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Mathematical Simulations and Analysis of Predator and Prey Model Dynamics with its Functional Response in Ecological Systems

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Abstract
The scope of this research includes the development of mathematical models, simulation and Analysis of Prey Model Dynamics with its functional response in an ecological system. The simulation results showed that the functional response affects the dynamics of prey and predator population in the presence of human disturbance with a monotone decrease in the population of the prey; the dynamics of the system led to cycles of predator-prey interactions with human action, demonstrating the classical predator-prey relationship; the noise contributes to the irregularities in the population dynamics thereby making the simulation more realistic and accounting for external factors that may influence the populations; the periodic force introduced oscillations or variations in the predator-prey populations over time and there is a phase shift continuum as the predator population grows and decline, allowing the prey population to recover as a result of the functional response present in the ecological system. Finally, the recommends the optimization of functional response and human disturbance, adaptive management of the ecosystem, advocates incorporating environmental data into numerical models for better dynamism, a collaboration between ecologists, mathematicians, and environmental scientists for better understanding of inter-plays and conduct educational outreaches to raise awareness about the importance of functional response and human action in ecological systems.

Keywords: Mathematical Simulation, Analysis, Functional Response, Ecological System, Predator-Prey

Introduction

The predator-prey model is the building block of the bio and ecosystems as bio-masses are grown out of their resource masses Das and Samanta (2012), Boyce et al. (2021). This model involves at least two species (the predator, for example, is a cat; and the prey, for example, is a rat). In their lifespan, the species involve, compete, develop or evolve and scatter or disperse for the purpose of scrambling for resources for sustenance. Based on their specific settings, the predator-prey can take the forms of parasite-host, tumour cells (virus)–immune system, resource–consumer, plant-herbivore etc. This is in agreement with the submission of Nkuturum (2012). The predator-prey embarks on the business of one species's loss is another's gain; this agrees with the writings of Taufiq and Agustito (2020). These interactions may have applications outside the ecosystems. Predation is the process of removing individuals from a lower trophic level to prevent monopoly competitive success among the prey. It allows increased diversity through the cropping principle Boyce et al. (2021) Lokta-Volterra Model (2024). This effect is demonstrated by removing top predators which results in drastic reductions in prey diversity as successful competitors freed from predation preempt resources – Molla et al. (2021). It has a major effect on the size of a population when the death rate exceeds the birth rate in a population. If predators are very effective at hunting their prey, the result is often a decrease in the size of the prey population. But a decrease in the prey population in turn affects the predator population. Wolves and Lions preying on ungulates, and Cats preying on Rats have their take limited by the effective defences of the prey animals such that their predation cannot interrupt the rapid population growth of the prey when food and decrease in the prey population in turn affects the predator population. Wolves and Lions preying on ungulates, and Cats preying on Rats have their take or catch limited by the effective defenses of the prey animals such that their predation cannot interrupt the rapid population growth of the prey when food and population dynamics produce exponential increase, but relatively high predator densities accentuate population crashes that follow, these are in line with thoughts of Mukhopadhyay and Bhattacharyya (2016), and Haque et al. (2023).

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Predation can be a powerful determinant of community structure as submitted by Nkuturum (2012), Sibenaller (2012), and Sun et al. (2012); with a dynamic influence on the numbers and quality of both predator and prey. This acts as an important agent of natural selection in both groups. However, there is an imbalance in the diversity of the ecosystem, which is a system of plants, animals and other organisms interacting within themselves and non-living components of their environment. For example, a lake or forest. There are "natural" and "managed" (that is farms or market gardens) ecosystems. Today, few ecosystems remain untouched by human activities. Managed ecosystems are essential to our survival by reducing competition through the removal of non-useful species (that is weeds). People are able to intensify food and other natural materials production. These processes often reduce species diversity. But there are instances where human management actually increases species diversity. No simple relationship exists between the ecosystem and ecological processes. An ecological system is open, thereby allowing interaction that is non-linear and may be noisy Sivakumar and Senthamarai (2020). The model will explain the interaction between the species and their natural environment which is the ecological system.

Materials and Methods

Model Formulation

The numerical analysis of predator-prey relationship with its functional response can be described using a system of differential equations. The mathematical model can be expressed as follows:

$$\frac{dR}{dt} = a_1 R(t) - a_2 R(t)C(t) - a_3 R(t)$$

$$\frac{dC}{dt} = -b_1 C(t) + b_2 R(t)C(t) - b_3 C(t)$$

Explanation of the Parameter Values Used

The given system of equations represents a basic prey-predator model, describing the dynamics of a prey population (R) and a predator population (C) in an ecosystem. Here's an explanation of the parameters used in the equations:

a_1: This parameter represents the intrinsic growth rate of the prey population (R) in the absence of predators. It indicates how fast the prey population can increase under favourable conditions.

a_2: This parameter represents the predation rate, indicating how quickly the predator population (C) consumes the prey population (R). It signifies the impact of predators on the prey population.

a_3: This parameter represents the rate at which the prey population (R) experiences natural death or mortality unrelated to predation or human disturbance. It accounts for factors that reduce the prey population, such as disease, accidents, or resource limitations.

b_1: This parameter represents the natural death rate of the predator population (C) in the absence of prey. It indicates how quickly predators die when there's no prey available to sustain them.

b_2: This parameter represents the efficiency with which predator population (C) converts consumed prey into its growth. It quantifies how effectively the predator population utilizes the consumed prey to increase its numbers.

b_3: This parameter represents the natural death rate of the predator population (C) unrelated to prey consumption or human disturbance. It accounts for factors that reduce the predator population, similar to parameter 'c' for the prey population.

Assumptions

1. No Immigration/Emigration: The model assumes that there is no movement of prey or predator populations into or out of the system (no immigration or emigration). This simplifies the model to a closed ecological system.
2. Continuous Interaction: The interactions between prey and predators are assumed to be continuous, without discrete events. In reality, interactions may be discrete, such as individual predation events.
3. Instantaneous Response: The model assumes that prey and predators respond instantaneously to changes in population sizes and other factors. This simplification ignores time delays in response due to factors like reproductive rates.
4. No Age or Stage Structure: The model considers all individuals in the populations as homogeneous and does not differentiate between different age groups or life stages. In reality, age and life stage can impact population dynamics.
5. No Other Predators: The model assumes that predators only consume the prey species in question and do not have any other sources of food.
6. Constant Parameters: The parameters (a_1,a_2,a_3,b_1,b_2,b_3) are assumed to be constant over time and not influenced by the population dynamics themselves. In reality, some parameters could change due to interactions between the species or due to external factors.

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7. No Environmental Variation: The model assumes a constant and unchanging environment, ignoring fluctuations in factors like habitat quality, resource availability, or climate.

Techniques Used to Solve the Model

In ecological research, mathematical models play a crucial role in understanding the dynamics of species interactions within ecosystems. The prey-predator model with human disturbance is a classic example of such a model, providing insights into how external factors can influence population dynamics. MATLAB, a versatile programming environment, offers an excellent platform for simulating and analyzing such models. In this walk-through, we will illustrate how to use MATLAB to simulate the prey-predator model with human disturbance and visualize the results.

1. Step 1: Define the Model Equations:
   Start by defining the differential equations that describe the dynamics of the prey (R) and predator (C) populations.
2. Step 2: Set Parameters and Initial Conditions:
   Assign values to the parameters (a_1,a_2,a_3,b_1,b_2,b_3) and define the initial populations of prey (R) and predator (C).
3. Step 3: Define the Differential Equations:
   Create MATLAB functions that define the differential equations. These functions will take the current time (t), prey population (R), and predator population (C) as inputs and calculate the corresponding rates of change.
4. Step 4: Numerical Integration:
   Use MATLAB's built-in numerical solver, ODE45, to integrate the differential equations over a specified time span.
5. Step 5: Visualize the Results:

Finally, plot the simulation results using MATLAB's plotting functions. Using MATLAB, we have successfully simulated the dynamics of a prey-predator model and visualized the results. MATLAB's computational capabilities and visualization tools make it a valuable tool for understanding complex ecological systems and their behaviours over time. This tutorial provides a basic introduction to running simulations in MATLAB; more advanced features and techniques can be incorporated as needed for specific research questions.

Results

The main components of this session are the presentation of the results of the simulations, discussion findings and the interpretation of the findings about the study of objectives. The simulation of the model will be done with Mathlab.

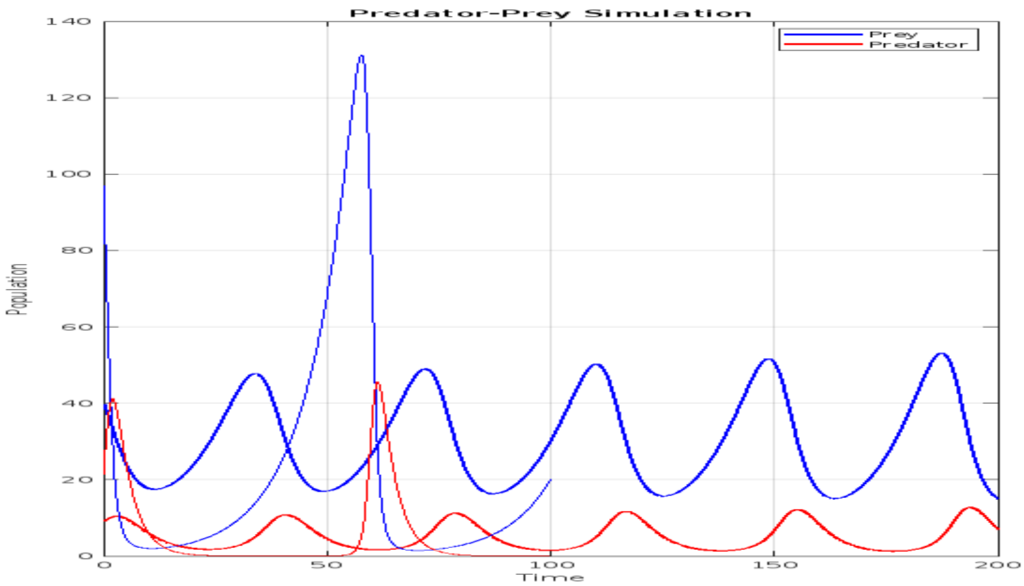

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Fig1: Simulation of The Predator-Prey Population Against Time

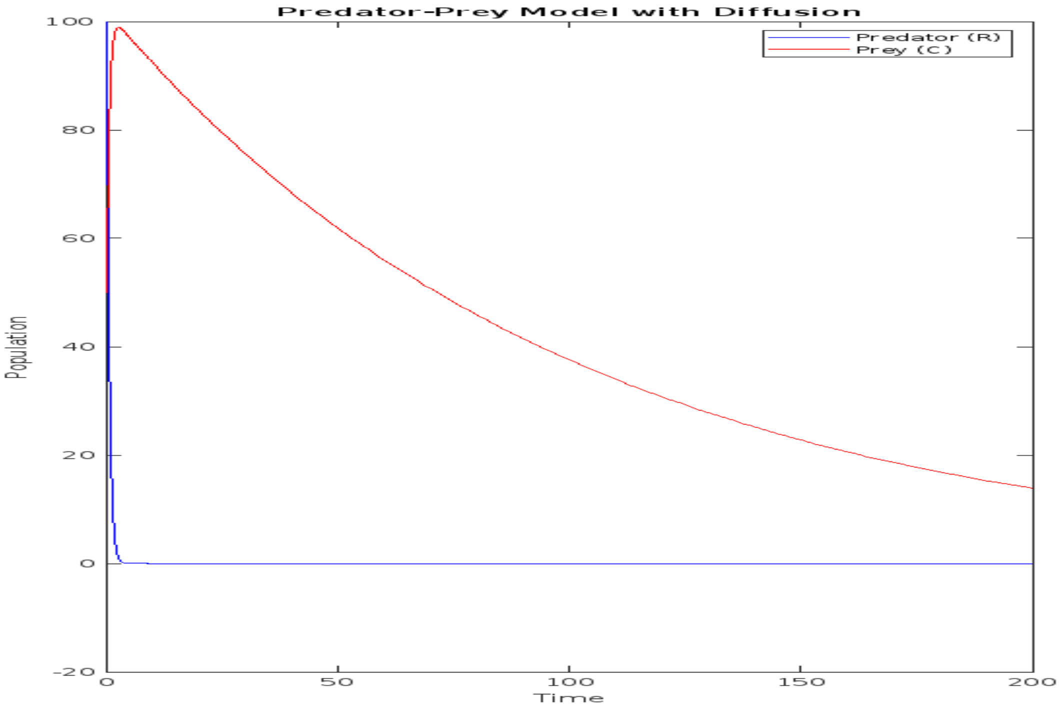

Fig 2: **simulation of the predator-prey against time with diffusion**

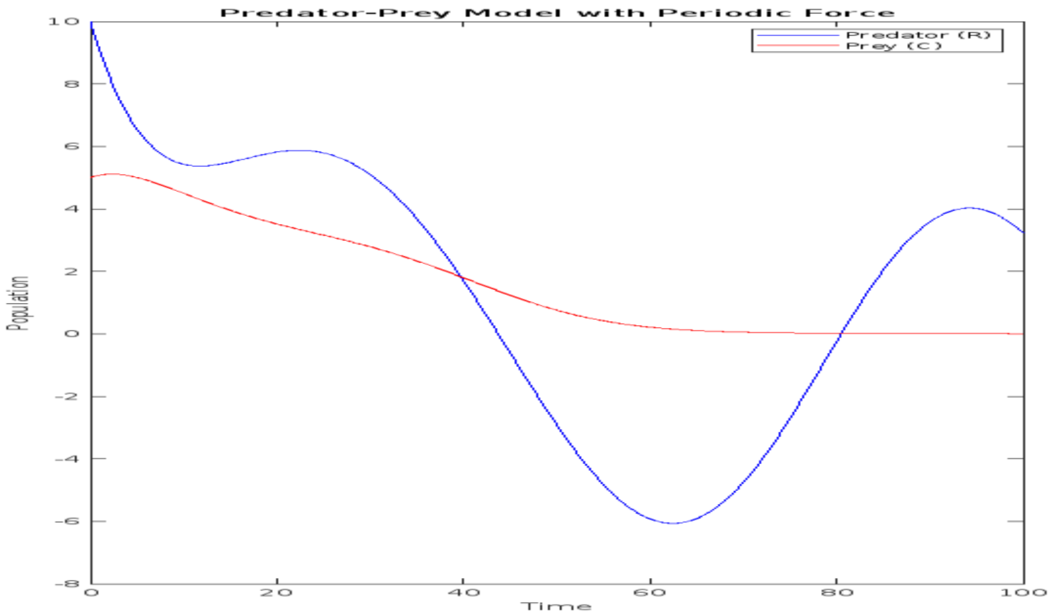

Fig 3: **simulation of the predator-prey against time with periodic force**

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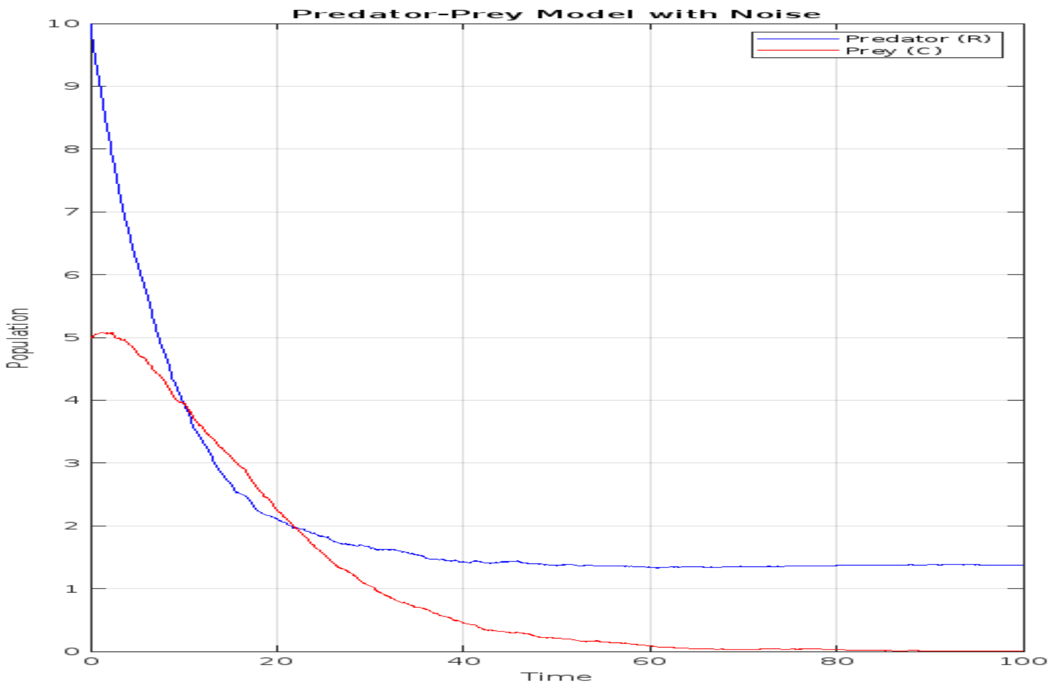

Fig 4: **simulation of the predator-prey against time with noise**

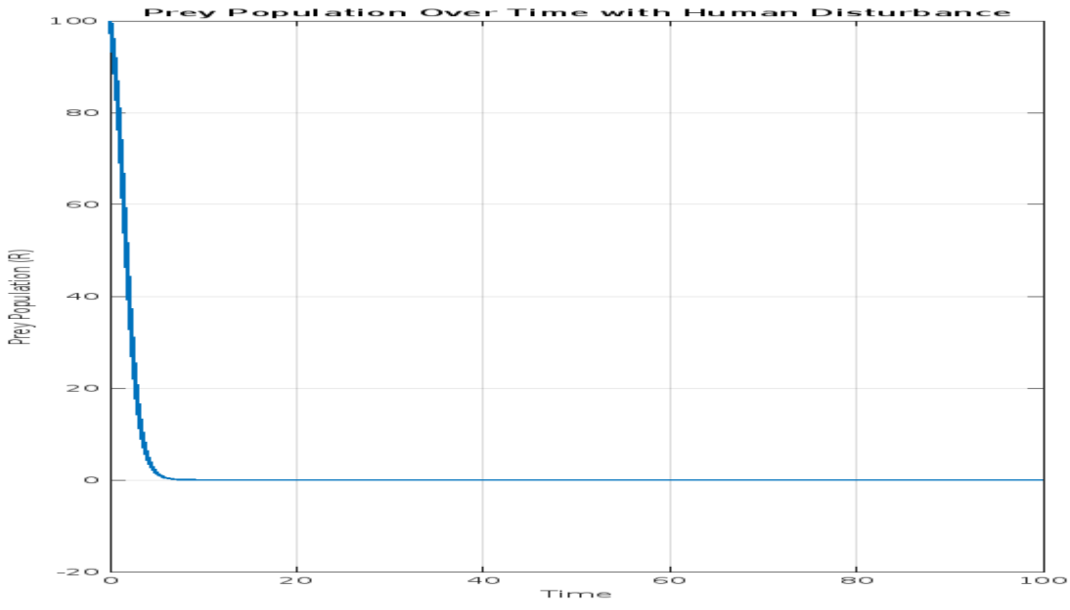

Fig 5: **simulation of the predator-prey against time with human disturbance**

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Discussion

From Figure 1 (simulation of the predator-prey population against time), the explanation for the simulation is that it models the classic predator-prey relationship, where the prey population grows, but as it increases, it becomes more attractive to predators, causing the predator population to grow. As the predator population grows, it starts to limit the prey population by increased predation. This, in turn, causes the predator population to decline, allowing the prey population to recover, and the cycle continues. From Figure 2 (simulation of the predator-prey against time with diffusion), the predator population (red curve) may exhibit oscillations, influenced by the interplay of birth, death, and diffusion dynamics. The prey population (blue curve) may show responses to predation, reproduction, and diffusion effects. The dynamics of the system could lead to cycles of predator-prey interactions, demonstrating the classical predator-prey relationship. In the graph, observe how the predator and prey populations evolve. The peaks and troughs in the populations can indicate cycles of predation and recovery. From Figure 3 (simulation of the predator-prey against time with periodic force), the graph will show how the predator and prey populations change over time in the presence of the periodic force. The periodic force can introduce oscillations or variations in the predator-prey populations over time. Interpret the graph by observing the dynamics of the predator (R) and prey (C) populations in response to the periodic force.

From Figure 4 (simulation of the predator-prey against time with noise), the added noise introduces random variations in both predator and prey populations. This reflects the inherent unpredictability or stochastic nature of ecological systems. The noise contributes to the irregularities in the population dynamics, making the simulation more realistic and accounting for external factors that may influence the populations. From Figure 5 (simulation of the predator-prey against time with human disturbance), in the graph, we should observe the dynamics of the prey population over time, considering the impact of human disturbance and analyze how the populations interact and evolve based on the given graph. The disturbances from humans affect the prey population, and you can observe how these dynamics unfold in the graph. The summary of the findings of the study can be given as follows with regard to the objective of this research;

i. As the predator population grows, it starts to limit the prey population by increased predation. This, in turn, causes the predator population to decline, allowing the prey population to recover, and the cycle continues.
ii. In the graph, we observe how the predator and prey populations evolve, the dynamics of the system could lead to cycles of predator-prey interactions, demonstrating the classical predator-prey relationship.
iii. The periodic force can introduce oscillations or variations in the predator-prey populations over time. Interpret the graph by observing the dynamics of the predator (R) and prey (C) populations in response to the periodic force.
iv. The noise contributes to the irregularities in the population dynamics, making the simulation more realistic and accounting for external factors that may influence the populations.
v. The disturbances from humans affect the prey population, we observe how these dynamics unfold in the graph.

Conclusion

This study on simulation and numerical analysis of the predator-prey relationship with functional response has helped to gain a better understanding of the fundamental mechanisms that drive ecosystem dynamics and aids in the conversation and management of the forcing function. This showed that with the presence of the functional response, the predator-prey dynamics oscillates completely in an ecosystem.

Recommendations

In a numerical analysis of a predator-prey system with a relationship to a functional response, consider the following recommendations:

i. Optimization of functional response: Explore ways to optimize the functional response, to enhance ecosystem stability or achieve specific conservation goals, considering ecological and mathematical perspectives.
ii. Adaptive Management Strategies: Develop adaptive management strategies that can respond to changes in the functional response, enabling a more flexible and resilient approach to ecosystem management.
iii. Incorporate Environmental Data: Integrate additional environmental data into the numerical models to better capture the dynamics of the f functional response, and its influence on predator-prey interactions.
iv. Collaborative Research: Foster collaboration between ecologists, mathematicians, and environmental scientists to gain a comprehensive understanding of the interplay between the functional response, and predator-prey dynamics.

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v. Educational Outreach: Conduct educational outreach programs to raise awareness among the public about the importance of the functional response in ecological systems and the need for sustainable management practices.

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