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Mathematical Analysis of Yeast Species Using the Laplace Decomposition Method

[*1]Abanum, G. C., & [2]Eli, I. C.,
[1]Department of Mathematics, Dennis Osadebay University, Asaba, Nigeria
[2]Department of Mathematics/Statistics, Federal University Otuoke, Yenagoa, Nigeria

[*]Corresponding author email: abanum.godspower@dou.edu.ng

Abstract
This paper examined a mathematical model for the growth of yeast species 1 and 2 through numerical analysis. The approximate solutions in the form of infinite series were obtained by applying the Laplace Decomposition Method. With the use of the Maple 18 program, numerical justification was carried out on the model parameter values to produce the desired outcomes. According to the findings, yeast species 1 and 2 were shown to decline when growth rate coefficients dropped from 50% to 90%, and increased when growth rate coefficients increased from 120% to 160%. When yeast species 1 and 2's growth rate coefficients dropped from 50%, 70%, and 90%, a loss in biodiversity was seen; similarly, when their growth rate coefficients increased from 120%, 140%, and 160%, a gain in biodiversity was observed.

Keywords: Mathematical model, Laplace Decomposition Agricultural Assets, Biomass, Infinite Series

Introduction

Dynamical systems originate from a system of functional differential equations of neutral type or hyperbolic partial differential equations (Hale, 1969). However, according to Rajendra (2021), dynamical systems are defined as the study of the long-term behaviour of evolving systems. The modern theory of dynamical systems originated at the end of the 19th century with fundamental questions concerning the stability and evolution of the solar system. Attempts to answer those questions led to the development of a rich and powerful field with applications to physics, biology, meteorology, astronomy, and other fields. Despite its origins in Newtonian mechanics, the concept of dynamical systems has been thoroughly explored by various researchers in recent years due to its profound impact on natural science and engineering.

Eli and Abanum (2020) compared the analytical and numerical results of a dynamic system's stability analysis. Using a mathematical model of biology, they developed a system of ordinary differential equations involving sickle cell, HIV, and T cells. By employing an analytical characteristic equation, the eigenvalues were derived to test for the trivial steady-state solution or points. Lastly, they ran a numerical simulation to investigate the reliability of the outcome. Godspower et al. (2020) focused on the investigation of typical agricultural assets' biodiversity. They used numerical modelling techniques, specifically ODE 45, to determine how biodiversity would grow as a result of variations in the growth rates. Similar research was conducted by (Solomonovic et al. 1998; Bertoin, 2016), who proposed recovery models of economic agriculture-industry interaction to examine the features of bifurcation and stability as well as the persistence and extinction of economic and environmental wealth. The author applied numerical analysis and a special situation of negative recovery. Eward and Ford, (2002) investigated the stability and bounds of differential equations. Their study addresses the stability and boundedness of solutions as they relate to the qualitative behaviour of solutions to differential equations.

Solomonovic et al. (1998) investigated the stability analysis problem for a new class of discrete-time recurrent neural networks with mixed time delays. The mixed time delays, which are composed of both discrete and distributed time delays, are addressed for the first time when analyzing the asymptotic stability for discrete-time neural networks. The existence of the equilibrium point was first proven under mild conditions by constructing a new Lyapnuov-Krasovskii functional, and a linear matrix inequality (LMI) approach was developed to determine sufficient conditions for the discrete-time neural networks to be globally asymptotically stable. As an extension, they also take into consideration the stability analysis problem for the same class of neural networks but with state-dependent stochastic problems. Rajendra (2021) reviewed the standard dynamical system approach to biomedical science. Their research aims to address the needs of both the present and the future for the interaction of different scientific and technological fields with dynamical systems. They talk about a variety of models for interacting populations, including discrete population models for single species, continuous population models for single species, Lotka-Voltera systems, and realistic predator-prey models. They create models that encapsulate the key elements of diverse interactions, facilitating a deeper comprehension of the results. Analyzing biomedical concerns through one of the most significant branches of mathematics provides a comprehensive viewpoint using numerical simulation methods with ODE45.

Eli and Ekaka-a (2021) used ODE45 numerical simulation tools to investigate how discrete time delays affect a dynamical system's stability. The outcome demonstrated the overwhelming instability of the dynamical system. Forecasting the expansion of yeast species was not taken into consideration in the research contributions of (Abanun et al., 2024; Liu et al., 2009; Yan & Ekaka-a, 2011; Suha et al., 2023; Anya et al., 2022; Najib & Hassan, 2021) and many other researchers in the fields of ecological modelling and mathematical biology. In light of this, the Laplace Adomian Decomposition Method (LADM) is being used to investigate the effect of decreasing and increasing the growth rate coefficients of yeast species 1 and 2. Finding the numerical solution for the differential model system and solving non-linear dynamical systems are two tasks that the Laplace Adomian Decomposition Method (LADM) can achieve effectively and simply (Bazuaye & Omoregbe, 2022, Abanum et al., 2024). It was Adomain, (1988) who first proposed this method. This study uses a method that breaks down a solution into an infinite series that quickly converges to the exact solution.

Model Assumptions

For this study, we shall consider the following assumptions:

i. The growth of yeast species 1 and yeast species 2 depends on the difference between the survival rate and the death rate.
ii. The interaction within each species, also known as the self-interaction coefficient of the intra-competition process, may also have an impact on the expansion of these two species.
iii. The inter-competition coefficient, which indicates how much each species contributes to preventing the growth of another, may also have an impact on the expansion of these two species.
iv. The beginning data values of yeast species 1 and yeast species 2, when the growing season is measured in weeks, may also have an impact on the growth of these two competing species.

Mathematical Formulations

We have taken into consideration the multi-parameter continuous dynamical system of a nonlinear first-order Ordinary Differential Equation (Eli & Ekaka-a, 2021)for this study.

$$\frac{dx}{dt} = \propto_1 x - \beta_1 x^2 - \gamma_1 xy \quad 1$$
$$\frac{dy}{dt} = \propto_2 y - \beta_2 y^2 - \gamma_2 xy \quad 2$$

$x(t)$ denotes the biomass of yeast specy 1 (candida albican) at time t in the unit of weeks.
$y(t)$ denotes the biomass of yeast specy 2 (candida parapsilosis) at time t in the unit of weeks.
$\propto_1$ and $\propto_2$ specify the growth rate of yeast species 1 and 2 respectively.
β_1 and β_2 specify the intra-competition coefficient of yeast species 1 and 2 respectively.
γ_1 and γ_2 denote the competition of yeast species 1 and yeast 2 respectively where γ_1 is the contribution of the yeast species to inhibit the growth of species 2 as γ_2 is the contribution of the yeast species 2 to inhibit the growth of species 1.
At a unique positive steady-state solution,

$$\frac{dx}{dt} = \frac{dy}{dt} = 0,$$

So that equation (1) and (2) becomes

$$\propto_1 x - \beta_1 x^2 - \gamma_1 xy = 0 \quad 3$$

$$\propto_2 y - \beta_2 y^2 - \gamma_2 xy = 0 \quad 4$$

From equ (1)

$x(\propto_1 - \beta_1 x - \gamma_1 y) = 0 \Longrightarrow x = 0$ or

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$\propto_1 - \beta_1 x - \gamma_1 y = 0$

$\therefore \beta_1 x = \propto_1 - \gamma_2 y$

$\therefore x = \frac{\propto_1}{\beta_1} - \frac{\gamma_1 y}{\beta_1}$ 5

Similarly, from equation 4

$y(\propto_2 - \beta_2 y - \gamma_2 x) = 0$

$\Longrightarrow y = 0$

or

$\propto_2 - \beta_2 y - \gamma_2 x = 0$

$\therefore \beta_2 y = \propto_2 - \gamma_2 x$

$\therefore y = \frac{\propto_2}{\beta_2} - \frac{\gamma_2 x}{\beta_2}$ 6

Now, substituting (5) into (6) we get

$y = \frac{\propto_2}{\beta_2} - \frac{\gamma_2}{\beta_2}\left(\frac{\propto_1}{\beta_2} - \frac{\gamma_1 y}{\beta_1}\right)$ 7

$$\therefore y = \frac{\propto_2}{\beta_2} - \frac{\gamma_2 \propto_1}{\beta_1\beta_2} + \frac{\gamma_1\gamma_2 y}{\beta_1\beta_2}$$

$$\Longrightarrow y - \frac{\gamma_1\gamma_2 y}{\beta_1\beta_2} = \frac{\propto_2}{\beta_2} - \frac{\gamma_2 \propto_1}{\beta_1\beta_2}$$

$$y\left(1 - \frac{\gamma_1\gamma_2 y}{\beta_1\beta_2}\right) = \frac{\propto_2 \beta_1 - \gamma_2 \propto_1}{\beta_1\beta_2}$$

$$y\left(\frac{\beta_1\beta_2 - \gamma_1\gamma_2}{\beta_1\beta_2}\right) = \frac{\propto_2 \beta_1 - \gamma_2 \propto_1}{\beta_1\beta_2}$$

$$y(\beta_1\beta_2 - \gamma_1\gamma_2) = \propto_2 \beta_1 - \gamma_2 \propto_1$$

$\therefore y = \frac{\propto_2\beta_1 - \gamma_2\propto_1}{\beta_1\beta_2 - \gamma_1\gamma_2}$ 8

To get x, put 8 into 5, so that

$x = \frac{\propto_1}{\beta_1} - \frac{\gamma_1}{\beta_1}\left(\frac{\propto_2\beta_1 - \gamma_2\propto_2}{\beta_1\beta_2 - \gamma_1\gamma_2}\right)$ 9

Thus, the trivial steady state solution is $(x, y) = (0, 0)$, while the non-trivial steady state solution is when $x \neq 0$ and $y \neq 0$

$\therefore (x, y)\left\{\left[\frac{\propto_1}{\beta_1} - \frac{\gamma_1}{\beta_1}\left(\frac{\propto_2\beta_1 - \gamma_2\propto_2}{\beta_1\beta_2 - \gamma_1\gamma_2}\right)\right], \left[\frac{\propto_2\beta_1 - \gamma_2\propto_1}{\beta_1\beta_2 - \gamma_1\gamma_2}\right]\right\}$ 10

Applications of the Laplace Decomposition Method

Laplace transform is a mathematical tool used to convert a system of differential equations to a system of algebraic equations, (Abanum et al, 2024). It transforms a variable (such as x, y in space or at time t) to a parameter(s) a 'constant' under certain conditions. It transforms one variable at a time. Applying the Laplace transform on both sides of the model (1) to (2) above, we obtain the system of equations.

$$\mathcal{L}\left[\frac{dx}{dt}\right] = \mathcal{L}[\alpha_1 x - \beta_1 x^2 - \gamma_1 xy] \quad 11$$

$$\mathcal{L}\left[\frac{dx}{dt}\right] = Sx(t) - S(0) = S\mathcal{L}x[t] - x(0) \quad 12$$

$$S\mathcal{L}x[t] - x(0) = \alpha_1\mathcal{L}[x] - \beta_1\mathcal{L}[x]^2 - \gamma_1\mathcal{L}[xy]$$

$$S\mathcal{L}x[t] = x(0) + \alpha_1\mathcal{L}[x] - \beta_1\mathcal{L}[x]^2 - \gamma_1\mathcal{L}[xy]$$

$$\mathcal{L}x[t] = \frac{x(0)}{s} + \frac{\alpha_1}{s}\mathcal{L}[x] - \frac{\beta_1}{s}\mathcal{L}[x]^2 + \frac{\gamma_1}{s}\mathcal{L}[xy] \quad 13$$

Assume $\mathcal{L}x[t]$ as an infinite series

$$\mathcal{L}x[t] = \mathcal{L}\left[\sum_{i=0}^{\infty} x(t)\right] \quad 14$$

Also, we decompose the nonlinearity solution using the Adomian technique

$$xy = \frac{1}{i!}\frac{d^i}{\lambda^i}\left[\sum_{j=0}^{i}\lambda^j x_j \sum_{j=0}^{i}\lambda^j y_j\right]_{\lambda=0} = \sum_{i=0}^{\infty} A_i \quad 15$$

For $\quad i = 0, \quad j = 0$

$$A_0 = x_0(t)y_0(t)$$

For $\quad i = 1, \quad j = 0,\ 1$

$$\left.\begin{aligned} A_1 &= x_0(t)y_1(t) + x_1(t)y_0(t) \\ A_2 &= x_0(t)y_2(t) + x_1(t)y_1(t) + x_2(t)y_0(t) \\ A_3 &= x_0(t)y_3(t) + x_1(t)y_2(t) + x_2(t)y_1(t) + x_3(t)y_0(t) \end{aligned}\right\} \quad 16$$

Substituting into equ 13

$$\mathcal{L}\left[\sum_{i=0}^{\infty} x(t)\right] = \frac{x(0)}{s} + \frac{\alpha_1}{s}\mathcal{L}\left[\sum_{i=0}^{\infty} x(t)\right] - \frac{\beta_1}{s}\mathcal{L}\left[\sum_{i=0}^{\infty} x(t)\right]^2 - \frac{\gamma}{s}\mathcal{L}\left[\sum_{i=0}^{\infty} A_i\right]$$

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$using\ initial\ condition\ x(0) \geq 0 \geq n_1$

$$\mathcal{L}\left[\sum_{i=0}^{\infty} x(t)\right] = \frac{n_i}{s} + \frac{\alpha_1}{s}\mathcal{L}\left[\sum_{i=0}^{\infty} x(t)\right] - \frac{\beta_1}{s}\mathcal{L}\left[\sum_{i=0}^{\infty} x(t)\right]^2 - \frac{\gamma}{s}\mathcal{L}\left[\sum_{i=0}^{\infty} A_i\right] \quad 17$$

$$\mathcal{L}[x_0] = \frac{n_i}{s} x_0 = n_1$$

$$\mathcal{L}[x_1] = \frac{\alpha_1}{s\psi}\mathcal{L}[x_0] - \frac{\beta_1}{s\psi}\mathcal{L}[x_0]^2 - \frac{\gamma}{s\psi}\mathcal{L}[A_0]$$

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$\mathcal{L}[x_1] = \frac{\alpha}{S^{\psi}}\mathcal{L}[x_1] - \frac{\beta_1}{S^{\psi}}\mathcal{L}[x_1]^2 - \frac{\gamma}{S^{\psi}}\mathcal{L}[A_1]$

⋮

$$\mathcal{L}[x_{k+1}] = \frac{\alpha_1}{S^4}\mathcal{L}[x_k] - \frac{\beta_1}{S^4}\mathcal{L}[x_k]^2 - \frac{\gamma}{S^4}\mathcal{L}[A_k]$$

To get the solutions x_1, x_2

To get x_1

Using

$$\mathcal{L}[x_1] = \frac{\alpha_1}{S^4}\mathcal{L}[x_0] + \frac{\beta_1}{S^4}\mathcal{L}[x_0]^2 - \frac{\gamma_1}{S^4}\mathcal{L}[A_0]$$

$x_0 = n_1\ ,\ y_0 = n_2$

$A_0 = x_0 y_0 = n_1 n_2$

Then, we have

$$\mathcal{L}[x_1] = \frac{\alpha_1}{S^{\psi}}\mathcal{L}[n_1] - \frac{\beta_1}{S^{\psi}}\mathcal{L}[n_1]^2 - \frac{\gamma_1}{S^{\psi}}\mathcal{L}[n_1 n_2]$$

Taking the inverse Laplace transform

$$\mathcal{L}^{-1}\big[\mathcal{L}[x_1]\big] = \mathcal{L}^{-1}\left(\frac{\alpha_1 n_1}{S^{\psi}}\mathcal{L}[1] - \frac{\beta_1 {n_1}^2}{S^{\psi}}\mathcal{L}[1] - \frac{\gamma_1 n_1 n_2}{S^{\psi}}\mathcal{L}[1]\right)$$

$$x_1 = \mathcal{L}^{-1}\left[\frac{\alpha_1 n_1}{S^{\psi}}\cdot\frac{1}{S} - \frac{\beta_1 {n_1}^2}{S^{\psi}}\cdot\frac{1}{S} - \frac{\gamma_1 n_1 n_2}{S^{\psi}}\cdot\frac{1}{S}\right]$$

Since $\mathcal{L}[1] = \frac{1}{S}$

$$x_1 = \mathcal{L}^{-1}\left[\frac{\alpha_1 n_1}{S^{\psi+1}} - \frac{\beta_1 {n_1}^2}{S^{\psi+1}} - \frac{\gamma_1 n_1 n_2}{S^{\psi+1}}\right]$$

$$x = \alpha_1 n_1 \mathcal{L}^{-1}\left(\frac{1}{S^{\psi+1}}\right) - \beta_1 {n_1}^2 \mathcal{L}^{-1}\left(\frac{1}{S^{\psi+1}}\right) - n_1 n_2 \mathcal{L}^{-1}\left(\frac{1}{S^{\psi+1}}\right)$$

From the Laplace property, $\mathcal{L}\big[t^{\psi}\big] = \frac{\psi!}{S^{\psi+1}} \Longrightarrow \mathcal{L}^{-1}\left[\frac{1}{S^{4+1}}\right] = \frac{t^{\psi}}{\psi!}$

Hence

$$x_1 = \alpha_1 n_1 \frac{t^{\psi}}{\psi!} - \beta_1 {n_1}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 n_1 n_2 \frac{t^{\psi}}{\psi!} \qquad 18$$

Similarly,

$$y_1 = \alpha_2 n_2 \frac{t^{\psi}}{\psi!} - \beta_2 {n_2}^2 \frac{t^{\psi}}{\psi!} - \gamma_2 n_1 n_2 \frac{t^{\psi}}{\psi!} \qquad 19$$

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$$x_2 = \alpha_1 \cdot x_1 \cdot \frac{t^{\psi}}{\psi!} - \beta_1 (x_1 \cdot x_1) \cdot \frac{t^{\psi}}{\psi!} - \gamma_1 (A_1) \cdot \frac{t^{\psi}}{\psi!} \quad 20$$

$$y_2 = \alpha_2 \cdot y_1 \cdot \frac{t^{\psi}}{\psi!} - \beta_2 (y_1 \cdot y_1) \frac{t^{\psi}}{\psi!} - \gamma_2 (x_1 y_1) \cdot \frac{t^{\psi}}{\psi!} \quad 21$$

$$x_3 = \alpha_1 \cdot x_2 \cdot \frac{t^{\psi}}{\psi!} - \beta_1 (x_2 \cdot x_2) \frac{t^{\psi}}{\psi!} - \gamma_1 (x_2 y_2) \cdot \frac{t^{\psi}}{\psi!} \quad 22$$

$$y_3 = \alpha_2 \cdot y_2 \cdot \frac{t^{\psi}}{\psi!} - \beta_2 (y_2 \cdot y_2) \frac{t^{\psi}}{\psi!} - \gamma_2 (x_2 y_2) \cdot \frac{t^{\psi}}{\psi!} \quad 23$$

With the following precise model parameters from Eli and Ekaka-a, (2021)

$\alpha_1 = 0.1,\ \beta_1 = 0.0014,\ \gamma_1 = 0.0012,\ \alpha_2 = 0.08,\ \beta_2 = 0.001, \gamma_2 = 0.0009$

$x = x_1 + x_2 + x_3$

$$X = 0.1872\,t + 0.01872\,t^2 - 0.000099335808\,t^3 + 0.1\left(0.01872\,t^2 - 0.000099335808\,t^3\right) t - 0.0014\left(0.01872\,t^2 - 0.000099335808\,t^3\right)^2 t - 0.0012\left(0.01872\,t^2 - 0.000099335808\,t^3\right)\left(0.017904\,t^2 - 0.000100360872\,t^3\right) t$$

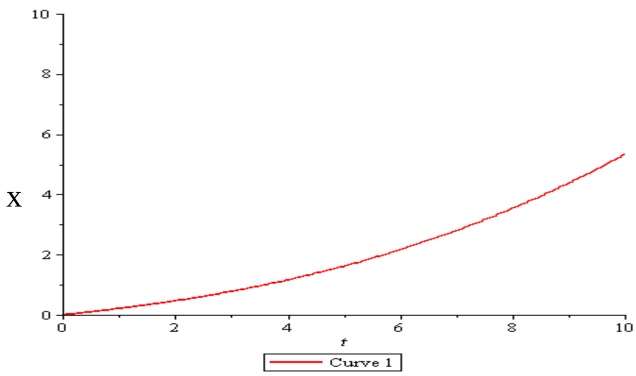

Figure 1. *Plot of numerical solution of yeast specy 1 biomass x(t) corresponding to a different time (t) in years*

$y = y_1 + y_2 + y_3$

$$Y = 0.2238\,t + 0.017904\,t^2 - 0.000100360872\,t^3 + 0.08\left(0.017904\,t^2 - 0.000100360872\,t^3\right) t - 0.001\left(0.017904\,t^2 - 0.000100360872\,t^3\right)^2 t - 0.0012\left(0.01872\,t^2 - 0.000099335808\,t^3\right)\left(0.017904\,t^2 - 0.000100360872\,t^3\right) t$$

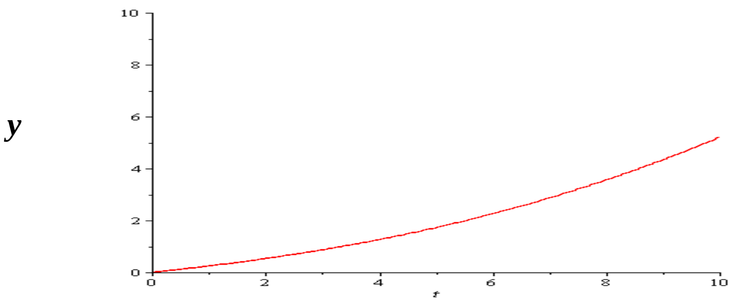

Figure 2. *Plot of numerical solution of yeast specy 2 biomass y(t) corresponding to a different time (t) in years*

$\alpha_1 = 0.1 * 0.50$

$$x_1 = \alpha_1 n_1 \frac{t^\psi}{\psi!} - \beta_1 {n_1}^2 \frac{t^\psi}{\psi!} - \gamma_1 n_1 n_2 \frac{t^\psi}{\psi!}$$

$X1 := 0.0872\, t$

$$x_2 = \alpha_1 x_1 \frac{t^\psi}{\psi!} - \beta_1 {x_1}^2 \frac{t^\psi}{\psi!} - \gamma_1 x_1 y_1 \frac{t^\psi}{\psi!}$$

$X2 := 0.0043600\, t^2 - 0.000034063808\, t^3$

$$x_3 = \alpha_1 x_2 \frac{t^\psi}{\psi!} - \beta_1 {x_2}^2 \frac{t^\psi}{\psi!} - \gamma_1 x_2 y_2 \frac{t^\psi}{\psi!}$$

$$X3 := 0.050\,(0.0043600\, t^2 - 0.000034063808\, t^3)\, t - 0.0014\,(0.0043600\, t^2 - 0.000034063808\, t^3)^2\, t - 0.0012\,(0.0043600\, t^2 - 0.000034063808\, t^3)\,(0.017904\, t^2 - 0.000100360872\, t^3)\, t$$

$$x = x_1 + x_2 + x_3$$

$$X := 0.0872\, t + 0.0043600\, t^2 - 0.000034063808\, t^3 + 0.050\,(0.0043600\, t^2 - 0.000034063808\, t^3)\, t - 0.0014\,(0.0043600\, t^2 - 0.000034063808\, t^3)^2\, t - 0.0012\,(0.0043600\, t^2 - 0.000034063808\, t^3)\,(0.017904\, t^2 - 0.000100360872\, t^3)\, t$$

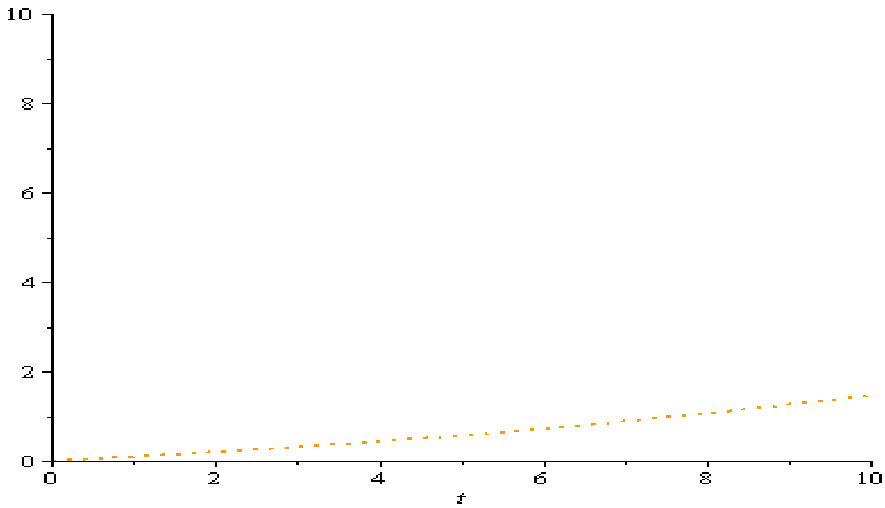

Figure 3. *Plot of numerical solution of yeast specy 1 biomass x(t) corresponding to a different time (t) in years when* α_1 *is varied at* 50%

$\alpha 1 := 0.1 \cdot 0.70 :$

$$x_1 = \alpha_1 n_1 \frac{t^\psi}{\psi!} - \beta_1 {n_1}^2 \frac{t^\psi}{\psi!} - \gamma_1 n_1 n_2 \frac{t^\psi}{\psi!}$$

$X1 := 0.1272\, t$

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$$x_2 = \alpha_1 x_1 \frac{t^\psi}{\psi!} - \beta_1 {x_1}^2 \frac{t^\psi}{\psi!} - \gamma_1 x_1 y_2 \frac{t^\psi}{\psi!}$$

$X2 := 0.0089040\, t^2 - 0.000056812608\, t^3$

$$x_3 = \alpha_1 x_2 \frac{t^\psi}{\psi!} - \beta_1 {x_2}^2 \frac{t^\psi}{\psi!} - \gamma_1 x_2 y_2 \frac{t^\psi}{\psi!}$$

$$X3 := 0.070\,(0.0089040\, t^2 - 0.000056812608\, t^3)\, t - 0.0014\,(0.0089040\, t^2 - 0.000056812608\, t^3)^2\, t - 0.0012\,(0.0089040\, t^2 - 0.000056812608\, t^3)\,(0.017904\, t^2 - 0.000100360872\, t^3)\, t$$

$$x = x_1 + x_2 + x_3$$

$$X := 0.1272\, t + 0.0089040\, t^2 - 0.000056812608\, t^3 + 0.070\,(0.0089040\, t^2 - 0.000056812608\, t^3)\, t - 0.0014\,(0.0089040\, t^2 - 0.000056812608\, t^3)^2\, t - 0.0012\,(0.0089040\, t^2 - 0.000056812608\, t^3)\,(0.017904\, t^2 - 0.000100360872\, t^3)\, t$$

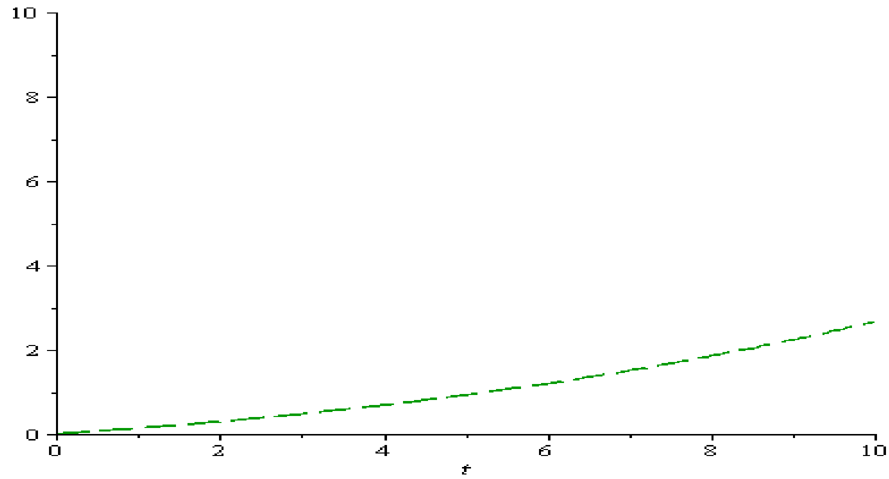

Figure 4. *Plot of numerical solution of yeast specy 1 biomass x(t) corresponding to a different time (t) in years when* α_1 *is varied at* 70%

$\alpha 1 := 0.1 \cdot 0.90 :$

$$x_1 = \alpha_1 n_1 \frac{t^\psi}{\psi!} - \beta_1 {n_1}^2 \frac{t^\psi}{\psi!} - \gamma_1 n_1 n_2 \frac{t^\psi}{\psi!}$$

$X1 := 0.1672\, t$

$$x_2 = \alpha_1 x_1 \frac{t^\psi}{\psi!} - \beta_1 {x_1}^2 \frac{t^\psi}{\psi!} - \gamma_1 x_1 y_1 \frac{t^\psi}{\psi!}$$

$X2 := 0.0150480\, t^2 - 0.000084041408\, t^3$

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$$x_3 = \alpha_1 x_2 \frac{t^{\psi}}{\psi!} - \beta_1 {x_2}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 x_2 y_2 \frac{t^{\psi}}{\psi!}$$

$$X3 := 0.090\left(0.0150480\,t^2 - 0.000084041408\,t^3\right)t - 0.0014\left(0.0150480\,t^2 - 0.000084041408\,t^3\right)^2 t - 0.0012\left(0.0150480\,t^2 - 0.000084041408\,t^3\right)\left(0.017904\,t^2 - 0.000100360872\,t^3\right)t$$

$$x = x_1 + x_2 + x_3$$

$$X := 0.1672\,t + 0.0150480\,t^2 - 0.000084041408\,t^3 + 0.090\left(0.0150480\,t^2 - 0.000084041408\,t^3\right)t - 0.0014\left(0.0150480\,t^2 - 0.000084041408\,t^3\right)^2 t - 0.0012\left(0.0150480\,t^2 - 0.000084041408\,t^3\right)\left(0.017904\,t^2 - 0.000100360872\,t^3\right)t$$

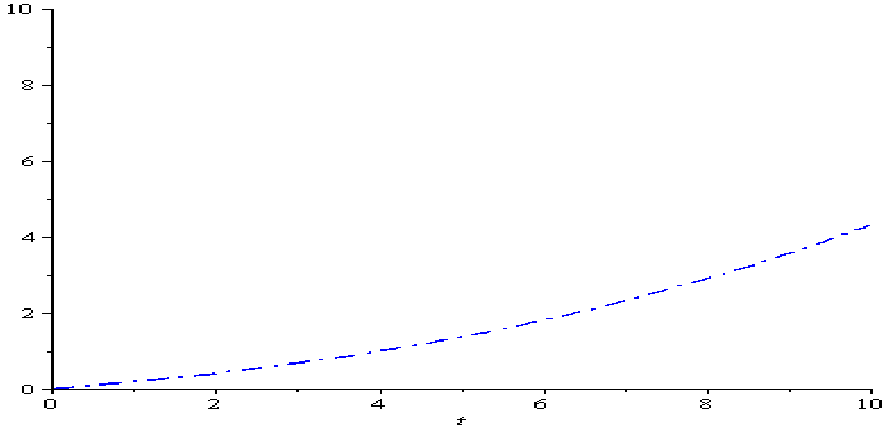

Figure 5. *Plot of numerical solution of yeast specy 1 biomass x(t) corresponding to a different time (t) in years when* α_1 *is varied at* 90%

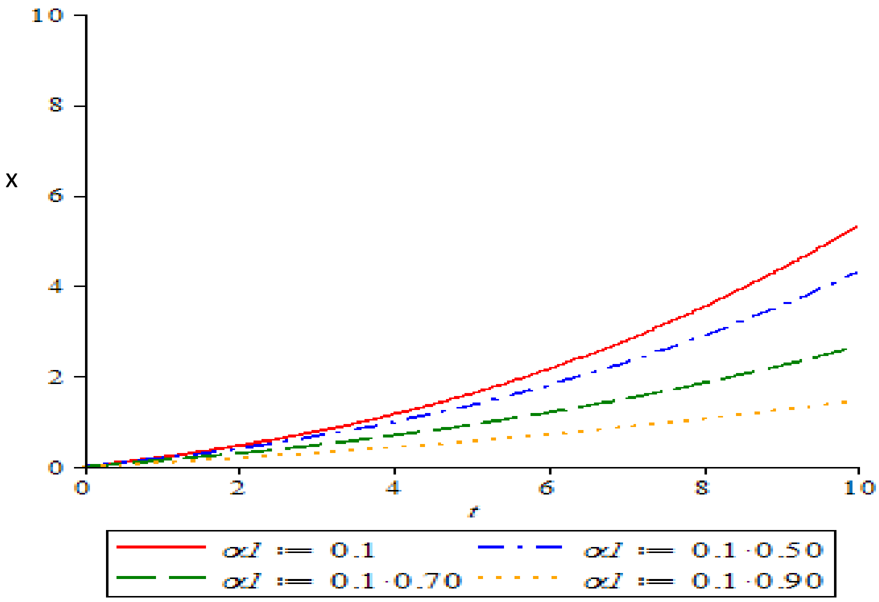

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Figure 6. *Summary plot of numerical solution of yeast specy 1 biomass x(t) corresponding to different time (t) in years*

$\alpha 1 := 0.1 \cdot 1.20:$

$$x_1 = \alpha_1 n_1 \frac{t^{\psi}}{\psi!} - \beta_1 n_1{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 n_1 n_2 \frac{t^{\psi}}{\psi!}$$

$X1 := 0.2272\, t$

$$x_2 = \alpha_1 x_1 \frac{t^{\psi}}{\psi!} - \beta_1 x_1{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 x_1 y_1 \frac{t^{\psi}}{\psi!}$$

$X2 := 0.0272640\, t^2 - 0.000133284608\, t^3$

$$x_3 = \alpha_1 x_2 \frac{t^{\psi}}{\psi!} - \beta_1 x_2{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 x_2 y_2 \frac{t^{\psi}}{\psi!}$$

$$X3 := 0.120\left(0.0272640\, t^2 - 0.000133284608\, t^3\right) t - 0.0014\left(0.0272640\, t^2 - 0.000133284608\, t^3\right)^2 t - 0.0012\left(0.0272640\, t^2 - 0.000133284608\, t^3\right)\left(0.017904\, t^2 - 0.000100360872\, t^3\right) t$$

$$x = x_1 + x_2 + x_3$$

$$X := 0.2272\, t + 0.0272640\, t^2 - 0.000133284608\, t^3 + 0.120\left(0.0272640\, t^2 - 0.000133284608\, t^3\right) t - 0.0014\left(0.0272640\, t^2 - 0.000133284608\, t^3\right)^2 t - 0.0012\left(0.0272640\, t^2 - 0.000133284608\, t^3\right)\left(0.017904\, t^2 - 0.000100360872\, t^3\right) t$$

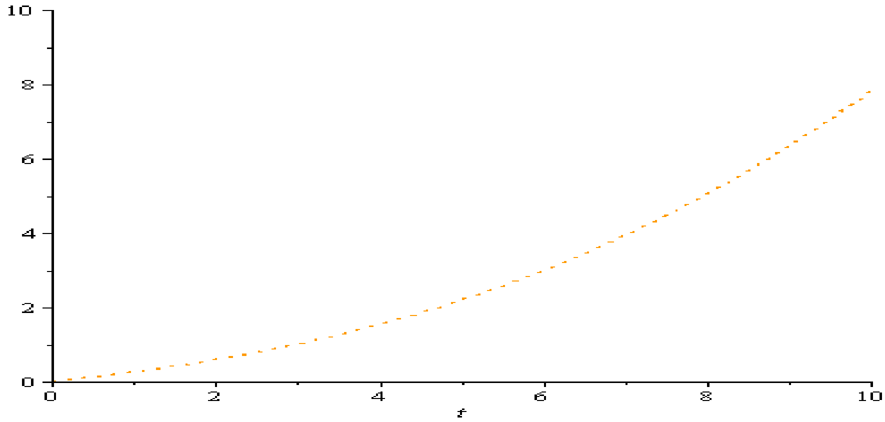

Figure 7. *Plot of numerical solution of yeast specy 1 biomass x(t) corresponding to a different time (t) in years when α_1 is varied at 120%*

$\alpha 1 := 0.1 \cdot 1.40:$

$$x_1 = \alpha_1 n_1 \frac{t^{\psi}}{\psi!} - \beta_1 n_1{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 n_1 n_2 \frac{t^{\psi}}{\psi!}$$

$X1 := 0.2672\, t$

$$x_2 = \alpha_1 x_1 \frac{t^{\psi}}{\psi!} - \beta_1 x_1{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 x_1 y_1 \frac{t^{\psi}}{\psi!}$$

$X2 := 0.0374080\, t^2 - 0.000171713408\, t^3$

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$$x_3 = \alpha_1 x_2 \frac{t^{\psi}}{\psi!} - \beta_1 x_2{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 x_2 y_2 \frac{t^{\psi}}{\psi!}$$

$$X3 := 0.140\,(0.0374080\,t^2 - 0.000171713408\,t^3)\,t - 0.0014\,(0.0374080\,t^2 - 0.000171713408\,t^3)^2\,t - 0.0012\,(0.0374080\,t^2 - 0.000171713408\,t^3)\,(0.017904\,t^2 - 0.000100360872\,t^3)\,t$$

$$x = x_1 + x_2 + x_3$$

$$X := 0.2672\,t + 0.0374080\,t^2 - 0.000171713408\,t^3 + 0.140\,(0.0374080\,t^2 - 0.000171713408\,t^3)\,t - 0.0014\,(0.0374080\,t^2 - 0.000171713408\,t^3)^2\,t - 0.0012\,(0.0374080\,t^2 - 0.000171713408\,t^3)\,(0.017904\,t^2 - 0.000100360872\,t^3)\,t$$

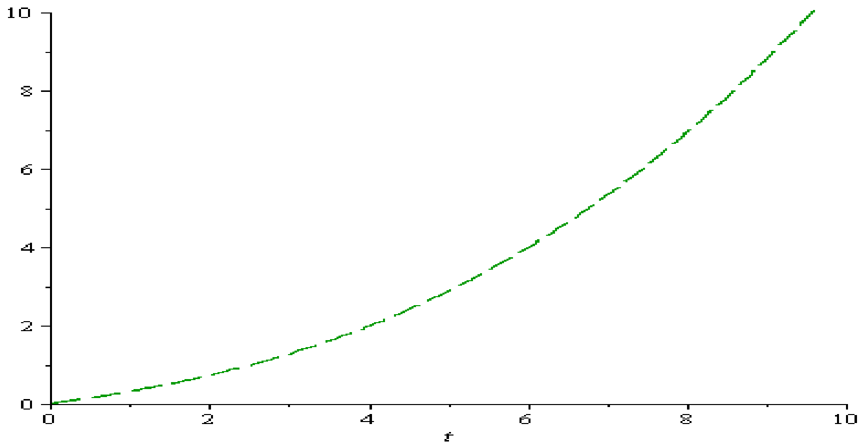

Figure 8. *Plot of numerical solution of yeast specy 1 biomass x(t) corresponding to a different time (t) in years when α_1 is varied at* 140%

$\alpha 1 := 0.1 \cdot 1.60 :$

$$x_1 = \alpha_1 n_1 \frac{t^{\psi}}{\psi!} - \beta_1 n_1{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 n_1 n_2 \frac{t^{\psi}}{\psi!}$$

$$X1 := 0.3072\,t$$

$$x_2 = \alpha_1 x_1 \frac{t^{\psi}}{\psi!} - \beta_1 x_1{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 x_1 y_1 \frac{t^{\psi}}{\psi!}$$

$$X2 := 0.0491520\,t^2 - 0.000214622208\,t^3$$

$$x_3 = \alpha_1 x_2 \frac{t^{\psi}}{\psi!} - \beta_1 x_2{}^2 \frac{t^{\psi}}{\psi!} - \gamma_1 x_2 y_2 \frac{t^{\psi}}{\psi!}$$

$$X3 := 0.160\,(0.0491520\,t^2 - 0.000214622208\,t^3)\,t - 0.0014\,(0.0491520\,t^2 - 0.000214622208\,t^3)^2\,t - 0.0012\,(0.0491520\,t^2 - 0.000214622208\,t^3)\,(0.017904\,t^2 - 0.000100360872\,t^3)\,t$$

$$x = x_1 + x_2 + x_3$$

$$X := 0.3072\,t + 0.0491520\,t^2 - 0.000214622208\,t^3 + 0.160\,(0.0491520\,t^2 - 0.000214622208\,t^3)\,t - 0.0014\,(0.0491520\,t^2 - 0.000214622208\,t^3)^2\,t - 0.0012\,(0.0491520\,t^2 - 0.000214622208\,t^3)\,(0.017904\,t^2 - 0.000100360872\,t^3)\,t$$

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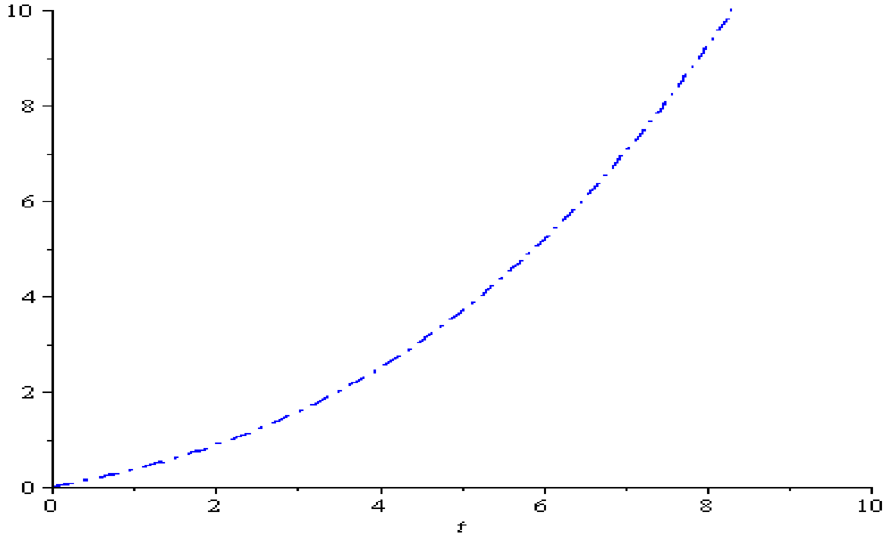

Figure 9. *Plot of numerical solution of yeast specy 1 biomass x(t) corresponding to a different time (t) in years when* α_1 *is varied at* 160%

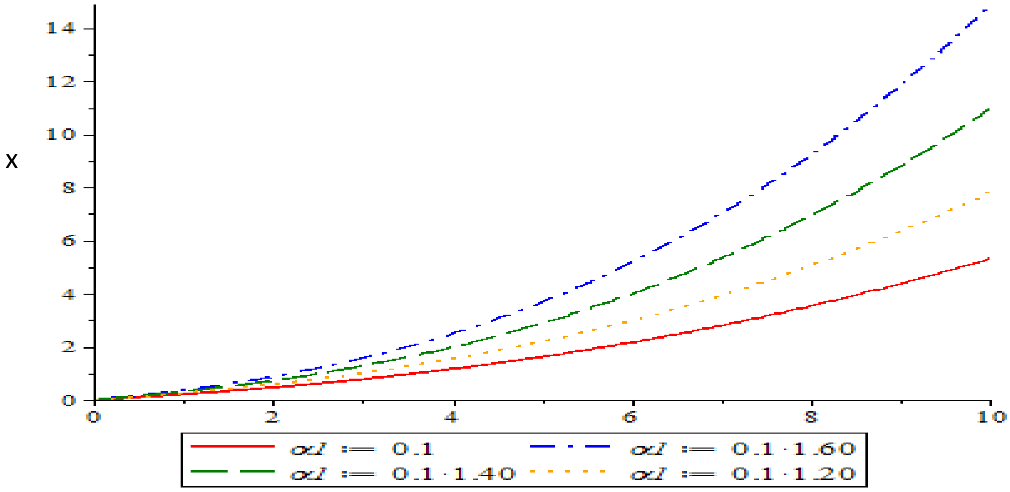

Figure 10. *Summary plot of numerical solution of yeast specy 1 biomass x(t) corresponding to different time (t) in years*

$\alpha 2 := 0.08 \cdot 0.50 :$

$$y_1 = \alpha_2 n_2 \frac{t^\psi}{\psi!} - \beta_2 {n_2}^2 \frac{t^\psi}{\psi!} - \gamma_2 n_1 n_2 \frac{t^\psi}{\psi!}$$

$Y1 := 0.1038\, t$

$$y_2 = \alpha_2 \cdot y_1 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_1 \cdot y_1) \frac{t^\psi}{\psi!} - \gamma_2 (x_1 y_1) \cdot \frac{t^\psi}{\psi!}$$

$Y2 := 0.00415200\, t^2 - 0.000049039272\, t^3$

$$y_3 = \alpha_2 \cdot y_2 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_2 \cdot y_2) \frac{t^\psi}{\psi!} - \gamma_2 (x_2 y_2) \cdot \frac{t^\psi}{\psi!}$$

$$Y3 := 0.0400\left(0.00415200\,t^2 - 0.000049039272\,t^3\right)t - 0.001\left(0.00415200\,t^2 - 0.000049039272\,t^3\right)^2 t - 0.0012\left(0.0491520\,t^2 - 0.000214622208\,t^3\right)\left(0.00415200\,t^2 - 0.000049039272\,t^3\right)t$$

$$Y := Y1 + Y2 + Y3;$$

$$Y := 0.1038\,t + 0.00415200\,t^2 - 0.000049039272\,t^3 + 0.0400\left(0.00415200\,t^2 - 0.000049039272\,t^3\right)t - 0.001\left(0.00415200\,t^2 - 0.000049039272\,t^3\right)^2 t - 0.0012\left(0.0491520\,t^2 - 0.000214622208\,t^3\right)\left(0.00415200\,t^2 - 0.000049039272\,t^3\right)t$$

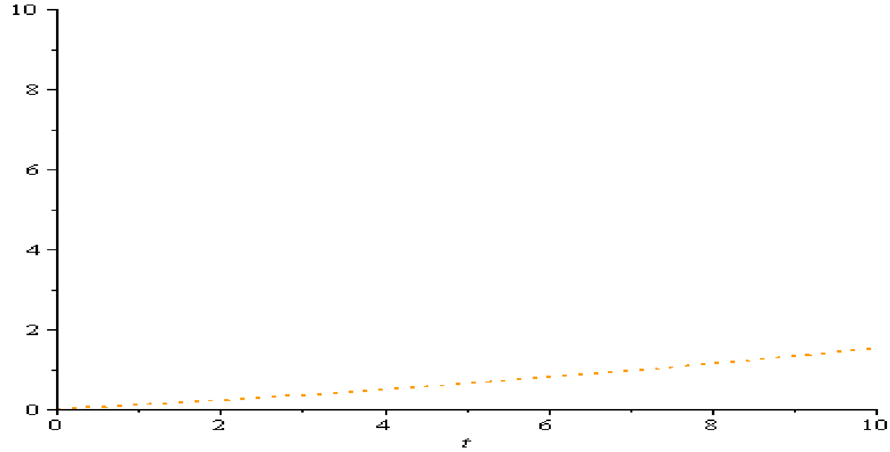

Figure 11. *Plot of numerical solution of yeast specy 2 biomass y(t) corresponding to a different time (t) in years when α_2 is varied at* 50%

$$\alpha 2 := 0.08 \cdot 0.70 :$$

$$y_1 = \alpha_2 n_2 \frac{t^\psi}{\psi!} - \beta_2 {n_2}^2 \frac{t^\psi}{\psi!} - \gamma_2 n_1 n_2 \frac{t^\psi}{\psi!}$$

$$Y1 := 0.1518\,t$$

$$y_2 = \alpha_2 \cdot y_1 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_1 \cdot y_1) \frac{t^\psi}{\psi!} - \gamma_2 (x_1 y_1) \cdot \frac{t^\psi}{\psi!}$$

$$Y2 := 0.00850080\,t^2 - 0.000079002792\,t^3$$

$$y_3 = \alpha_2 \cdot y_2 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_2 \cdot y_2) \frac{t^\psi}{\psi!} - \gamma_2 (x_2 y_2) \cdot \frac{t^\psi}{\psi!}$$

$$Y3 := 0.0560\left(0.00850080\,t^2 - 0.000079002792\,t^3\right)t - 0.001\left(0.00850080\,t^2 - 0.000079002792\,t^3\right)^2 t - 0.0012\left(0.0491520\,t^2 - 0.000214622208\,t^3\right)\left(0.00850080\,t^2 - 0.000079002792\,t^3\right)t$$

$$y = y_1 + y_2 + y_3$$

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$Y := 0.1518\,t + 0.00850080\,t^2 - 0.000079002792\,t^3 + 0.0560\left(0.00850080\,t^2 - 0.000079002792\,t^3\right)t - 0.001\left(0.00850080\,t^2 - 0.000079002792\,t^3\right)^2 t - 0.0012\left(0.0491520\,t^2 - 0.000214622208\,t^3\right)\left(0.00850080\,t^2 - 0.000079002792\,t^3\right)t$

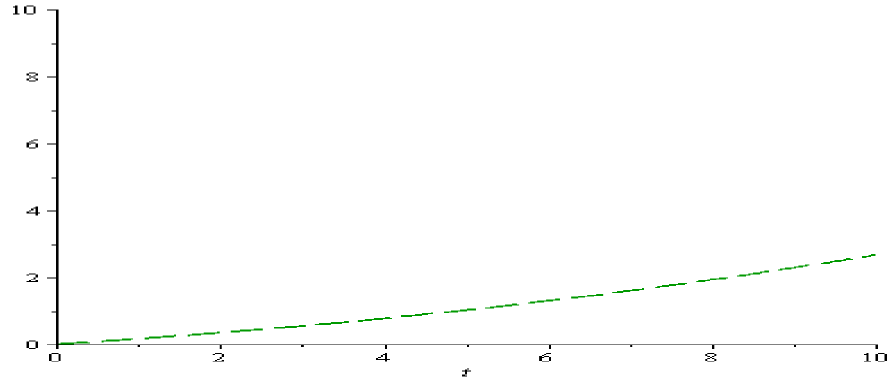

Figure 12. *Plot of numerical solution of yeast specy 2 biomass y(t) corresponding to a different time (t) in years when α_2 is varied at 70%*

$\alpha 2 := 0.08 \cdot 0.90:$

$$y_1 = \alpha_2 n_2 \frac{t^\psi}{\psi!} - \beta_2 {n_2}^2 \frac{t^\psi}{\psi!} - \gamma_2 n_1 n_2 \frac{t^\psi}{\psi!}$$

$Y1 := 0.1998\,t$

$$y_2 = \alpha_2 \cdot y_1 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_1 \cdot y_1) \frac{t^\psi}{\psi!} - \gamma_2 (x_1 y_1) \cdot \frac{t^\psi}{\psi!}$$

$Y2 := 0.01438560\,t^2 - 0.000113574312\,t^3$

$$y_3 = \alpha_2 \cdot y_2 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_2 \cdot y_2) \frac{t^\psi}{\psi!} - \gamma_2 (x_2 y_2) \cdot \frac{t^\psi}{\psi!}$$

$Y3 := 0.0720\left(0.01438560\,t^2 - 0.000113574312\,t^3\right)t - 0.001\left(0.01438560\,t^2 - 0.000113574312\,t^3\right)^2 t - 0.0012\left(0.0491520\,t^2 - 0.000214622208\,t^3\right)\left(0.01438560\,t^2 - 0.000113574312\,t^3\right)t$

$$y = y_1 + y_2 + y_3$$

$Y := 0.1998\,t + 0.01438560\,t^2 - 0.000113574312\,t^3 + 0.0720\left(0.01438560\,t^2 - 0.000113574312\,t^3\right)t - 0.001\left(0.01438560\,t^2 - 0.000113574312\,t^3\right)^2 t - 0.0012\left(0.0491520\,t^2 - 0.000214622208\,t^3\right)\left(0.01438560\,t^2 - 0.000113574312\,t^3\right)t$

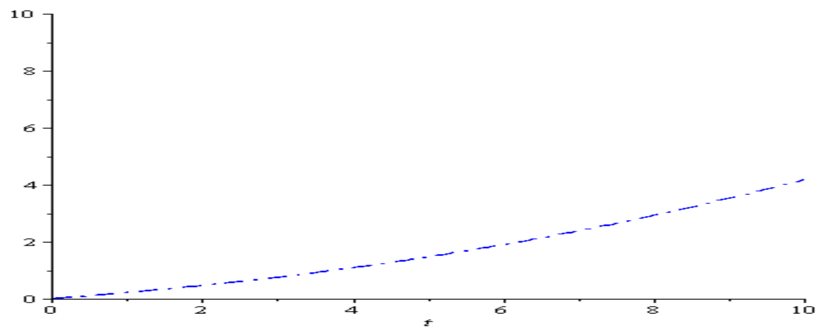

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Figure 13. *Plot of numerical solution of yeast specy 2 biomass y(t) corresponding to a different time (t) in years when α_2 is varied at 90%*

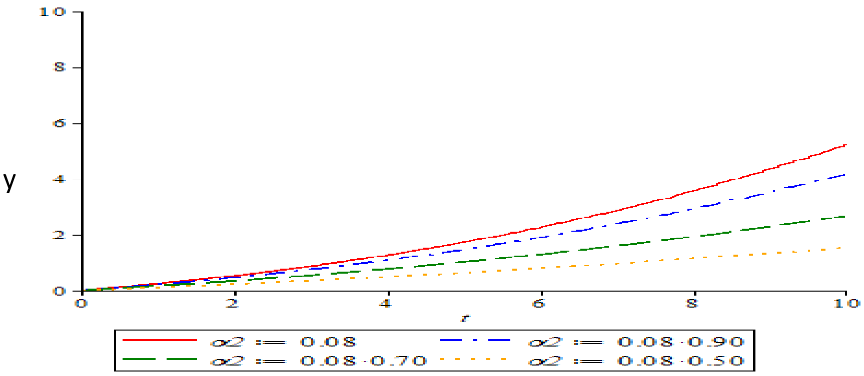

Figure 14. *Summary plot of numerical solution of yeast specy 2 biomass y(t) corresponding to different time (t) in years*

$\alpha 2 := 0.08 \cdot 1.20:$

$$y_1 = \alpha_2 n_2 \frac{t^\psi}{\psi!} - \beta_2 {n_2}^2 \frac{t^\psi}{\psi!} - \gamma_2 n_1 n_2 \frac{t^\psi}{\psi!}$$

$Y1 := 0.2718\, t$

$$y_2 = \alpha_2 \cdot y_1 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_1 \cdot y_1) \frac{t^\psi}{\psi!} - \gamma_2 (x_1 y_1) \cdot \frac{t^\psi}{\psi!}$$

$Y2 := 0.02609280\, t^2 - 0.000174071592\, t^3$

$$y_3 = \alpha_2 \cdot y_2 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_2 \cdot y_2) \frac{t^\psi}{\psi!} - \gamma_2 (x_2 y_2) \cdot \frac{t^\psi}{\psi!}$$

$$Y3 := 0.0960\left(0.02609280\, t^2 - 0.000174071592\, t^3\right) t - 0.001\left(0.02609280\, t^2 - 0.000174071592\, t^3\right)^2 t - 0.0012\left(0.0491520\, t^2 - 0.000214622208\, t^3\right)\left(0.02609280\, t^2 - 0.000174071592\, t^3\right) t$$

$$y = y_1 + y_2 + y_3$$

$$Y := 0.2718\, t + 0.02609280\, t^2 - 0.000174071592\, t^3 + 0.0960\left(0.02609280\, t^2 - 0.000174071592\, t^3\right) t - 0.001\left(0.02609280\, t^2 - 0.000174071592\, t^3\right)^2 t - 0.0012\left(0.0491520\, t^2 - 0.000214622208\, t^3\right)\left(0.02609280\, t^2 - 0.000174071592\, t^3\right) t$$

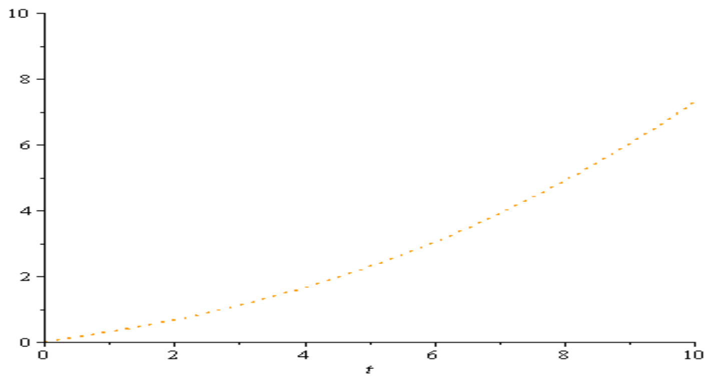

Figure 15. *The plot of numerical solution of yeast specy 2 biomass y(t) corresponding to a different time (t) in years when* α_2 *is varied at* 120%

$\alpha 2 := 0.08 \cdot 1.40:$

$$y_1 = \alpha_2 n_2 \frac{t^\psi}{\psi!} - \beta_2 {n_2}^2 \frac{t^\psi}{\psi!} - \gamma_2 n_1 n_2 \frac{t^\psi}{\psi!}$$

$Y1 := 0.3198\, t$

$$y_2 = \alpha_2 \cdot y_1 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_1 \cdot y_1) \frac{t^\psi}{\psi!} - \gamma_2 (x_1 y_1) \cdot \frac{t^\psi}{\psi!}$$

$Y2 := 0.03581760\, t^2 - 0.000220163112\, t^3$

$$y_3 = \alpha_2 \cdot y_2 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_2 \cdot y_2) \frac{t^\psi}{\psi!} - \gamma_2 (x_2 y_2) \cdot \frac{t^\psi}{\psi!}$$

$$Y3 := 0.1120 \left(0.03581760\, t^2 - 0.000220163112\, t^3\right) t - 0.001 \left(0.03581760\, t^2 - 0.000220163112\, t^3\right)^2 t - 0.0012 \left(0.0491520\, t^2 - 0.000214622208\, t^3\right) \left(0.03581760\, t^2 - 0.000220163112\, t^3\right) t$$

$$y = y_1 + y_2 + y_3$$

$$Y := 0.3198\, t + 0.03581760\, t^2 - 0.000220163112\, t^3 + 0.1120 \left(0.03581760\, t^2 - 0.000220163112\, t^3\right) t - 0.001 \left(0.03581760\, t^2 - 0.000220163112\, t^3\right)^2 t - 0.0012 \left(0.0491520\, t^2 - 0.000214622208\, t^3\right) \left(0.03581760\, t^2 - 0.000220163112\, t^3\right) t$$

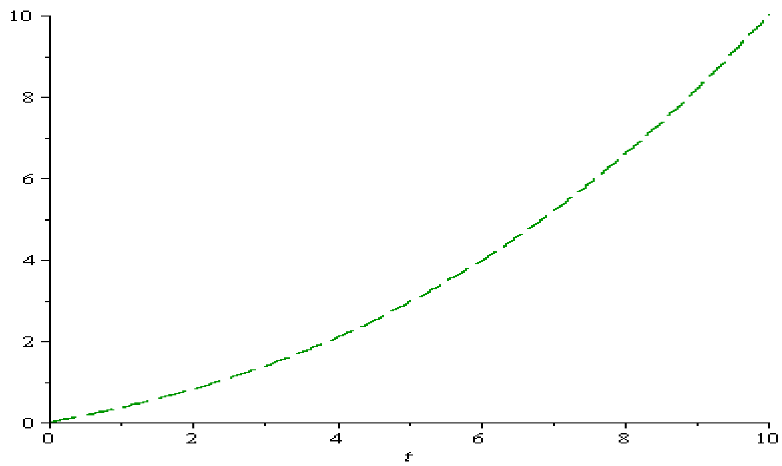

Figure 16. *Plot of numerical solution of yeast specy 2 biomass y(t) corresponding to a different time (t) in years when* α_2 *is varied at* 140%

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$\alpha 2 := 0.08 \cdot 1.60 :$

$$y_1 = \alpha_2 n_2 \frac{t^\psi}{\psi!} - \beta_2 n_2{}^2 \frac{t^\psi}{\psi!} - \gamma_2 n_1 n_2 \frac{t^\psi}{\psi!}$$

$$Y1 := 0.3678\, t$$

$$y_2 = \alpha_2 \cdot y_1 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_1 \cdot y_1) \frac{t^\psi}{\psi!} - \gamma_2 (x_1 y_1) \cdot \frac{t^\psi}{\psi!}$$

$$Y2 := 0.04707840\, t^2 - 0.000270862632\, t^3$$

$$y_3 = \alpha_2 \cdot y_2 \cdot \frac{t^\psi}{\psi!} - \beta_2 (y_2 \cdot y_2) \frac{t^\psi}{\psi!} - \gamma_2 (x_2 y_2) \cdot \frac{t^\psi}{\psi!}$$

$$Y3 := 0.1280 \left(0.04707840\, t^2 - 0.000270862632\, t^3\right) t - 0.001 \left(0.04707840\, t^2 - 0.000270862632\, t^3\right)^2 t - 0.0012 \left(0.0491520\, t^2 - 0.000214622208\, t^3\right) \left(0.04707840\, t^2 - 0.000270862632\, t^3\right) t$$

$$y = y_1 + y_2 + y_3$$

$$Y := 0.3678\, t + 0.04707840\, t^2 - 0.000270862632\, t^3 + 0.1280 \left(0.04707840\, t^2 - 0.000270862632\, t^3\right) t - 0.001 \left(0.04707840\, t^2 - 0.000270862632\, t^3\right)^2 t - 0.0012 \left(0.0491520\, t^2 - 0.000214622208\, t^3\right) \left(0.04707840\, t^2 - 0.000270862632\, t^3\right) t$$

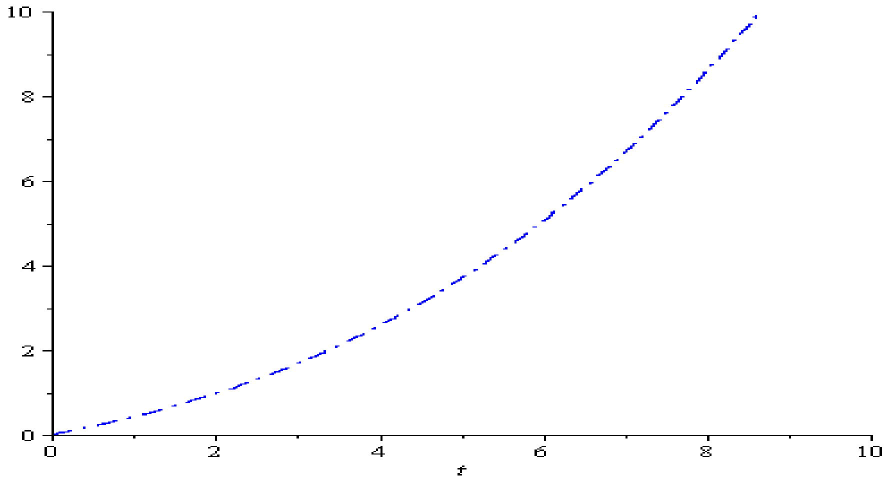
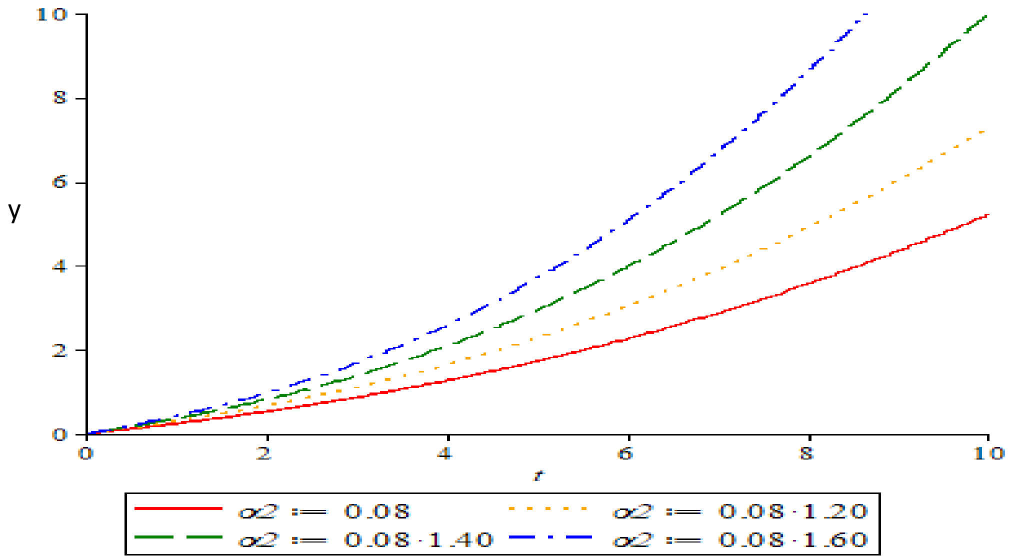


Figure 17. *Plot of numerical solution of yeast specy 2 biomass y(t) corresponding to a different time (t) in years when* α_2 *is varied at* 160%



sition method.

Figure 18. *Summary plot of numerical solution of yeast specy 2 biomass y(t) corresponding to different time (t) in years*

Discussion

Using a mathematical tool called Maple 18 software, the behaviour of the several yeast species 1 and 2 under examination at different model parameter values is implemented and is visually displayed in Figures 1 through 18. The solution trajectories of the first scenario are less than those of the second scenario, and so on. As the percentage of $\propto_1$ increases, there is an improved biomass of yeast species 1 called x(t), which gets closer to the biomass of yeast species 1 at a fixed model parameter. The effects of increasing $\propto_1$, from 120% to 160%, are shown in Figs. 7 to 10. These figures illustrate the amount to which the estimated proportion grows as a result of the growth rate variation known as $\propto_1$. When all model parameters are fixed at 100%, the solution trajectories or biomass of yeast species 1 for the first scenario are greater than that of yeast species 1. However, as growth rates are increased from 120% to 160%, the values increase, indicating that the biomass of yeast species 1 has maintained an improved x(t), providing evidence of biodiversity gain.

Additionally, we noticed that the solution trajectories of the first scenario are less than those of the second scenario as we increase the decreased growth rates of yeast species 2 ($\propto_2$) from 50% to 90% on the biomass of yeast species 2. Similarly, as the percentage of $\propto_1$ increases, there is an improved biomass of yeast species 2 called y(t), which gets closer to the biomass of yeast species 2 at a fixed model parameter. The effects of increasing $\propto_2$ by 120% to 160% are shown in Figs. 15 to 18, which illustrates the extent to which the estimated percentage grows as a result of the growth rate variation known as $\propto_1$.

When all model parameters are fixed at 100%, the solution trajectories or biomass of yeast species 1 for the first scenario are greater than that of yeast species 1, and as growth rates are increased from 120% to 160%, the values increase. This indicates that the biomass of yeast species 2 has maintained an improved y(t), indicating a gain in biodiversity.

Conclusion

In this paper, we applied the Laplace Decomposition Method to investigate the behaviour of yeast species on biodiversity scenarios due to the variation of $\propto_1$ and $\propto_2$. We observed a biodiversity loss when $\propto_1$ and $\propto_2$ are decreased together from 20% to 99%. Similarly, when $\propto_1$ and $\propto_2$ are increased from 110% to 120%, a biodiversity gain was observed.

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