

Modelling Stochastic Adverse Effects of Economic Turbulence on Performance of Nigerian University Lecturers and Students: A Numerical Investigation

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Abstract

It is a well-known fact that the prosperity and economic growth of any nation depends on the contributions of its University Education. Despite its convincing and impactful contributions, Nigerian University Education is consistently faced with Economic turbulence. Economic turbulence in Nigerian Universities is recently characterized and fueled by the Federal Government contributory economic policies of introduction of IPPIS, 2023 CBN redesigned naira note policy, 2023 Fuel subsidy removal which caused inflation and high exchange rate, negligence in honouring FG-ASUU 2009 agreements for University growth and school fees increment in the Nigerian Universities which result to adverse effects of decaying reading or research culture and low performance - level volatilities and uncertainties among Nigerian University Lecturers and their students. This paper aims to investigate and proffer solutions to the undesirable effects of these economic turbulences on Nigerian University Lecturers and students' performance through a numerical investigation. These adverse effects are able to generate future delay and volatility-noise in the financial market which influences the Nigerian University Lecturers and students performance. The Government's contributory economic policies which result in adverse effects of economic turbulence are modeled as Econometric Stochastic Time-Delay Differential Equation (ESTDDE). The modelled equation is solved using a Family of Seventh Order Implicit Hybrid Extended Block Adams Moulton Methods (FSOHEBAMM) with the help of new sequences for evaluations of the delay term and the noise term. Numerically, through mathematical experiments and the results obtained after solving some examples of the modelled equation, this study recommended the best ways to overcome the adverse effects of economic turbulence which affect the Nigerian University Lecturers and students performance.

Keywords: Economic Turbulence; Nigerian University; Performance; Government; Absolute volatility Error

Introduction

The adverse effects of Economic Turbulence as a result of the Federal Government's contributory economic policies in Nigerian Universities cannot be over-emphasized as it has resulted in financial hardship, suffering, lack of motivation, brain drain and decaying reading or research culture and low performance among lecturers and students, lots of school-drop-outs which causes high rate of crimes and insecurity, "Japa" (migration) syndrome which leads to capital flight and massive failure in academic performance and production of half-becked graduands which affect the rate of productivity and human capital development. According to Stefan and Galina (2019) and Zakari (2017), economic turbulence is an extreme economic phenomenon caused by forces which should have a certain degree of intensity or energy exhibiting consistent chaotic behaviour. The most important economic objective of any University in Nigeria is how to achieve improved economic growth and minimize the poverty rate among its nation's citizens through innovative, adaptive and applicable knowledge that impacts its students. Isola and Alani (2012) stated that the means of acquiring and adopting new knowledge remains a great confirmatory factor for economic growth in Nigeria. To achieve these important objectives, improved contributions to human capital development through the aspect of good education is paramount and a key component of economic growth and enhanced productivity in Nigeria which will influence the performance of the Lecturers and their students (Adamu, 2003; Chete, & Adeoye, 2002).

There is a big distraction in Nigerian Universities where the performance of the Lecturers and the students are

rated based on lectures covered, number of years spent during the programme and students' grades without new innovations. On the contrary, in developed countries, the performance of Lecturers and students is rated based on real-life problems solved through the acquired knowledge and innovations otherwise performance becomes an academic exercise and library beautification without real-life applications and solutions. According to Obaji (2006) and Olaofe (2005), the ability of teachers to adapt to new innovations in the teaching field is linked to the intensive training received as it is a major factor for determining of quality education and performance level of teachers. The performance of lecturers and students in Nigerian Universities under the present economic turbulence is a cause for concern and solutions to it are the major objective of this study. The variations and unpredictable changes over a stochastic period in Nigeria University activities are influenced by a complex interplay of Federal Government contributory economic policies. The Federal Government contributory economic policies such as the introduction of IPPIS, the 2023 CBN redesigned naira note policy, the 2023 Fuel subsidy removal which caused inflation and high exchange-rate, negligence in honouring FG-ASUU 2009 agreements for University growth and school fees increment in the Nigerian Universities caused economic turbulence which resulted to many adverse effects of low performance - level volatilities and uncertainties among Nigerian University Lecturers and their students. The existing literature on this concept revealed that no research has been carried out mathematically in addressing the adverse effects of economic turbulence on Nigerian University Lecturers and students performance through a numerical approach which left a huge research gap. There is an urgent need to understand, evaluate and provide solutions to the recent economic turbulence in Nigerian Universities which are very crucial for the performance of Nigerian University Lecturers and their students.

To tackle the adverse effects of Economic Turbulence as a result of the Federal Government's contributory economic policies in Nigerian Universities which influence performance of the Nigerian University Lecturers and their students, this study applied the Family of Seventh Order Implicit Hybrid Extended Block Adams Moulton Methods (FSOHEBAMM) in finding the numerical solution of some Econometric Stochastic Time-Delay Differential Equation (ESTDDE) with new mathematical expression developed by Osu et al. (2023) for delay and noise terms computations. ESTDDE is an econometric stochastic process that depends on the current state as the drift part and the volatility state as the diffusion part of the modelled equation for economic turbulence. A volatility term is an uncertainty event of any probability variables $\{X_t, t \in T\}$ where X_t is the volatility-time and T is the time variations of various turbulences in different fields of study. Ugbebor (1991) defined turbulence as a contingent process for the collation of stochastic variables on a set of discrete time points controlled by laws of uncertainties. Akhtari et al. (2015) formulated a structural model equation of Stochastic Time-Delay Differential Equation (STDDE) which contains the present state and the uncertainty state as presented below;

$$\begin{aligned} dy(t) &= A(y(t), y(t - \tau), t)dt + \beta(y(t), y(t - \tau), t)d\Phi(t) \text{ for } t > 0, \tau > 0 \\ y(t) &= \Phi(t), \quad \text{for } t > 0 \end{aligned} \quad (1)$$

Modifying the econometric model developed by Aigbedion et al. (2016), a multiple regression equation containing one econometric dependent variable (Nigerian University Lecturers and students performance) and five econometric independent variables (components of economic turbulence), with a constant, parameters and error term was obtained and presented as:

$$NULSP = \alpha + \beta_1 IPPIS + \beta_2 CBN + \beta_3 FSR + \beta_4 FGASUU + \beta_5 SFI + \varepsilon_i \quad (2)$$

where the econometric variables constitute the constructs of this study which are characterized by the Federal Government contributory economic policies such as the introduction of IPPIS, the 2023 CBN redesigned naira note policy, 2023 Fuel subsidy removal which caused inflation and high exchange rate, negligence in honouring FG-ASUU 2009 agreements for University growth, School fees increment in the Nigerian Universities and Nigerian University Lecturers and Students performance. Incorporating the econometric multiple regression equation (2) into the Stochastic Time-Delay Differential Equation (STDDE) (1), then the developed equation for this study becomes Econometric Stochastic Time-Delay Differential Equation (ESTDDE) showing the computational functions of the econometric variables of the form;

$$\begin{aligned} dNULSP(t) &= A(NULSP(t), NULSP(t - \tau), t)dt + B(NULSP(t), NULSP(t - \tau), t)d|\varepsilon_i|(t) \\ &\quad \text{for } t > 0, \tau > 0 \\ NULSP(t) &= \Phi(t), \quad \text{for } t > 0 \end{aligned} \quad (3)$$

where $\Phi(t)$ is the fundamental function, A, B are drift and volatility coefficients, $NULSP(t)$ denotes the Nigerian University Lecturers and Students performance, t is the time of psychological and ethical delay effect measured in months, τ is known as the delay, $(t - \tau)$ indicates the delay term and $NULSP(t - \tau)$ describe the delay term

of Nigerian University Lecturers and Students performance. The variable $|\varepsilon_i|(t)$ represents the absolute volatility errors of economic turbulence with its differential correlation $d|\varepsilon_i|(t)$ as the volatility term with the result of the delay term for Nigerian University Lecturers and Students performance $NULSP(t) = B(NULSP(t), NULSP(t - \tau), t)d|\varepsilon_i|(t)$ on the volatility part of (3). The drift part of (3) $NULSP(t) = A(NULSP(t), NULSP(t - \tau), t)dt$ is predictive which handles the average performance rate of Nigerian University Lecturers and Students without any economic turbulence. The above-developed econometric model is stochastically in nature containing all the constructs of this study as presented in (3).

Materials and Methods

Formulation of the Method

The discrete schemes of $k = 2, 3$ and 4 for Family of Seventh Order Implicit Hybrid Extended Block Adams Moulton Methods (FSOHEBAMM) were derived by Chibuisi et al. (2022) through matrix inversion techniques on the k -step multistep collocation method and displayed as;

$k = 2$ FSOHEBAMM

$$\begin{aligned}
 y_a &= y_{a+1} - \frac{67031}{249480} b_{x_a} - \frac{6037}{3528} b_{x_{a+1}} + \frac{47129}{2520} b_{x_{a+2}} - \frac{130048}{2835} b_{x_{a+\frac{9}{4}}} + \frac{14296}{315} b_{x_{a+\frac{5}{2}}} - \frac{510464}{24255} b_{x_{a+\frac{11}{4}}} + \frac{28817}{7560} b_{x_{a+3}} \\
 y_{a+2} &= y_{a+1} - \frac{799}{249480} b_{x_a} + \frac{543}{1960} b_{x_{a+1}} + \frac{3307}{840} b_{x_{a+2}} - \frac{4096}{567} b_{x_{a+\frac{9}{4}}} + \frac{216}{35} b_{x_{a+\frac{5}{2}}} - \frac{20992}{8085} b_{x_{a+\frac{11}{4}}} + \frac{3313}{7560} b_{x_{a+3}} \\
 y_{a+\frac{9}{4}} &= y_{a+1} - \frac{40825}{12773376} b_{x_a} - \frac{250055}{903168} b_{x_{a+1}} + \frac{520375}{129024} b_{x_{a+2}} - \frac{63755}{9072} b_{x_{a+\frac{9}{4}}} + \frac{12325}{2016} b_{x_{a+\frac{5}{2}}} - \frac{200125}{77616} b_{x_{a+\frac{11}{4}}} + \frac{168625}{387072} b_{x_{a+3}} \\
 y_{a+\frac{5}{2}} &= y_{a+1} - \frac{43}{13440} b_{x_a} + \frac{1737}{6272} b_{x_{a+1}} + \frac{18021}{4480} b_{x_{a+2}} - \frac{724}{105} b_{x_{a+\frac{9}{4}}} + \frac{219}{35} b_{x_{a+\frac{5}{2}}} - \frac{636}{245} b_{x_{a+\frac{11}{4}}} + \frac{1961}{4480} b_{x_{a+3}} \\
 y_{a+\frac{11}{4}} &= y_{a+1} - \frac{5831}{1824768} b_{x_a} + \frac{567}{2048} b_{x_{a+1}} + \frac{123823}{30720} b_{x_{a+2}} - \frac{44933}{6480} b_{x_{a+\frac{9}{4}}} + \frac{1029}{160} b_{x_{a+\frac{5}{2}}} - \frac{6559}{2640} b_{x_{a+\frac{11}{4}}} + \frac{119707}{276480} b_{x_{a+3}} \\
 y_{a+3} &= y_{a+1} - \frac{20}{6237} b_{x_a} + \frac{611}{2205} b_{x_{a+1}} + \frac{1264}{315} b_{x_{a+2}} - \frac{19456}{2835} b_{x_{a+\frac{9}{4}}} + \frac{1984}{315} b_{x_{a+\frac{5}{2}}} - \frac{54272}{24255} b_{x_{a+\frac{11}{4}}} + \frac{487}{945} b_{x_{a+3}} \quad (4)
 \end{aligned}$$

$k = 3$ FSOHEBAMM

$$\begin{aligned}
 y_a &= y_{a+2} - \frac{3841}{13230} b_{x_a} - \frac{5506}{3465} b_{x_{a+1}} + \frac{962}{2205} b_{x_{a+2}} - \frac{2582}{945} b_{x_{a+3}} + \frac{14464}{2205} b_{x_{a+\frac{7}{2}}} - \frac{434176}{72765} b_{x_{a+\frac{15}{4}}} + \frac{997}{630} b_{x_{a+4}} \\
 y_{a+1} &= y_{a+2} + \frac{127}{13230} b_{x_a} - \frac{3461}{9240} b_{x_{a+1}} - \frac{5659}{5880} b_{x_{a+2}} + \frac{9307}{7560} b_{x_{a+3}} - \frac{1864}{735} b_{x_{a+\frac{7}{2}}} + \frac{159232}{72765} b_{x_{a+\frac{15}{4}}} - \frac{467}{840} b_{x_{a+4}} \\
 y_{a+3} &= y_{a+2} + \frac{2}{1323} b_{x_a} - \frac{551}{27720} b_{x_{a+1}} + \frac{6967}{17640} b_{x_{a+2}} + \frac{8963}{7560} b_{x_{a+3}} - \frac{3032}{2205} b_{x_{a+\frac{7}{2}}} + \frac{77312}{72765} b_{x_{a+\frac{15}{4}}} - \frac{629}{2520} b_{x_{a+4}} \\
 y_{a+\frac{7}{2}} &= y_{a+2} + \frac{181}{125440} b_{x_a} - \frac{471}{24640} b_{x_{a+1}} + \frac{24429}{62720} b_{x_{a+2}} + \frac{391}{280} b_{x_{a+3}} - \frac{927}{980} b_{x_{a+\frac{7}{2}}} + \frac{2416}{2695} b_{x_{a+\frac{15}{4}}} - \frac{3921}{17920} b_{x_{a+4}}
 \end{aligned}$$

$$y_{a+\frac{15}{4}} = y_{a+2} + \frac{6419}{4423680} b_{x_a} - \frac{77861}{4055040} b_{x_{a+1}} + \frac{287567}{737280} b_{x_{a+2}} + \frac{1536983}{1105920} b_{x_{a+3}} - \frac{9359}{11520} b_{x_{a+\frac{7}{2}}} + \frac{1526}{1485} b_{x_{a+\frac{15}{4}}} - \frac{335111}{1474560} b_{x_{a+4}}$$

$$y_{a+4} = y_{a+2} + \frac{19}{13230} b_{x_a} - \frac{2}{105} b_{x_{a+1}} + \frac{286}{735} b_{x_{a+2}} + \frac{1322}{945} b_{x_{a+3}} - \frac{128}{147} b_{x_{a+\frac{7}{2}}} + \frac{8192}{6615} b_{x_{a+\frac{15}{4}}} - \frac{29}{210} b_{x_{a+4}} \quad (5)$$

k = 4 FSOHEBAMM

$$y_a = y_{a+3} - \frac{1013}{3360} b_{x_a} - \frac{11601}{7840} b_{x_{a+1}} - \frac{45}{112} b_{x_{a+2}} - \frac{689}{560} b_{x_{a+3}} + \frac{1017}{1120} b_{x_{a+4}} - \frac{464}{735} b_{x_{a+\frac{9}{2}}} + \frac{153}{1120} b_{x_{a+5}}$$

$$y_{a+1} = y_{a+3} + \frac{53}{5670} b_{x_a} - \frac{808}{2205} b_{x_{a+1}} - \frac{409}{315} b_{x_{a+2}} - \frac{307}{945} b_{x_{a+3}} - \frac{43}{630} b_{x_{a+4}} + \frac{256}{3969} b_{x_{a+\frac{9}{2}}} - \frac{1}{63} b_{x_{a+5}}$$

$$y_{a+2} = y_{a+3} - \frac{311}{90720} b_{x_a} + \frac{2647}{70560} b_{x_{a+1}} - \frac{485}{1008} b_{x_{a+2}} - \frac{10331}{15120} b_{x_{a+3}} + \frac{2561}{10080} b_{x_{a+4}} - \frac{3056}{19845} b_{x_{a+\frac{9}{2}}} + \frac{61}{2016} b_{x_{a+5}}$$

$$y_{a+4} = y_{a+3} - \frac{151}{90720} b_{x_a} + \frac{1063}{70560} b_{x_{a+1}} - \frac{361}{5040} b_{x_{a+2}} + \frac{8021}{15120} b_{x_{a+3}} + \frac{7169}{10080} b_{x_{a+4}} - \frac{4336}{19845} b_{x_{a+\frac{9}{2}}} + \frac{353}{10080} b_{x_{a+5}}$$

$$y_{a+\frac{9}{2}} = y_{a+3} - \frac{151}{107520} b_{x_a} + \frac{3231}{250880} b_{x_{a+1}} - \frac{225}{3584} b_{x_{a+2}} + \frac{9029}{17920} b_{x_{a+3}} + \frac{35703}{35840} b_{x_{a+4}} + \frac{22}{735} b_{x_{a+\frac{9}{2}}} + \frac{153}{7168} b_{x_{a+5}}$$

$$y_{a+5} = y_{a+3} - \frac{11}{5670} b_{x_a} + \frac{38}{2205} b_{x_{a+1}} - \frac{5}{63} b_{x_{a+2}} + \frac{517}{945} b_{x_{a+3}} + \frac{533}{630} b_{x_{a+4}} + \frac{9472}{19845} b_{x_{a+\frac{9}{2}}} + \frac{61}{315} b_{x_{a+5}} \quad (6)$$

Evaluation of Fundamental Properties of the Proposed Method

Following the steps developed by Lambert (1973) and Dahlquist (1956), the fundamental conditions for the convergence and stability of the proposed method are examined.

Order and Error Constant

As FSOHEBAMM is one of the classes of the Linear Multistep Method (LMM), Lambert (1973) analyzed that LMM is said to be of order z if $C_0 = C_1 = 0, \dots, C_z = 0$ but $C_{z+1} \neq 0$ and C_{z+1} is the error constant.

The order and error constants for (4) are analyzed and presented as follows;

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = c_7 = (0 \ 0 \ 0 \ 0 \ 0 \ 0)^T \text{ but}$$

$$\left(-\frac{3751}{3386880}, -\frac{89}{1128960}, -\frac{872075}{11098128384}, -\frac{79}{1003520}, -\frac{5929}{75497472}, -\frac{67}{846720} \right)^T$$

Therefore, (4) is of order $z = 7$ and error constants,

$$-\frac{3751}{3386880}, -\frac{89}{1128960}, -\frac{872075}{11098128384}, -\frac{79}{1003520}, -\frac{5929}{75497472}, -\frac{67}{846720}$$

For (5), the result is displayed as

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = c_7 = (0 \ 0 \ 0 \ 0 \ 0 \ 0)^T \text{ but}$$

$$\left(-\frac{787}{211680}, \frac{317}{376320}, \frac{773}{3386880}, \frac{1721}{8028160}, \frac{244853}{1132462080}, \frac{1}{4704} \right)^T$$

Therefore, (5) is of order $z = 7$ and error constants,

$$-\frac{787}{211680}, \frac{317}{376320}, \frac{773}{3386880}, \frac{1721}{8028160}, \frac{244853}{1132462080}, \frac{1}{4704}$$

For (6), the result is presented as

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = c_7 = (0 \ 0 \ 0 \ 0 \ 0 \ 0)^T \text{ but}$$

$$\left(-\frac{57}{7840}, \frac{11}{10584}, -\frac{191}{211680}, -\frac{131}{211680}, -\frac{4017}{8028160}, -\frac{41}{52920} \right)^T$$

Therefore, (20) is of order $z = 7$ and error constants,

$$-\frac{57}{7840}, \frac{11}{10584}, -\frac{191}{211680}, -\frac{131}{211680}, -\frac{4017}{8028160}, -\frac{41}{52920}$$

Consistency

According to Lambert (1973), a numerical method is said to be consistent if the order $z \geq 1$. Since the order of FSOHEBAMM as computed is $z \geq 1$, hence the consistency is satisfied.

Zero Stability Computation

In Dahlquist (1956), a numerical method is said to be zero stable if no roots $n_i, i = 1, 2, 3, \dots, m$ of the initial characteristic polynomial $P(n)$ developed as $P(n) = \det(nT_u^{(v)} - T_v^{(v)})$ is greater than 1 which satisfies $|n_i| \leq 1$ and the roots n is simple or distinct where $T_u^{(v)}$ and $T_v^{(v)}$, are the matrices of the initial characteristic polynomial obtained from (4), (5) and (6).

Applying the developed initial characteristic polynomial, (4) is presented and analyzed as:

$$P(n) = \det(nT_2^{(1)} - T_1^{(1)}) = |nT_2^{(1)} - T_1^{(1)}| = 0. \quad (7)$$

With Maple (18) software application,

$$P(n) = -n^5(n + 1) \Rightarrow -n^5(n + 1) = 0 \\ \Rightarrow n_1 = -1, n_2 = 0, n_3 = 0, n_4 = 0, n_5 = 0, n_6 = 0.$$

From $|n_i| < 1, i = 1, 2, 3, 4, 5, 6$, (4) is zero stable.

For (5), the initial characteristics polynomial is:

$$P(n) = \det(nT_2^{(2)} - T_1^{(2)}) = |nT_2^{(2)} - T_1^{(2)}| = 0. \quad (8)$$

With the help of Maple (18) software:

$$P(n) = -n^5(n + 1) \Rightarrow -n^5(n + 1) \\ \Rightarrow n_1 = -1, n_2 = 0, n_3 = 0, n_4 = 0, n_5 = 0, n_6 = 0.$$

From $|n_i| < 1, i = 1, 2, 3, 4, 5, 6$, (5) is zero stable.

For (6), using the same procedure:

$$P(n) = \det(nT_2^{(3)} - T_1^{(3)}) = |nT_2^3 - T_1^{(3)}| = 0. \quad (9)$$

The application of Maple (18) software shows:

$$P(n) = -n^5(n + 1) \Rightarrow -n^5(n + 1)$$

$$\Rightarrow n_1 = -1, n_2 = 0, n_3 = 0, n_4 = 0, n_5 = 0, n_6 = 0.$$

From the $|n_i| < 1, i = 1, 2, 3, 4, 5, 6$ is zero stable.

Convergence

Lambert (1973) and Dahlquist (1956) developed the required axioms of consistency and zero-stability for a numerical method to be convergent and since the derived discrete schemes (4), (5) and (6) of FSOHEBAMM are all consistent and zero stable, therefore the method is convergent.

Region of Absolute Equilibrium

The G - and R - regions of absolute equilibrium of (4), (5) and (6) are plotted using the software's of Maple 18 and MATLAB and are displayed in figure 1 to 4 below:

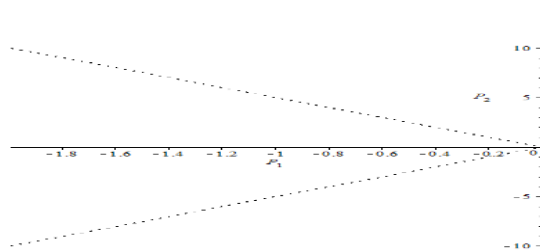


Figure 1: G - equilibrium in (4)

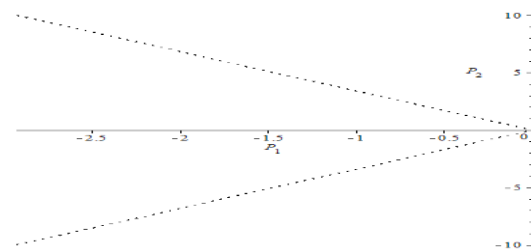


Figure 2: G - equilibrium in (5)

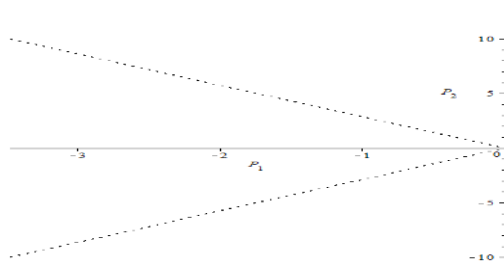


Figure 3: G - equilibrium in (6)

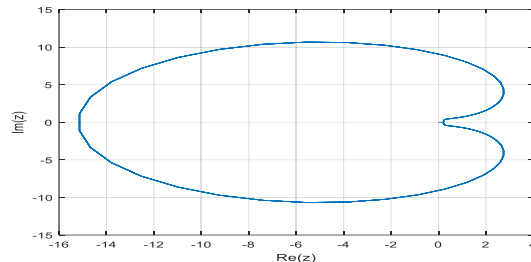


Figure 4: R - equilibrium in (4)

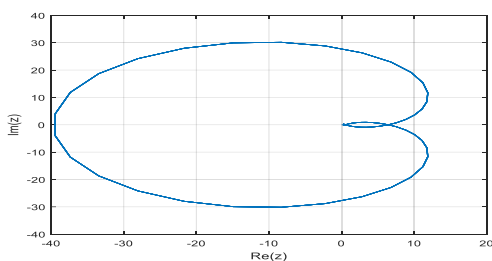
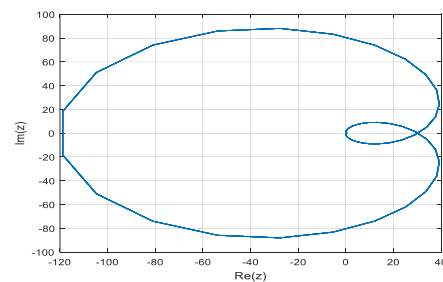


Figure 5: R - equilibrium in (5)

Figure



6: R - equilibrium in (6)

Computational Experiment

In this section, two examples of the modelled equation of this study shall be solved using the proposed method with the incorporation of the evaluated solutions of the delay and noise or volatility terms using the mathematical expressions developed by Osu et al. (2023) to obtain the numerical solutions of $dNULSP(t)$.

Numerical Examples

Example 1

$$dNULSP(t) = 1000 \left(dNULSP(t) - dNULSP \left(t - (\ln(1000 - 1)) \right) \right) dt + \left(dNULSP(t) - dNULSP \left(t - (\ln(1000 + 1)) \right) \right) d|\varepsilon_i|(t), 0 \leq t \leq 30.$$

$$dNULSP(t) = e^{-t}, t \geq 0. \text{ Exact Solution, } dNULSP(t) = e^{-t}.$$

Example 2

$$dNULSP(t) = \cos(t) \left((dNULSP(dNULSP(t) - 2)) dt + ((dNULSP(dNULSP(t) - 2)) d|\varepsilon_i|(t)), 0 \leq t \leq 30.$$

$$dNULSP(t) = 1, t \geq 0. \text{ Exact Solution, } dNULSP(t) = 1 + \sin(t).$$

Results

The above examples of model equations of this study were solved using the derived equations (4), (5) and (6) of FSOHEBAMM and the absolute volatility errors results obtained were computed and displayed as attachments in the appendix section for tables 1 to 2.

Graphical Display of Absolute Volatility Errors

Using the software of R and R – studio, the graphs of Absolute Volatility Error Results of FSOHEBAMM for Examples 1 and 2 above in tables 1 to 2 are plotted and presented as;

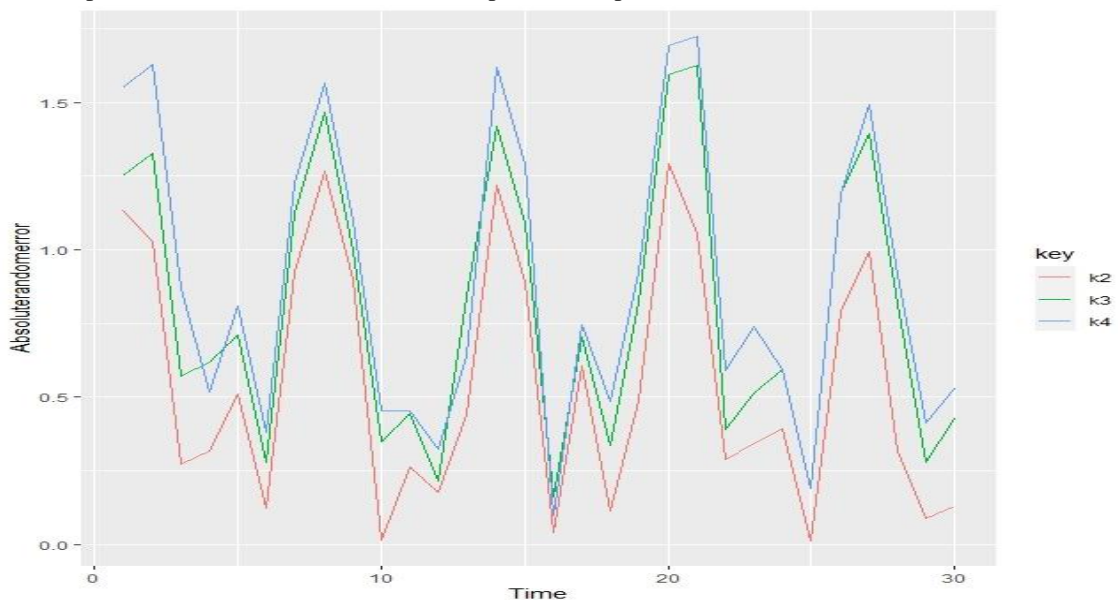


Figure 7: Absolute Volatility Error Results for Example 2 using FSOHEBAMM (as seen in the pigmented lines) against Time of delay in months. The pigmented lines represent the stochastic display of the method for step numbers $k = 2, 3$ and 4 with different Absolute Volatility Errors representing the economic turbulence characterized by Federal Government contributory economic policies on the performance of Nigerian University Lecturers and students.

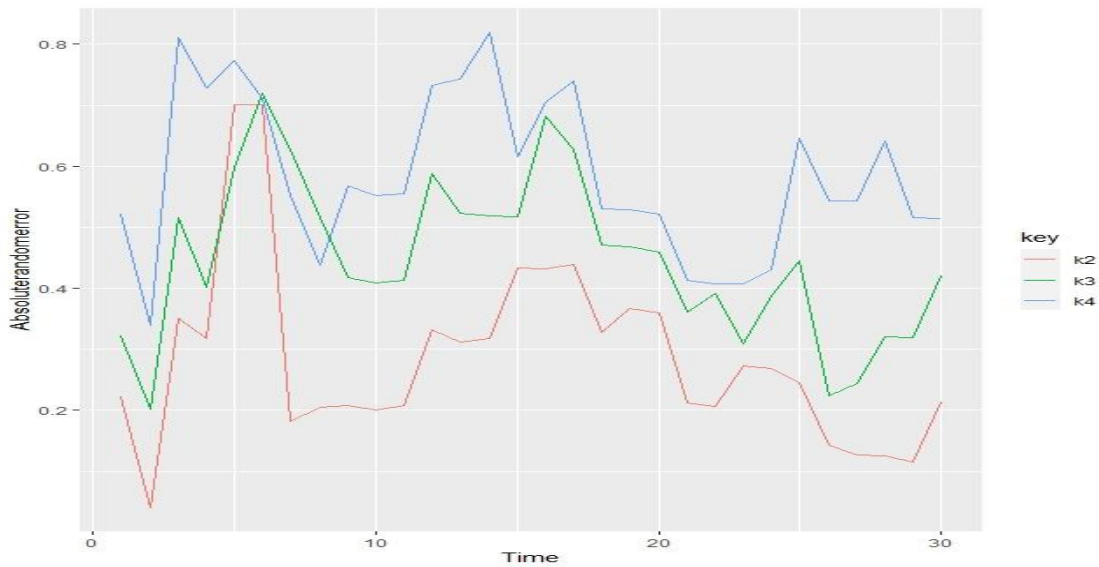


Figure 8: Absolute Volatility Error Results for Example 2 using FSOHEBAMM (as seen in the pigmented lines) against Time of delay in months. The pigmented-lines represent the stochastic display of the method for step numbers $k = 2, 3$ and 4 with different Absolute Volatility Errors representing the economic turbulence characterized by Federal Government contributory economic policies on the performance of Nigerian University Lecturers and students.

Comparison of Results

In the search of finding the numerical solution of Econometric Stochastic Time-Delay Differential Equation (ESTDDE), most scholars such as Osu et al. (2021), Buckwar (2000), Bahar (2019), Wang and Gan (2011) and Zhang et al. (2009) used interpolation system in solving the delay term and volatility term for approximate solutions and suffered setbacks in obtaining the Minimum Absolute Volatility Errors (MAVEs) at the Lowest Computer Processing Unit Time (LCPUT) which affected the accuracy of their numerical methods. Osu et al. (2023) obtained the analytic solution of a deterministic and stochastic model of time-varying investment returns with random parameters and revealed that the lower the absolute random errors of a stochastic process, the lower the risk effect of economic fluctuations in any financial model. The difficulty detected by the above scholars and the recent economic turbulence characterized by the Federal Government contributory economic policies such as the introduction of IPPIS, the 2023 CBN redesigned naira note policy, 2023 Fuel subsidy removal which caused inflation and high exchange rate, negligence in honouring FG-ASUU 2009 agreements for University growth, School fees increment in the Nigerian Universities and the low performance of Nigerian University Lecturers and Students and its adverse effects left a huge research gap which is the motivation behind this study. To tackle these challenges, the proposed method of this study with the help of new mathematical expressions developed by Osu et al. (2023) was used to obtain the absolute random errors. Comparing the Minimum Absolute Volatility Errors (MAVEs) of the proposed method with other methods in Buckwar (2000), Bahar (2019), and Osu et al. (2021), the advantage of the proposed method of this study was determined and presented below:

Table 3: Comparing the Minimum Absolute Volatility Errors (MAVEs) of FSOHEBAMM for $k = 2, 3$ and 4 with Buckwar (2000), Bahar (2019), Osu et al. (2021) for fixed step size $b = 0.01$ using Example 1

Computational Method	Compared MAVEs with Buckwar (2000), Bahar (2019), Osu et al. (2021)
FSOHEBAMM MAVE $k = 2$	2.23E-03
FSOHEBAMM MAVE $k = 3$	4.23E-02
FSOHEBAMM MAVE $k = 4$	5.56E-02
CSSEMM MAVE $k = 2$ Buckwar (2000)	4.76E-01
CSSEMM MAVE $k = 3$ Buckwar (2000)	9.17E-01

CSSEMM MAVE k = 4 Buckwar (2000)	1.62E-01
EMM MAVE k = 2 Bahar (2019)	1.84E+00
EMM MAVE k = 3 Bahar (2019)	2.47E-01
EMM MAVE k = 4 Bahar (2019)	9.73E-01
BSM MAVE k = 2 Osu et al (2021)	7.04E-01
BSM MAVE k = 3 Osu et al (2021)	7.05E-01
BSM MAVE k = 4 Osu et al (2021)	7.06E-01

Table 4: Comparing the Minimum Absolute Volatility Errors (MAVEs) of FSOHEBAMM for k = 2, 3 and 4 with Buckwar (2000), Bahar (2019), Osu et al. (2021) for fixed step size $b = 0.01$ using Example 2

Computational Method	Compared MAVEs with Buckwar (2000), Bahar (2019), Osu et al. (2021)
FSOHEBAMM MAVE k = 2	3.18E-04
FSOHEBAMM MAVE k = 3	1.18E-02
FSOHEBAMM MAVE k = 4	2.79E-02
CSSEMM MAVE k = 2 Buckwar (2000)	4.76E-01
CSSEMM MAVE k = 3 Buckwar (2000)	9.17E-01
CSSEMM MAVE k = 4 Buckwar (2000)	1.62E-01
EMM MAVE k = 2 Bahar (2019)	1.84E+00
EMM MAVE k = 3 Bahar (2019)	2.47E-01
EMM MAVE k = 4 Bahar (2019)	9.73E-01
BSM MAVE k = 2 Osu et al (2021)	7.04E-01
BSM MAVE k = 3 Osu et al (2021)	7.05E-01
BSM MAVE k = 4 Osu et al (2021)	7.06E-01

Graphical Display for Compared Solutions

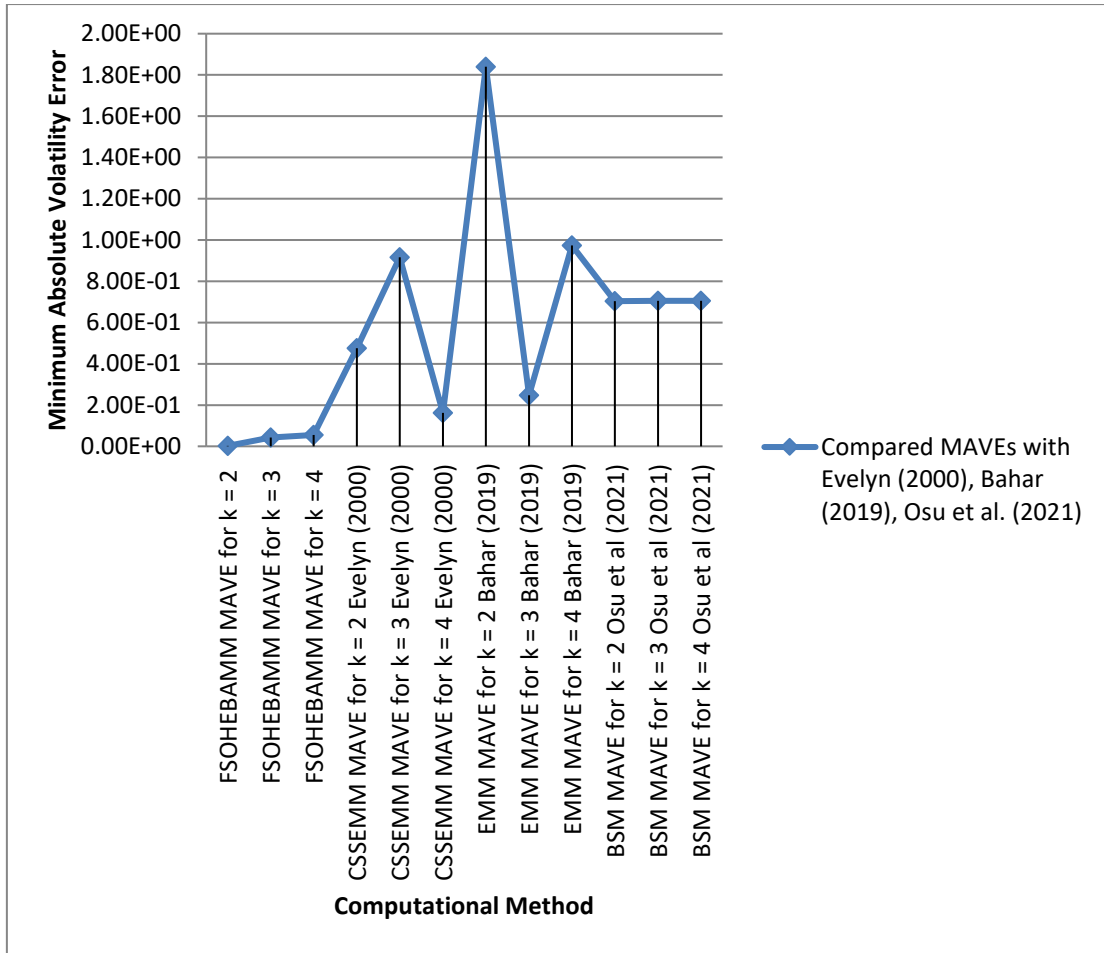


Figure 11: Compared MAVEs of FSOHEBAMM for k = 2, 3 and 4 with Buckwar (2000), Bahar (2019), Osu et al (2021) using Example 1

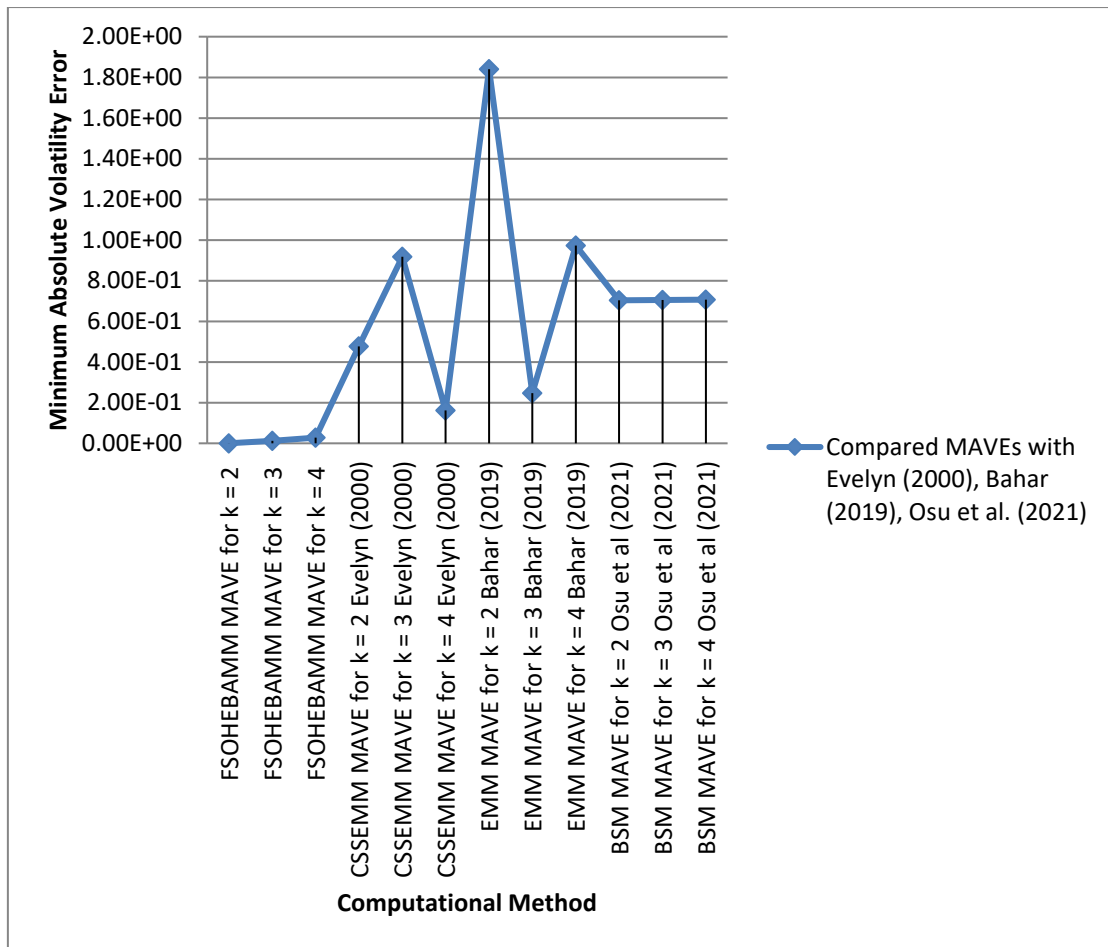


Figure 12: Compared MAVEs of FSOHEBAMM for $k = 2, 3$ and 4 with Buckwar (2000), Bahar (2019), Osu et al. (2021) using Example 2

The numerical solutions of the proposed numerical method of this study on the developed econometric model as computed and shown in tables 1 to 2 and figures 7 to 8 revealed the volatilities in the performance level of the University Lecturers and the students under the economic turbulence characterized by Federal Government contributory economic policies such as the introduction of IPPIS, 2023 CBN redesigned naira note policy, 2023 Fuel subsidy removal which caused inflation and high exchange-rate, negligence in honouring FG-ASUU 2009 agreements for University growth and School fees increment in the Nigerian Universities and can widely be used as an assessment model to reveal the education-economic growth rate for Nigerian Universities during economic turbulence in Nigeria and any other parts of the world.

Conclusion

This study has demonstrated how the recent Federal Government contributory economic policies such as the introduction of IPPIS, 2023 CBN redesigned naira note policy, 2023 Fuel subsidy removal which caused inflation and high exchange-rate, negligence in honouring FG-ASUU 2009 agreements for University growth and School fees increment in the Nigerian Universities have caused economic turbulence in the Nigerian Universities resulting into adverse effects of low performance among lecturers and students, financial hardship, suffering, lack of motivation, brain-drain and decaying reading or research culture and losts of school-drop-outs which causes high rate of crimes and insecurity, japa syndrome which leads to capital flight and massive failure in academic performance and production of half-becked graduands which affect the rate of productivity and human capital development. The study also demonstrated that Family of Seventh Order Implicit Hybrid Extended Block Adams Moulton Methods (FSOHEBAMM) is suitable for solving some Econometric Stochastic Time-Delay Differential Equation (ESTDDE) with new mathematical expression developed by Osu et al. (2023) for delay and noise terms computations. In tables 1 to 2 and figures 7 to 8, the computational solutions of step number $k = 2$ FSOHEBAMM produced better and faster solutions than step number $k = 3, 4$ by giving the Least Minimum

Absolute Volatility Errors (LMAVEs) at a Lower Computational Processing Unit Time (LCPUT). Comparing the computational solutions of this method with other methods in literature as shown in Tables 3 to 4 and Figures 7 to 8, the discrete schemes of FSOHEBAMM performed better by giving the Least Minimum Absolute Volatility Errors (LMAVEs) than other methods that applied interpolation procedures in computing the delay term and the noise term. The lower the Absolute Volatility Error (ARE), the lower the economic turbulence and also the lower the economic turbulence on Nigerian Universities, the lower its adverse effects on lecturers and students. More research should be applied using the proposed method for step numbers $k = 5, 6, 7, \dots$ in obtaining the numerical solutions of ESTDDE.

Recommendations

Following the numerical result obtained and discussion of findings of the proposed method in solving some examples of ESTDDE, this study recommends that the Nigeria Government should;

1. enforce full implementation of the removal of the Nigerian Universities from IPPIS.
2. Conduct a comprehensive feasibility study on the positive and negative impact of fuel subsidy removal.
3. Come up with compensating measures to cushion the adverse effects of economic turbulence.
4. Ensure quick implementation and payment of the officially approved 2024 National minimum wage increment to Nigerian workers.
5. Formulate and implement an economic policy for a price control system on goods and services.
6. Enforce quick and full implementation of FG-ASUU 2009 agreements for the innovative growth of Nigerian Universities.
7. Ensure total reduction of student school fees across all the public Universities in the country to eradicate the adverse effects of economic turbulence in Nigerian Universities.

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Appendix

Table 1: Absolute Volatility Errors of Example 1 using the FSOHEBAMM for $k = 2, 3$ & 4.

t	K = 2 Absolute Volatility Error	K = 3 Absolute Volatility Error	K = 4 Absolute Volatility Error
1	0.322170381	0.522170391	0.622170394
2	0.444863395	0.64486339	0.744863391
3	0.52065847	0.820658466	0.920658465
4	0.642473835	0.742473801	0.842473802
5	0.44449147	0.644491492	0.744491476
6	0.33928579	0.639285771	0.739285782
7	0.431482072	0.531481947	0.63148194
8	0.222780846	0.422780894	0.622780882
9	0.313807802	0.513807777	0.613807778
10	0.404792038	0.604792017	0.790438341
11	0.395817435	0.595817436	0.795817437
12	0.586914319	0.7869143	0.886914297
13	0.278093238	0.478093157	0.67809317
14	0.369357386	0.569357413	0.769357405
15	0.360707677	0.460707663	0.660707662
16	0.252143701	0.552143669	0.752143676
17	0.243664767	0.543664776	0.643664776
18	0.435270194	0.535270196	0.735270197
19	0.426959154	0.626959128	0.726959128
20	0.518730741	0.718730752	0.91873075
21	0.310584255	0.581058425	0.710584246
22	0.302518805	0.502518788	0.602518798
23	0.294533606	0.594533611	0.894533603
24	0.486627871	0.686627866	0.886627861
25	0.27880083	0.578800776	0.678800786
26	0.371051575	0.47105159	0.671051587
27	0.363379503	0.463379492	0.563379495

28	0.255783744	0.455783727	0.555783741
29	0.448263564	0.548263584	0.648263569
30	0.240818211	0.44081821	0.640818218

CPUT of FSOHEBAMM for $k = 2$ is 0.003s, $k = 3$ is 0.05s and $k = 4$ is 0.06s.

Table 2: Absolute Volatility Errors of Example 2 using the FSOHEBAMM for $k = 2, 3$ & 4.

t	K = 2 Absolute Volatility Error	K = 3 Absolute Volatility Error	K = 4 Absolute Volatility Error
1	0.24065388	0.429917144	0.629500144
2	0.278830916	0.578830908	0.378655331
3	0.277361151	0.306768578	0.400126528
4	0.082366973	0.116132299	0.124538783
5	0.13031049	0.236995236	0.13835184
6	7.1089E-05	0.329056196	0.522853401
7	0.256471767	0.151125121	0.441863736
8	0.370493021	0.277008978	0.755340254
9	0.179402522	0.027419743	0.168805171
10	0.087500065	0.091257607	0.197215056
11	0.015878236	0.128289342	0.259872576
12	0.202027826	0.089721817	0.178464396
13	0.058188744	0.145369945	0.129425605
14	0.322321523	0.510015515	0.694084363
15	0.111382421	0.065357958	0.349404015
16	0.453475425	0.338167282	0.402485416
17	0.103467863	0.426056033	0.253957893
18	0.47713948	0.296421498	0.027875032
19	0.147105489	0.475110589	0.106065722
20	0.548018814	0.119828539	0.661968313
21	0.324534506	0.375704536	0.631353053
22	0.75218858	0.682041596	0.834679047
23	0.233716936	0.095700965	0.35186571
24	0.160716276	0.412998136	0.535288236
25	0.241282868	0.011761666	0.664052305
26	0.044477956	0.196759814	0.480151794
27	0.217782061	0.618391039	0.693406723
28	0.000318195	0.147143678	0.254543404
29	0.087317185	0.364987869	0.267097907
30	0.316476811	0.671873017	0.970163419

CPUT of FSOHEBAMM for $k = 2$ is 0.004s, $k = 3$ is 0.06s and $k = 4$ is 0.07s