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Exploring Key Results on Soft Multigroups

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Abstract
In (1999), Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty. In (2011), Alkhazaleh et al introduced the notion of soft multiset as a generalization of Molodtsov's soft set. In the work of Nazmul and Samanta (2015), the concept of Soft multigroup was introduced. This paper continues the study of soft multigroup which has been explored over some time. Onoyima et al. (2024) defined the idea of a soft multigroupoid but the definition was incomplete. We propose the complete definition of soft multigroupoid and the necessary and sufficient condition for a soft multigroupoid to be a soft multigroup was given. It is shown that the union of two soft multigroups is again a soft multigroup if they are comparable. Finally, more results in the framework of soft multigroup were established.

Keywords; Soft Set, Soft Multiset, Soft Multigroups, Normal Soft Multigroups, Soft Multigroupoid

Introduction

The theory of multiset is an important generalization of classical set theory which has emerged by violating a basic property of classical set that an object can be a member of a set only once. That is, a multiset which has replaced a variety of terms viz; heap, list, sample, bag, fire sets and weighted set is a set where an object can occur more than once. Multisets are very important structures arising in many areas of mathematics and computer science such as data queries. For more, see Yager, (1987), Miyamoto, (2003), Blizard, (1989) etc. Molodtsov (1999) initiated a novel idea of soft set theory which is completely a new approach for modelling vagueness and uncertainty. Soft set theory has a very rich potential for applications in several directions few of which has been indicated in Molodsov, (1999). Some authors have also generalized the concept of multisets in the setting of fuzzy sets to form fuzzy multisets and soft multisets [(Zadeh, 1965), (Mujumdar & Samanta, 2012), (Miyamoto, 2003) and (Alkhazaleh et al, 2003)]. The notion of multigroup was proposed by (Nazmul et al, 2013) as an algebraic structure of multisets that extended the concept of group. The idea is consistent with other standard groups in (Ejegwa & Ibrahim, 2017; Ibrahim & Ejegwa, 2017; Barlotti & Strambach, 1991; Dresher & Ore, 1938; Prenowitz, 1943). The idea of multigroup by Nazmul et al. (2013) is well accepted due to the fact that it agrees with other non-classical groups and defined over multiset. see (Ejegwa & Ibrahim, 2017; Ibrahim & Ejegwa, 2017; Barlotti & Strambach, 1991) for details of multigroup. Again, the theory of group is one of the most essential algebraic structures in modern mathematics. Several authors have introduced the notion of group theory in fuzzy sets, soft set, fuzzy soft set, soft group, fuzzy multigroup see (Alkhazaleh et al., 2003; Singh et al., 2007; Biswas, 1989; Prenowitz, 1943). It's natural to establish the properties of group structures in soft multiset frame works and investigate their basic properties.

Preliminaries

In this section, we review some existing definitions for the sake of completeness and reference.

Definition 2.1. (Maji et al., 2003) ***(Soft Set)***. A pair (F, A) is called a soft set over X where F is a mapping given by $F: A \to P(X)$ and $A \subseteq E$.

Definition 2.2. (Molodtsov, 1999). ***(Union of Soft Sets).*** Let (F, A) and (G, B) be two soft sets over X. Then their union is a soft set (H, C) over X where $C = A \cup B$ and for all $\alpha \in C$,

$$H(\alpha) = \begin{cases} F(\alpha) if\ \alpha \in A/B \\ G(\alpha) if\ \alpha \in B/A \\ F(\alpha) \cup G(\alpha)\ if\ \alpha \in A \cap B \end{cases}$$

This relationship is written as $(F, A)\ \tilde{\cup}\ (G, B) = (H, C)$.

Definition 2.3. (Molodtsov, 1999). ***(Intersection of Soft Set)***. Let (F, A) and (G, B) be two soft sets over X. Then their intersection is a soft set (H, C) over X where $C = A \cap B$ and for all $\alpha \in C$, $H(\alpha) = F(\alpha) \cap G(\alpha)$. This relationship is written as $(F, A) \wedge (G, B) = (H, C)$.

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Definition 2.4. (Molodtsov, 1999). ***(Soft Subset).*** Let (F, A) and (H, B) be two soft sets over a common universe U, then we said that (H, B) is a soft subset of (F, A) if
$i.\ B \subseteq A$ and
$ii.\ H(e) \subseteq F(e)\ \forall\ e \in B$
We write $(H, B)\ \tilde{\subseteq}\ (F, A)$, (H, B) is said to be a soft super set of (F, A), if (F, A) is a subset of (H, B) we denote it by $(H, B)\ \tilde{\supseteq}\ (F, A)$.
Definition 2.5. (Aktas & Cagman, 2007). ***(Soft Group).*** Let G be a group and E be a set of parameters. For $A \subseteq E$, the pair (F, A) is called a soft group over G if and only if $F(\alpha) \leq G$ for all $\alpha \in A$ where F is a mapping of A into the set of subset of G.
Definition 2.6. (Aktas & Cagman, 2007). ***(Soft Subgroup).*** Let (F, A) and (H, K) be two soft group over G. Then (H, K) is a soft subgroup of (F, A) written as $(H, K)\ \tilde{<}\ (F, A)$ if

i. $K \subseteq A$,
ii. $H(x) \leq F(x)\ \ \forall\ x \in K$

Definition 2.7. (Blizzard, 1989). ***(Multiset).*** Let X be a set. A multiset A is characterized by a count function
$$C_A(x): X \longrightarrow \mathbb{N}.$$
Such that for $x \in Dom(A)$ implies $A(x) = C_A(x) > 0$, where $C_A(x)$ denotes the number of times an object x occurs in A. Whenever $C_A(x) = 0$ implies $x \notin Dom(A)$.the set of all multisets of X is denoted by $MS(X)$.
The root of a multiset A, denoted by A_* is defined as $A_* = \{x \in A: A(x) > 0\}$.
Definition 2.8. (Blizzard, 1989). ***(Submultiset).*** A multiset A is called a submultiset or a msubset of a multiset B denoted by $A \subseteq B$, if $C_A(x) \leq C_B(x)$, for all x.
A is a proper submultiset of B $(A \subset B)$ if $C_A(x) \leq C_B(x)$ for all x and there exists at least one x such that $C_A(x) < C_B(x)$.
Definition 2.9. (Blizzard, 1989). ***(Equal Multisets).*** Two multisets M_1 and M_2 are equal $(M_1 = M_2)$ if $(M_1 \subseteq M_2)$ and $(M_2 = M_1)$.
Definition 2.10. (Blizzard, 1989). ***(Intersection of Multisets).*** The intersection of two multisets M_1 and M_2 drawn from a set X is an mset M denoted by $M = M_1 \cap M_2$ such that for all $x \in X, C_M(x) = \min\{C_{M_1}(x),\ C_{M_2}(x)\}$.
Definition 2.11. (Blizzard, 1989). ***(Union of Multisets).*** The union of two multisets M_1 and M_2 drawn from a set X is an mset M denoted by $M = M_1 \cup M_2$ such that for all $x \in X, C_M(x) = \max\{C_{M_1}(x),\ C_{M_2}(x)\}$.
Definition 2.12. (Blizzard, 1989). ***(Sum of Multisets).***The addition or sum of two multisets M_1 and M_2 drawn from a set X results in a new multiset $M = M_1 \bigoplus M_2$ such that for all $x \in X$, $C_M(x) = C_{M_1}(x) + C_{M_2}(x)$.
Definition 2.13. (Alkhazaleh et al, 2003). ***(Soft Multiset).*** Let U be a universal multiset and E be a set of parameters. Then an ordered pair (F, E) is called a soft multiset where F is a mapping given by $F: A \rightarrow PW(U)$.
Definition. 2.14. (Alkhazaleh et al, 2003). ***(Soft Multi Subset).*** For two soft multisets (F, A) anUd (G, B) over a common universe U we say that (F, A) is a soft multi subset of (G, B) if
$i.\ A \subseteq B$ and
$ii.\ \forall\ \varepsilon \in A$, $F(\varepsilon)$ is a multi subset of $G(\varepsilon)$.
Definition 2.15. (Alkhazaleh et al, 2003). ***(Equal Soft Multisets)***. (F, A) and (G, B) over a common universe U are said to be equal if (F, A) is a soft multi subset of (G, B) and (G, B) is a soft multi subset of (F, A).
Definition 2.16. (Alkhazaleh et al, 2003).***(Intersection of Soft Multiset).*** The intersection of (F, A) and (G, B) over a common universe U is the soft multiset (H, C) where $C = A \cap B$ and for all $e \in C$,
$$H(e) = \begin{cases} F(e)\ \ if\ e \in A - B \\ G(e)\ if\ e \in B - A \\ F(e) \cap G(e)\ if\ e \in A \cap B \end{cases}$$
Definition. **2.17. (**Alkhazaleh et al, 2003).***(Union of Soft Multiset).*** The union of (F, A) and (G, B) over a common universe U is the soft multiset (H, C) where $C = A \cup B$ and for all $e \in C$,
$$H(e) = \begin{cases} F(e) if\ e \in A - B \\ G(e) if\ e \in B - A \\ F(e) \cup G(e)\ if\ e \in A \cap B \end{cases}$$
Definition. 2.18. (Nazmul et al, 2013). ***(Multigroup).*** Let X be a group. A multiset G over X is said to be a multigroup over X if the count function G or C_G satisfies the following two conditions:

i. $C_G(xy) \geq [C_G\ (x) \wedge C_G(y)],\ \forall\ x, y \in X$,
ii. $C_G\ (x^{-1}) \geq C_G(x),\ \forall\ x \in X$,

i.e., a multiset G is called a multigroup over X if $C_G(xy^{-1}) \geq [C_G\ (x) \wedge C_G(y)], \forall x, y \in X$. The set of all multigroups defined over X is denoted by $MG(X)$.
Definition. 2.19. (Nazmul et al, 2013). ***(Intersection and Union of Multigroup)***. Let$\{A_i\}_{i \in I},\ I = 1,2, \dots, n$ be an arbitrary family of multigroups of a group X. Then
$$C_{\cap A_i}(x) = \wedge\, C_{A_i}(x)\ \ \ \forall\ x \in X.$$

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$$C_{\cup A_i}(x) = \vee C_{A_i}(x) \quad \forall\, x \in X.$$

Definition. 2.20. (Nazmul et al, 2013). ***(Abelian Multigroup)***. Let $A \in MG(X)$. Then A is said to be abelian or commutative if for all $x, y \in X$, $C_A(xy) = C_A(yx)$.

Definition. 2.21. (Nazmul et al, 2013). ***(Submultigroup).*** Let $A \in MG(X)$. A submultiset B of A is called a submultigroup of A denoted by $B \subseteq A$ if B is a multigroup. A submultigroup B of A is a proper submultigroup denoted by $B \subset A$, if $B \subseteq A$ and $A \neq B$.

Definition. 2.22. (Ejegwa & Ibrahim, 2017). ***(Normal Submultigroup).*** Let $A, B \in MG(X)$ such that $A \subseteq B$. Then A is called a normal submultigroup of B if

$$C_A(xyx^{-1}) \geq C_A(y)\ \forall x\, y, \in X.$$

In general, $C_A(xyx^{-1}) = C_A(y)\ \forall x\, y, \in X$

Definition. 2.23. (Ibrahim & Ejegwa, 2017). ***(Automorphism of a Multigroup)***. Let X and Y be groups and let $A \in MG(X)$and $B \in MG(Y)$ respectively. Then a homomorphism f from X to X is called an automorphism of A onto A if f is both injective and surjective.

Definition. 2.24. (Ibrahim & Ejegwa, 2017). ***(Characteristic Submultigroup).*** Let $A, B \in MG(X)$ such that $A \subseteq B$. Then A is called a characteristic (f-invariant) submultigroup of B if $C_{\theta(A)}(x) = C_A(x)\quad \forall\, x \in X$ for every automorphism θ of X.

These basic definitions are necessary to accomplish the research objectives.

Definition. **2.25.** (Nazmul & Samanta, 2015). ***(Soft Multigroup)***. Let X be a group, M be a mgroup over X and $A \subseteq E$ be a set of parameters. A soft Mset (F, A) drawn from M is said to be a soft multigroup (shortly soft Mgroup) over M iff $F(\alpha)$ is a submultigroup of M, $\forall\, \alpha \in A$.

Definition. 2.26. (Nazmul & Samanta, 2015). ***(Soft Submultigroup).*** Let $(F_1,\ A_1)$ and $(F_2,\ A_2)$ be two soft multigroups over M. Then $(F_1,\ A_1)$ is said to be a soft submgroup of $(F_2,\ A_2)$ denoted by $(F_1,\ A_1) \mathrel{\tilde{\leqq}} (F_2,\ A_2)$ if $A_1 \subseteq A_2$ and $F_1(\alpha)$ is a submultigroup of $F_2(\alpha)$, $\forall\, \alpha \in A_1$.

Definition. **2.27.** (Nazmul & Samanta, 2015). ***(Soft Abelian Multigroup).*** A soft mgroup (F, A) over a mgroup M of a group X is called a soft abelian multigroup if $F(\alpha)$ is an abelian submgroup of M, $\forall\, \alpha \in A$.

Definition. **2.28.** (Nazmul & Samanta, 2015). Let (F, A) be a soft multigroup over M. Then

(i). (F, A) is said to be an identity soft mgroup over M if $F(\alpha) = [C_M(e)]_e$, $\forall\, \alpha \in A$, where e is the identity element of X.

(ii). (F, A) is said to be an absolute soft mgroup over M if $F(\alpha) = M$, $\forall\, \alpha \in A$.

Some Results on Soft Multigroups.

Some results and observations were explored in line with Nazmul and Samanta, (2015)

Definition. 3.1. ***(Normal Soft Submultigroups).*** Let $(F, A), (G, B) \in SMG(M)$ such that $(F, A) \sqsubseteq (G, B)$. Then (F, A) is termed a normal soft submultigroup of (G, B) if $\forall\, \alpha \in A$ and $\forall\, \beta \in B$, $C_{F(\alpha)}(xyx^{-1}) = C_{F(\alpha)}(x)\ \forall\, y \in F(\alpha)$ and $x \in G(\beta)$.

Theorem. 3.1. Let $(F, A), (G, B) \in SMG(X)$ such that $(F, A) \subseteq (G, B)$. Then the following statements are equivalent,

i. (F, A) is a normal soft submgroup of (G, B)
ii. $C_{F(\alpha)}(xyx^{-1}) = C_{F(\alpha)}(y)\ \forall\, x, y \in X$ and $\alpha \in A$
iii. $C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx)\ \forall\, x, y \in X$ and $\alpha \in A$.

Proof. $(i) \Rightarrow (ii)$. Assume (F, A) is a normal soft submultigroup of (G, B) from 3.1, it implies that $C_{F(\alpha)}(xyx^{-1}) = C_{F(\alpha)}(y)\ \forall\, x, y \in X$ and $\alpha \in A$.

$(ii) \Rightarrow (iii)$. Assume that $C_{F(\alpha)}(xyx^{-1}) = C_{F(\alpha)}(y)\ \alpha \in A$, it implies that $C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx)\ \ \forall\, x, y \in X$.

$(iii) \Rightarrow (i)$. Suppose that $\alpha \in A$, $C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx)\ \ \forall\, x, y \in X$, it then follows that (F, A) is a normal soft submultigroup of (G, B) since $(F, A) \subseteq (G, B)$

Proposition. 3.1. Let $(F, A) \in SMG(M)$. Then,

i. $C_{F(\alpha)}(e) \geq C_{F(\alpha)}(a)\quad \forall\, \alpha \in A, a \in M$
ii. $C_{F(\alpha)}(a^n) \geq C_{F(\alpha)}(a)\quad \forall\, \alpha \in A, a \in M.$
iii. $C_{F(\alpha)}(a^{-1}) = C_{F(\alpha)}(a)\quad \forall\, \alpha \in A, a \in M$

Proof. Let $(F, A) \in SMG(M)$ and $a, b \in M$, then for every $\alpha \in A$ we have

i. $C_{F(\alpha)}(e) = C_{F(\alpha)}(aa^{-1}) \geq \wedge\left[C_{F(\alpha)}(a), C_{F(\alpha)}(a^{-1})\right]$
$= C_{F(\alpha)}(a),\ \forall\, a, \in M$

ii. $C_{F(\alpha)}(a^n) \geq \wedge \left[C_{F(\alpha)}(a^{n-1}), C_{F(\alpha)}(a)\right] \geq C_{F(\alpha)}(a^{n-2}) \wedge C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(a)$
$\geq \left[C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(a) \wedge \ldots C_{F(\alpha)}(a)\right] = C_{F(\alpha)}(a).$

iii. Since $C_{F(\alpha)}(a^{-1}) \geq C_{F(\alpha)}(a)$
$= C_{F(\alpha)}([a^{-1}]^{-1}) \geq C_{F(\alpha)}(a^{-1}).$

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Hence $C_{F(\alpha)}(a^{-1}) \geq C_{F(\alpha)}(a)$

Proposition. 3.2. Let $(F,A) \in SMS(M)$. Then $(F,A) \in SMG(M)$ iff $\forall\, \alpha \in A$, $C_{F(\alpha)}(xy^{-1}) \geq \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)], \forall\, x, y \in M$.

Proof. Let $(F,A) \in SMG(X)$. Then $\forall\, \alpha \in A$, $C_{F(\alpha)}(xy^{-1}) \geq \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)]$, $\forall\, x, y \in X$. Therefore the condition is satisfied.

Conversely, let the given condition be satisfied. Now for $\forall\, \alpha \in A$, $C_{F(\alpha)}(e) = C_{F(\alpha)}(xx^{-1}) \geq \wedge\left[C_{F(\alpha)}(x), C_{F(\alpha)}(x)\right] = C_{F(\alpha)}(x)\ \forall\, x \in X ---------- i$

Again, $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(ex^{-1}) \geq \wedge\left[C_{F(\alpha)}(e), C_{F(\alpha)}(x)\right]$

$$= C_{F(\alpha)}(x)\ \forall\, x \in X -----ii$$

Also, $\forall\, \alpha \in A$, $C_{F(\alpha)}(xy) = C_{F(\alpha)}[x(y^{-1})^{-1} \geq \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y^{-1})]$

$$\geq \wedge\left[C_{F(\alpha)}(x), C_{F(\alpha)}(y)\right] ------- iii$$

Therefore, from (ii) and (iii) we have $(F,A) \in SMG(X)$.

Theorem. 3.2. Let M be a multigroup over a group X and (F_1, A), (F_2, B) be two soft mgroup over M. Then their union $(F_1, A)\grave{\cup}(F_2, B)$ is a soft multigroup over M if (F_1, A) and (F_2, B) are comparable.

Proof. Let $\alpha \in (A \cup B)$. Since (F_1, A) and (F_2, B) are comparable, then $F_1(\alpha) \leq F_2(\alpha)$ or $F_2(\alpha) \leq F_1(\alpha)\ \forall\, \alpha \in A$. Now since (F_1, A) is a soft multigroup, it implies that $F_1(\alpha)$ is a submultigroup of M and hence,

$C_{F_1(\alpha)}(xy) \geq \wedge\left[C_{F_1(\alpha)}(x), C_{F_1(\alpha)}(y)\right]$ and $C_{F_1(\alpha)}(x^{-1}) = C_{F_1(\alpha)}(x)$. Similarly, $F_2(\alpha)$ is a submultigroup of M and hence, $C_{F_2(\alpha)}(xy) \geq \wedge\left[C_{F_2(\alpha)}(x), C_{F_2(\alpha)}(y)\right]$

and $C_{F_2(\alpha)}(x^{-1}) = C_{F_2(\alpha)}(x), \forall\ x, y \in X$

Now, $C_{(F_1 \tilde{\cup} F_2)(\alpha)}(xy) = [C_{F_1(\alpha)}(xy) \wedge C_{F_2(\alpha)}(xy)] \geq C_{F_1(\alpha)}(x) \wedge C_{F_1(\alpha)}(y) \wedge C_{F_2(\alpha)}(x) \wedge C_{F_2(\alpha)}(y)$

$$= C_{(F_1 \tilde{\cup} F_2)(\alpha)}(x) \wedge C_{(F_1 \tilde{\cup} F_2)(\alpha)}(y)\ \forall\ x, y \in X.$$

Therefore, $(F_1 \grave{\cup} F_2)(\alpha)$ is a submultigroup of $\forall\, \alpha \in (A \cup B)$. Hence, $(F_1, A)\grave{\cup}(F_2, B)$ is a soft mgroup over X.

Theorem.3.3. Let (F,A) be a soft multigroup over M. If $C_{F(\alpha)}(x) \neq C_{F(\alpha)}(y)\ \forall\, x, y \in M$, then $C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx) = C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y), \forall\ \alpha \in A$

Proof. Let $x, y \in M$. Since $C_{F(\alpha)}(x) \neq C_{F(\alpha)}(y)$, it implies that $C_{F(\alpha)}(x) > C_{F(\alpha)}(y)$ or $C_{F(\alpha)}(y) > C_{F(\alpha)}(x)$. Suppose, $C_{F(\alpha)}(x) > C_{F(\alpha)}(y)$, then $C_{F(\alpha)}(xy) \geq C_{F(\alpha)}(y)$ and $C_{F(\alpha)}(y) = C_{F(\alpha)}(x^{-1}xy) \geq \wedge[C_{F(\alpha)}(x^{-1}), C_{F(\alpha)}(xy)]$

$$= \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(xy)] = C_{F(\alpha)}(xy)$$

It follows that $C_{F(\alpha)}(y) \geq C_{F(\alpha)}(xy) \geq \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)] = C_{F(\alpha)}(y)$.

Now we see that $C_{F(\alpha)}(xy) \geq \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)]$ and $\wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)] \geq C_{F(\alpha)}(xy)$

Thus, $C_{F(\alpha)}(xy) = \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)]$

Similarly, suppose $C_{F(\alpha)}(y) > C_{F(\alpha)}(x)$, we have that $C_{F(\alpha)}(yx) \geq C_{F(\alpha)}(x)$ and

$$C_{F(\alpha)}(x) = C_{F(\alpha)}(y^{-1}yx)$$
$$\geq \wedge[C_{F(\alpha)}(y^{-1}), C_{F(\alpha)}(yx)] = C_{F(\alpha)}(yx).$$

Thus we get $C_{F(\alpha)}(x) \geq C_{F(\alpha)}(yx) \geq \wedge[C_{F(\alpha)}(y), C_{F(\alpha)}(x)] = C_{F(\alpha)}(x)$

Clearly, $C_{F(\alpha)}(yx) = \wedge[C_{F(\alpha)}(y), C_{F(\alpha)}(x)]$

Therefore, $C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx) = C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y), \forall \alpha \in A$ and $\forall\, x, y \in X$.

Theorem. 3.4. Let $(F,A) \in SMG(X)$ and $a \in X$. Then, $C_{F(\alpha)}(ab) = C_{F(\alpha)}(e)$ if and only if $\forall\, \alpha \in A$, $C_{F(\alpha)}(a) = C_{F(\alpha)}(e)$.

Proof. Let $C_{F(\alpha)}(ab) = C_{F(\alpha)}(b)\ b \in X$. Then $C_{F(\alpha)}(a) = C_{F(\alpha)}(ae) = C_{F(\alpha)}(e)$.

Conversely, let $C_{F(\alpha)}(a) = C_{F(\alpha)}(e)$. Now since $C_{F(\alpha)}(e) \geq C_{F(\alpha)}(b)$, $b \in X$ we have $C_{F(\alpha)}(a) \geq C_{F(\alpha)}(b)$. Thus, $C_{F(\alpha)}(ab) \geq C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(b) = C_{F(\alpha)}(e) \wedge C_{F(\alpha)}(b)$. That is, $C_{F(\alpha)}(ab) \geq C_{F(\alpha)}(b)\ \forall\, b \in X$. But, $C_{F(\alpha)}(a) = C_{F(\alpha)}(a^{-1}ab) \geq C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(ab)$ and $C_{F(\alpha)}(a) \geq C_{F(\alpha)}(ab)$. It implies that $C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(ab) = C_{F(\alpha)}(ab) \leq C_{F(\alpha)}(b)$, $\forall\, b \in X$ and $\forall \alpha \in A$. And so, $C_{F(\alpha)}(b) \geq C_{F(\alpha)}(ab)\ \forall\, b \in X$. Hence, $C_{F(\alpha)}(ab) = C_{F(\alpha)}(b)$.

Theorem.3.5. Let (F,A) be a soft multigroup ove X. If for every $\forall\, \alpha \in A$, $C_{F(\alpha)}(a) < C_{F(\alpha)}(b)$ for some $a, b \in X$, then $C_{F(\alpha)}(ab) = C_{F(\alpha)}(a) = C_{F(\alpha)}(ba)$.

Proof. Suppose that $C_{F(\alpha)}(a) < C_{F(\alpha)}(b)$ for some $a, b \in X$, since $(F,A) \in SMG(X)$, then $C_{F(\alpha)}(ab) \geq C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(b) = C_{F(\alpha)}(a)$. Now, $C_{F(\alpha)}(a) = C_{F(\alpha)}(abb^{-1}) \geq C_{F(\alpha)}(ab) \wedge C_{F(\alpha)}(b^{-1}) = C_{F(\alpha)}(ab)$. Since $C_{F(\alpha)}(a) < C_{F(\alpha)}(b)$, $C_{F(\alpha)}(ab) < C_{F(\alpha)}(b)$.

Therefore, $C_{F(\alpha)}(ab) = C_{F(\alpha)}(a)$. Simirlarly, $C_{F(\alpha)}(ba) = C_{F(\alpha)}(a)$.

Theorem. 3.6. Let $(F,A) \in SMG(X)$ and $a, b \in X$. Then for every $\alpha \in A$, $C_{F(\alpha)}(ab) = C_{F(\alpha)}(b)$ if and only if $C_{F(\alpha)}(a) = C_{F(\alpha)}(e)$.

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Proof. Suppose $C_{F(\alpha)}(ab) = C_{F(\alpha)}(b)\ \ \forall\, b \in X$, then $C_{F(\alpha)}(a) = C_{F(\alpha)}(ae)$. Conversely, let $C_{F(\alpha)}(a) = C_{F(\alpha)}(e)$. since $C_{F(\alpha)}(e) \geq C_{F(\alpha)}(b)$ therefore we have $C_{F(\alpha)}(a) \geq C_{F(\alpha)}(b)$. Thus $C_{F(\alpha)}(ab) \geq C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(b) = C_{F(\alpha)}(e) \wedge C_{F(\alpha)}(b) = C_{F(\alpha)}(b)$.
That is, $C_{F(\alpha)}(ab) \geq C_{F(\alpha)}(b)\ \forall\, y \in X$. But $C_{F(\alpha)}(b) = C_{F(\alpha)}(aa^{-1}b) \geq C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(ab)$ and $C_{F(\alpha)}(a) \geq C_{F(\alpha)}(ab)$. This implies that $C_{F(\alpha)}(a) \wedge C_{F(\alpha)}(ab) = C_{F(\alpha)}(ab) \leq C_{F(\alpha)}(b),\ \ \forall\, b \in X$ and so $C_{F(\alpha)}(b) \geq C_{F(\alpha)}(ab)$. Hence, $C_{F(\alpha)}(ab) = C_{F(\alpha)}(b)\ \ b \in X$ and $\alpha \in A$.

Theorem. 3.7. Let (F, A) and (F, B) be a soft multigroup over M. Then the sum of (F, A) and (F, B) is a soft multigroup over X.

Proof. Let $x, y \in M$ and for any $\alpha \in A$ and $\beta \in B$ by definition of sum of two soft multigroups,

$$\begin{aligned} C_{(F.A)+(F.B)}(xy^{-1}) &= \wedge[C_{F(\alpha)}(xy^{-1}), C_{F(\beta)}(xy^{-1})] \\ &\geq \left(C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)\right) + (C_{F(\beta)}(x) \wedge C_{F(\beta)}(y)) \\ &= (C_{F(\alpha)}(x) + C_{F(\beta)}(x))\ \wedge (C_{F(\alpha)}(y) + C_{F(\beta)}(y)) \\ &= C_{F(\alpha)+F(\beta)}(x) \wedge\ C_{F(\alpha)+F(\beta)}(y) \end{aligned}$$

This implies, $C_{F(\alpha)+F(\beta)}(xy^{-1}) \geq C_{F(\alpha)+F(\beta)}(x) \wedge\ C_{F(\alpha)+F(\beta)}(y)$.

Hence, $(F, A) + (F, B)$ is a soft multigroup of X.

Theorem.3.8. Let (F, A) be a soft multigroup over M. Then, $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x^{n-1})\ \forall\, \alpha \in A, n \in \mathbb{N}$.

Proof. Let $x \in X$, $x \neq e$. Since x is finite, x has finite order say $n > 1$. Thus $x^n = e$ and so $x^{-1} = x^{n-1}$. Consequently, $F(\alpha)$ is finite since $(F, A) \in SMG(M)$, then we have

$$\begin{aligned} C_{F(\alpha)}(x^{-1}) &= C_{F(\alpha)}(x^{-1}e) = C_{F(\alpha)}(x^{n-1}x^n) \\ &\geq C_{F(\alpha)}(x^{n-1}) \wedge C_{F(\alpha)}(x^n) = C_{F(\alpha)}(x^{n-1}) \end{aligned}$$

$\Rightarrow C_{F(\alpha)}(x^{-1}) \geq C_{F(\alpha)}(x^{n-1})$.

and $C_{F(\alpha)}(x^{n-1}) = C_{F(\alpha)}(x^{n-1}x^n) = C_{F(\alpha)}\big((x^{n-2}x)x^n\big) \geq C_{F(\alpha)}(x^{n-2}x)\ \wedge C_{F(\alpha)}(x^n)$

$$\begin{aligned} &\geq C_{F(\alpha)}(x^{n-2})\ \wedge C_{F(\alpha)}(x) \geq C_{F(\alpha)}(x^{n-2}) \wedge \dots\ \wedge C_{F(\alpha)}(x) \\ &= C_{F(\alpha)}(x) = C_{F(\alpha)}(x^{-1}) \end{aligned}$$

$$\Longrightarrow C_{F(\alpha)}(x^{n-1}) \geq C_{F(\alpha)}(x^{-1})$$

Hence, $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x^{n-1})\ \forall\, \alpha \in A, \forall\, x \in M$.

Theorem. 3.9. Let $\{F_i,\ A_i\}_{i\in I}$ be a family of soft multigroups of M. If $\{F_i,\ A_i\}_{i\in I}$ have Sup/Inf assuming chain, then $\cup_{i\in I}\{F_i, A_i\}$ is a soft multigroup over M.

Proof. Let $F(\alpha) = \cup_{i\in I} f_i \alpha_i$, then $C_{F(\alpha)}(x) = \vee_{i\in I} C_{f_i(\alpha_i)}(x)$. We now show that
$C_{F(\alpha)}(xy^{-1}) \geq [C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)]\ \forall\, x, y \in M$.

If either $C_{F(\alpha)}(x) = 0$ or $C_{F(\alpha)}(y) = 0$, then the inequality holds. Let $C_{F(\alpha)}(x) > 0$ and $C_{F(\alpha)}(y) > 0$ then we have $\vee_{i\in I} C_{f_i(\alpha_i)}(x) > 0$, $\vee_{i\in I} C_{f_i(\alpha_i)}(y) > 0$.

Now by hypothesis, suppose $\exists\, a_0 \in I$ such that $C_{F(\alpha)_{a_0}}(x) = \vee_{i\in I} C_{f_i(\alpha_i)}(x)$ and also $\exists\, b_0 \in I$ such that $C_{F(\alpha)_{b_0}}(x) = \vee_{i\in I} C_{f_i(\alpha_i)}(x)$. Since $(F_i, \alpha_i)_{i\in I}$ have Sup/inf assuming chain, it follows that

i. $(F(\alpha))_{a_0} \subseteq\ (F(\alpha))_{b_0}$ or
ii. $(F(\alpha)_{b_0}) \subseteq\ (F(\alpha))_{a_0}$

i, Suppose $(F(\alpha))_{a_0} \subseteq\ (F(\alpha))_{b_0}$, that is, $C_{(F(\alpha))_{a_0}}(x) \leq C_{(F(\alpha))_{b_0}}(x)$, then

$$\begin{aligned} C_{F(\alpha)}(xy^{-1}) &= C_{(F(\alpha)_{b_0}}(xy^{-1}) \geq C_{(F(\alpha)_{b_0}}(x) \wedge C_{(F(\alpha)_{b_0}}(y) \\ &\geq C_{(F(\alpha))_{a_0}}(x) \wedge C_{(F(\alpha))_{a_0}}(y) = \vee_{i\in I}\, C_{F_{i(\alpha_i)}}(x)\ \wedge \vee_{i\in I}\, C_{F_{i(\alpha_i)}}(y) \\ &= C_{F(\alpha)}(x)\ \wedge C_{F(\alpha)}(y) \end{aligned}$$

ii. Suppose $(F(\alpha))_{b_0} \subseteq\ (F(\alpha))_{a_0}$ that is, $C_{(F(\alpha)_{b_0}}(x) \leq\ C_{(F(\alpha)_{a_0}}(x)$, then

$$\begin{aligned} C_{F(\alpha)}(xy^{-1}) &= C_{(F(\alpha)_{a_0}}(xy^{-1}) \geq C_{(F(\alpha)_{a_0}}(x) \wedge C_{(F(\alpha)_{a_0}}(y) \\ &\geq C_{(F(\alpha)_{b_0}}(x) \wedge C_{(F(\alpha)_{b_0}}(y) = \vee_{i\in I}\, C_{F_{i(\alpha_i)}}(x)\ \wedge \vee_{i\in I}\, C_{F_{i(\alpha_i)}}(y) \\ &= C_{F(\alpha)}(x)\ \wedge C_{F(\alpha)}(y) \end{aligned}$$

Hence, $F(\alpha) = \cup_{i\in I}(F_i \alpha_i) \in SMG(M)$.

Definition. (Onoyima et al., 2024**).** Let X be a group, M be a multigroup over X and $A \subseteq E$ be a set of parameter. A soft mset (F, A) drawn from M is said to be a **soft multigroupoid** over M iff $\forall\, \alpha \in A$, $F(\alpha)$ satisfy the condition $C_{F(\alpha)}(xy) \geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)\ \forall\, x, y \in M$

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Definition.3.2. Let X be a group, M be a multigroup over X and $A \subseteq E$ be a set of parameter. A soft mset (F, A) drawn from M is said to be a **soft multigroupoid** over M iff $\forall\, \alpha \in A$, $F(\alpha)$ satisfy the condition $C_{F(\alpha)}(xy) \geq \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)]\ \forall\, x, y \in X$ such that $x \neq y$.

Example. 3.1. Let $X = \{\pm 1, \pm i\}$ be a group of complex numbers, $A = \{\alpha_1, \alpha_2\}$ and $M = [\pm 1, \pm i]_{6544}$

if $F(\alpha_1) = [\pm 1, \pm i]_{4334}$ and $F(\alpha_2) = [\pm 1, \pm i]_{6223}$

$F(\alpha_1)$ is not a submultigroup of M similarly to $F(\alpha_2)$. Therefore (F, A) is a soft multigroupoid of M since $\forall\, x, y \in X, C_{F(\alpha)}(xy) \geq \wedge[C_{F(\alpha)}(x), C_{F(\alpha)}(y)]$.

Definition. 3.3. Let X be a group and $M \in MG(X)$. A soft multigroupoid (F, A) of M is called a soft multigroup over M if $\forall\, \alpha \in A$, $F(\alpha)$ satisfy the condition $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x)\ \forall\, x \in X$.

Remark. 3.1. Every soft multigroup is a soft multigroupoid but the converse is not true.

Theorem. 3.10. A soft multigroupoid (F, A) of a finite group X is a soft multigroup if $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x^{n-1})\ \forall\, x \in X$ and $n \in \mathbb{N}$.

Proof. Since (F, A) is a soft multihroupoid of X, then $C_{F(\alpha)}(xy) \geq \wedge\left(C_{F(\alpha)}(x),\ C_{F(\alpha)}(y)\right)\ \forall\, x, y \in X$ and $\alpha \in A$.

Suppose $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x^{n-1})\ \forall\, x \in X$ and $n \in \mathbb{N}$, then using the notion of soft submgroup repeatedly, we have

$$\begin{aligned} C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x^{n-2}x) &\geq \wedge\left(C_{F(\alpha)}(x^{n-2}),\ C_{F(\alpha)}(x)\right) \\ &\geq C_{F(\alpha)}(x) \wedge\ C_{F(\alpha)}(x) \wedge \dots \wedge C_{F(\alpha)}(x) = C_{F(\alpha)}(x) \end{aligned}$$

That is, $C_{F(\alpha)}(x^{-1}) \geq C_{F(\alpha)}(x)$

By the notion of soft multigroupoid, $C_{F(\alpha)}(x) = C_{F(\alpha)}((x^{-1})^{-1}) \geq\ C_{F(\alpha)}(x^{-1})$

It implies, $C_{F(\alpha)}(x) \geq C_{F(\alpha)}(x^{-1})$. Hence , $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x)$.

Therefore, (F, A) is a soft multigroup of $M \in MG(X)$.

Conclusion

The notion of soft multigroupoid was established and some of its related results are outlined. Finally, more results in soft multigroups were obtained. The concept of cuts of soft multigroups, factor soft multigroups, normalizer of soft multigroups and direct products of soft multigroups remain challenging in the frame work of soft multigroups.

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