and solving the resulting equations simultaneously. Since, these equations cannot be solved analytically, necessitating the use of computational techniques for numerical solution.

Results.

Simulation Study of the Stiigtle Distribution

In this analysis, 1000 samples were simulated from the STIIGTLE distribution through the quantile function defined in equation (20). The selected sample sizes were n=20, 50, 100, 250, 500, and 1000. These simulated datasets were then employed to estimate the model parameters and to calculate the correspondence bias and Root Mean Square Error (RMSE). The summarized results are presented in Tables 1 and 2. The Tables presents the parameter estimates obtained using we the MLE and MPS methods, together with their associated bias and root mean square error (RMSE), for the STIIGTLE at parameter values of α = 0.4, β =1, θ=0.5, and α = 0.5, β =1, θ=1.5, respectively. The results presented in the table show that increasing the sample size leads to a progressive reduction in both bias and RMSE, with these measures tending toward zero. This pattern indicates that the estimators improve in accuracy and stability as the sample size grows. Consequently, the parameter estimates can be regarded as efficient and consistent, since larger samples yield more precise estimation.

Table 1: MLEs, MPSs, biases and RMSE for some values of parameters

		Estimation Methods					
		MLE			MPS		
N	Parameters	Estimated Values	Bias	RMSE	Estimated Values	Bias	RMSE
20	$\alpha = 0.4$	0.4968	0.0968	0.3612	0.3891	-0.0109	0.2743
	$\beta = 1.0$	1.3376	0.3376	0.7796	1.0106	0.0106	0.5533
	$\theta = 0.5$	0.6380	0.1380	0.3765	0.6429	0.1429	0.3365
50	$\alpha = 0.4$	0.4493	0.0493	0.2524	0.3956	-0.0044	0.2076
	$\beta = 1.0$	1.1407	0.1407	0.4153	0.9897	-0.0103	0.3481
	$\theta = 0.5$	0.5860	0.0860	0.2718	0.5816	0.0816	0.2429
100	$\alpha = 0.4$	0.4244	0.0244	0.1811	0.4019	0.0019	0.1680
	$\beta = 1.0$	1.0737	0.0737	0.2312	0.9873	-0.0127	0.1916
	$\theta = 0.5$	0.5540	0.0540	0.1998	0.5482	0.0482	0.1911
250	$\alpha = 0.4$	0.4164	0.0164	0.1222	0.4101	0.0101	0.1217
	$\beta = 1.0$	1.0292	0.0292	0.1381	0.9885	-0.0115	0.1255
	$\theta = 0.5$	0.5206	0.0206	0.1388	0.5164	0.0164	0.1412
500	$\alpha = 0.4$	0.4107	0.0107	0.1003	0.4080	0.0080	0.0989
	$\beta = 1.0$	1.0153	0.0153	0.1001	0.9910	-0.0090	0.0930
	$\theta = 0.5$	0.5142	0.0142	0.1153	0.5095	0.0095	0.1126
1000	$\alpha = 0.4$	0.4112	0.0112	0.0813	0.4124	0.0124	0.0812
	$\beta = 1.0$	1.0043	0.0043	0.0703	0.9900	-0.0100	0.0671
	$\theta = 0.5$	0.5043	0.0043	0.0892	0.4988	-0.0012	0.0870

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Table 2: MLEs, MPSs, biases and RMSE for some values of parameters

		Estimation Methods					
		MLE			MPS		
N	Parameters	Estimated Values	Bias	RMSE	Estimated Values	Bias	RMSE
20	$\alpha = 0.5$	0.6208	0.1208	0.3985	0.4743	-0.0257	0.2810
	$\beta = 1.0$	1.2477	0.2477	0.6216	0.9667	-0.0333	0.4252
	$\theta = 1.5$	1.6937	0.1937	0.6747	1.7144	0.2144	0.5843
50	$\alpha = 0.5$	0.5457	0.0457	0.2569	0.4812	-0.0188	0.2057
	$\beta = 1.0$	1.1052	0.1052	0.3268	0.9700	-0.0300	0.2839
	$\theta = 1.5$	1.6560	0.1560	0.5745	1.6387	0.1387	0.4944
100	$\alpha = 0.5$	0.5158	0.0158	0.1829	0.4839	-0.0161	0.1616
	$\beta = 1.0$	1.0585	0.0585	0.1976	0.9804	-0.0196	0.1591
	$\theta = 1.5$	1.6169	0.1169	0.4460	1.6017	0.1017	0.3995
250	$\alpha = 0.5$	0.5027	0.0027	0.1235	0.4902	-0.0098	0.1155
	$\beta = 1.0$	1.0260	0.0260	0.1130	0.9884	-0.0116	0.1040
	$\theta = 1.5$	1.5672	0.0672	0.3195	1.5551	0.0551	0.3034
500	$\alpha = 0.5$	0.4936	-0.0064	0.0898	0.4873	-0.0127	0.0919
	$\beta = 1.0$	1.0155	0.0155	0.0793	0.9947	-0.0053	0.0755
	$\theta = 1.5$	1.5568	0.0568	0.2484	1.5532	0.0532	0.2523
1000	$\alpha = 0.5$	0.4919	-0.0081	0.0730	0.4913	-0.0087	0.0740
	$\beta = 1.0$	1.0095	0.0095	0.0574	0.9971	-0.0029	0.0549
	$\theta = 1.5$	1.5481	0.0481	0.2021	1.5381	0.0381	0.2023

Application of the New Model to Real-Life Data

To demonstrate the practical usefulness of the proposed distribution, it is fitted to real-life datasets. The goodness-of-fit of the model is compared with other related distributions using statistical measures such as, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Log-likelihood (LL) values. The datasets used in this research work are presented below.

Data Set 1

Data set 1, previously used by Gupta *et al.*, (2007) and Ismail *et al.* (2023), consists of the total skin folds of 202 athletes, collected at the Australian institute of sports, as follows:
28.0, 98, 89.0, 68.9, 109.0, 52.3, 52.8, 46.7, 82.7, 42.3, 109.1, 96.8, 98.3, 103.6, 110.2, 98.1, 57.0, 43.1, 71.1, 29.7, 96.3, 102.8, 80.3, 122.1, 71.3, 200.8, 80.6, 65.3, 78.0, 65.9, 38.9, 56.5, 104.6, 74.9, 90.4, 54.6, 131.9, 68.3, 52.0, 40.8, 34.3, 44.8, 105.7, 126.4, 83.0, 106.9, 88.2, 33.8, 47.6, 42.7, 41.5, 34.6, 30.9, 100.7, 80.3, 91.0, 156.6, 95.4, 43.5, 61.9, 35.2, 50.9, 31.8, 44.0, 56.8, 75.2, 76.2, 101.1, 47.5, 46.2, 38.2, 49.2, 49.6, 34.5, 37.5, 75.9, 87.2, 52.6, 126.4, 55.6, 73.9, 43.5, 61.8, 88.9, 31.0, 37.6, 52.8, 97.9, 111.1, 114.0, 62.9, 36.8, 56.8, 46.5, 48.3, 32.6, 31.7, 47.8, 75.1, 110.7, 70.0, 52.5, 67, 41.6, 34.8, 61.8, 31.5, 36.6, 76.0, 65.1, 74.7, 77.0, 62.6, 41.1, 58.9, 60.2, 43.0, 32.6, 48, 61.2, 171.1, 113.5, 148.9, 49.9, 59.4, 44.5, 48.1, 61.1, 31.0, 41.9, 75.6, 76.8, 99.8, 80.1, 57.9, 48.4, 41.8, 44.5, 43.8, 33.7, 30.9, 43.3, 117.8, 80.3, 156.6, 109.6, 50.0, 33.7, 54.0, 54.2, 30.3, 52.8, 49.5, 90.2, 109.5, 115.9, 98.5, 54.6, 50.9, 44.7, 41.8, 38.0, 43.2, 70.0, 97.2, 123.6, 181.7, 136.3, 42.3, 40.5, 64.9, 31.1, 55.7, 113.5, 75.7, 99,9, 91.2, 71.6, 103.6, 46.1, 51.2, 43.8, 30.5, 37.5, 96.9, 57.7, 125.9, 49.0, 143.5, 102.8, 46.3, 54.4, 58.3, 34.0, 112.5, 49.3, 67.2, 56.5, 47.6, 60.4, 34.9.

Fitting the Sine Type II Generalized Topp-Leone Exponential (STIIGTLE) Distribution

Here, the NSTIIGTLE distribution is applied to analyze both data set 1. An extensive comparison is performed by fitting the NSTIIGTLE distribution against other alternative competing distributions like TIHLEtEx, ToLEx, KEx and EtEx. The comparison seeks to highlight the adaptability and appropriateness of the new distribution and assess its fit to the experimental data sets compared to the comparative models. Both Maximum Likelihood Estimate (MLE) and Method of Maximum Product of Spacing (MPS) approach are applied, and computations are carried out using R, to

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ensure efficiency and simplicity.

The pdf of the distributions being compared are given by:

- Type I Half-Logistic Exponentiated Exponential (TIHLEtE) Distribution

$$f(x;\lambda,\alpha,\theta)=\frac{2\lambda\alpha\theta e^{-\theta x}\left[1-e^{-\theta x}\right]^{\alpha-1}\left[1-\left[1-e^{-\theta x}\right]^{\alpha}\right]^{\lambda-1}}{[1+[1-[1-e^{-\theta x}]^{\alpha}]^{\lambda}]^{2}}$$

- Topp-Leone Exponential (ToLE) Distribution
  $f(x;\alpha,\theta)=2\alpha\theta e^{-2\alpha x}[1-e^{-2\alpha x}]^{\theta-1}$
- Kumaraswamy Exponential (KEx) Distribution
  $f(x;\alpha,\lambda,\theta)=\alpha\lambda\theta e^{-\alpha x}(1-e^{-\alpha x})[1-[1-e^{-\alpha x}]^{\alpha}]^{\lambda-1}$
- Exponentiated Exponential (ExEx) Distribution

$$f(x;\alpha,\theta)=\alpha\theta e^{-\alpha x}[1-e^{-\alpha x}]^{\theta-1}$$

Comparison Results with the Competing Distributions

This section presents a comparison using the baseline exponential distribution, aiming to examine the impact of additional parameters on the distribution's applicability, effectiveness, and flexibility.

The MLEs, Log-likelihoods and Goodness of Fits Statistics of the models based on data set 1

Model	α	θ	λ	β	LL	AIC	BIC
NSTIIGTE	0.2720	0.0636	-	6.3719	-963.2616	1932.5230	1942.4480
TIHLEtEx	14.6653	0.0781	0.4881	-	-964.5330	1935.0660	1944.9910
ToLEx	5.4106	-	0.0168	-	-966.1600	1936.3200	1942.9370
KEx	4.6974	1.8955	0.0238	-	-966.0516	1938.1030	1948.0280
EtEx	0.0351	-	5.5725	-	-965.7797	1935.5590	1943.1760

According to Table 3, presents the Maximum Likelihood Estimation results for the NSTIIGTLE distribution alongside four other competing distributions, with the NSTIIGTLE distribution achieving the minimum AIC value of 1932.5230, BIC value of 1944.9910 and the highest LL value -963.2616. This shows that the NSTIIGTLE distribution provides the "best fit" and is the most suitable model for the dataset 1, as it outperforms the other competing distributions assessed based on the goodness of fits criteria, using the AIC, BIC and LL statistic.

Table 4: The MPSs, Log-likelihoods and Goodness of Fits Statistics of the models based on data set 1

Model	α	θ	λ	β	LL	AIC	BIC
NSTIIGTE	0.2855	0.0597	-	5.5272	-963.3895	1932.7790	1942.2080
TIHLEtEx	12.4534	0.0736	0.5092	-	-964.6751	1935.3500	1945.2750
ToLEx	6.9577	-	0.0188	-	-966.8036	1937.6070	1944.2240
KEx	11.7279	0.5910	0.0566	-	-966.8151	1939.6300	1949.5550
EtEx	0.0377	-	6.9741	-	-965.7955	1935.5910	1942.7040

According to Table 4, the Maximum Product of Spacing Estimates results for the NSTIIGTLE distribution and four other competing distributions are presented. Among these distributions, the NSTIIGTLE distribution has the minimum AIC value of 1932.7790, BIC value of 1942.2080 and the highest LL value -963.4895. This shows that the NSTIIGTLE distribution provides the best fit and is the most suitable model for the dataset 1. The distribution's superior AIC value, BIC value and LL value, show that it outperforms the other four competitive distributions in terms of goodness-of-fit, making it a more appropriate choice for this dataset.

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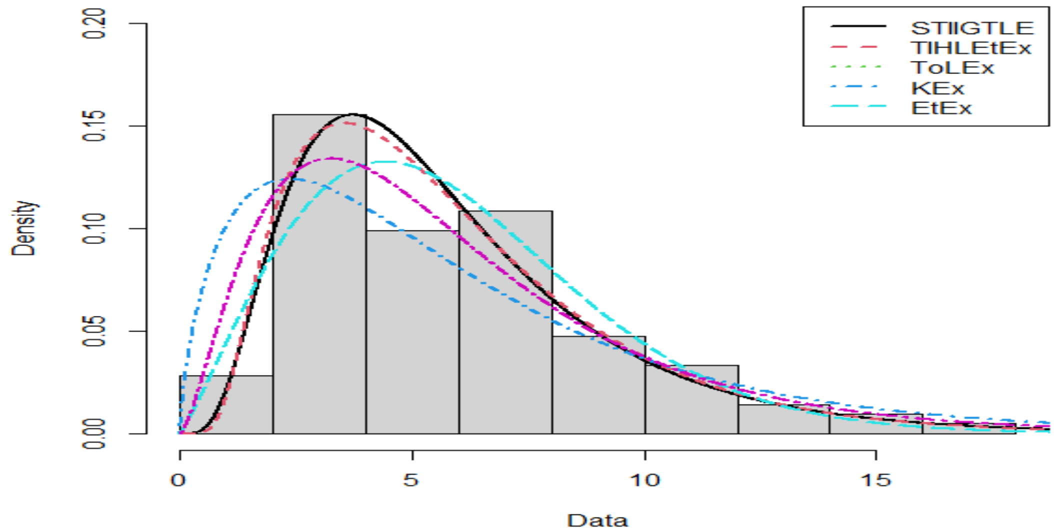

Figure 5: Fitted pdfs for the NSTIIGTLE, TIHLEtEx, ToLEx, KEx and EtEx models to the data set 1.

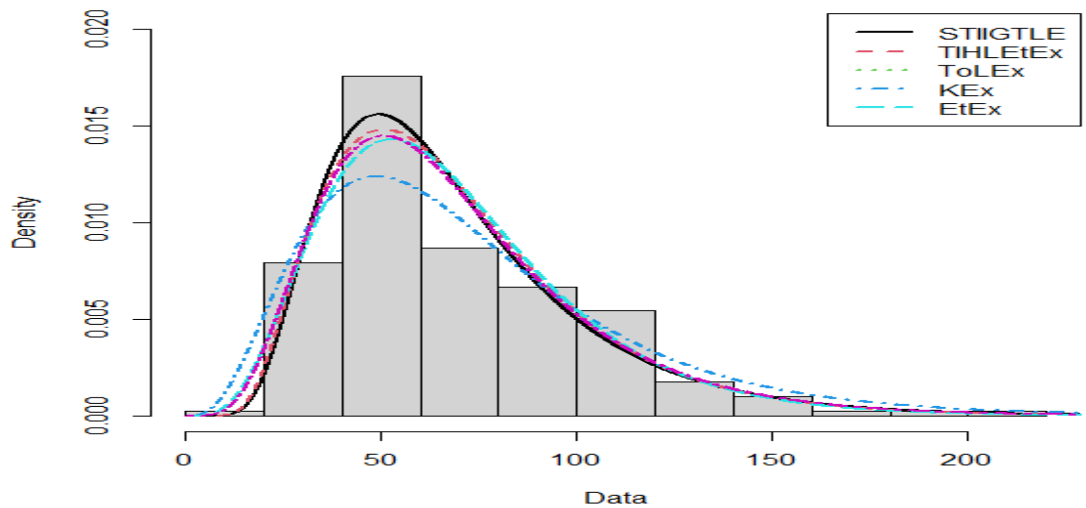

Figure 6: Fitted pdfs for the NSTIIGTLE, TIHLEtEx, ToLEx, KEx and EtEx models to the data set 2.

Figures 5 and 6 provide a visual representation of the NSTIIGTLE distribution alongside its competitive distributions. From this visual inspection, it is clear that the NSTIIGTLE distribution provide a better fit to the data than the other distributions. This visual evidence further reinforces the conclusion that the NSTIIGTLE distribution is superior in accurately modeling the datasets under study.

Summary

This research focused on constructing and examining the theoretical and practical aspects of a new probability model known as the Sine Type II Generalized Topp-Leone Exponential Distribution. The Development of this model was

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driven by the inadequately of standard lifetime distributions, particularly the exponential distribution, in representing datasets that exhibit asymmetry, variability in dispersion, and non-constant failure behavior.
Key mathematical characteristics of the model were derived as foundation for understanding the behavior of the distribution under different conditions.
To assess its practical relevance, the model was applied to real-world data involving total skinfold measurements of 202 athletes sourced from the Australian Institute of Sport. The outcome of the application showed that the proposed distribution was able to capture the underlying structure of the dataset more accurately than several conventional models, demonstrating the usefulness in empirical data analysis.

Conclusion

The formulation of the Sine Type II Generalized Topp-Leone Exponential Distribution adds to the expanding literature on flexible statistical models. By incorporating a sine-based transformation into the generalized Topp-Leone structure, combined with an exponential baseline, the resulting distribution gains enhanced adaptability in modeling diverse data patterns.
The theoretical investigation confirmed that the model possesses well-defined statistical properties and is suitable for analyzing data with non-standard shapes, which are often encountered in applied research.
The empirical application further supports its usefulness, as the model demonstrated improved fitting performance when applied to the athlete skinfold dataset. This indicates that the distribution is not only theoretically valid but also practically effective in handling real-world data.

Recommendations

Based on the outcome of this research, the following suggestions are offered:

1. The model should be considered when analyzing datasets that displays skewness, irregular variability, or non-constant hazard behavior.
2. Scholars in applied statistics, biomedical research, and sport science are encourage to test the model on different datasets to further establish its robustness and versatility.
3. Future work should consider alternative estimation procedures such as Bayesian inference, bootstrap methods, to improve parameter estimation accuracy.

Compliance with ethical standards

Disclosure of Conflicts of Interest

No conflict of interest to be disclosed

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