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Application of Group Theory in Analyzing Operation Pipeline Systems

***Festus, U., & Nwagor, P.**
Department of Mathematics/Statistics, Ignatius Ajuru University of Education, Rivers State, Nigeria

***Corresponding author email**: festusudochukwu457@gmail.com

Abstract
Considering the barrier to effective communication, and in order to enhance network operations, this research work presents an algebraic approach, taking into account the obstacle to efficient communication computation. This research effort, which is diverse in content but consistent with results from previous works in this area of study, applies group theory to address the pipeline dilemma facing telecommunications businesses. The purpose of the pipeline entity group is to analyze the operation of pipeline entities and to transmit information within the pipeline system. By the application of group theory, the result is justifiable for the symmetric and translation operations pipeline entity described to be a member of the group elements of the order 12 pipeline entity group. The transformation is classified into six (6) dimensional types and is defined as a characteristics transformation matrix that satisfies the property of group associativity, closure, also the associative, distributive, and inverse transformation properties of group based on the expression of the pipeline entity group model, two-dimensional natures, symmetry principal, 3 Ordered model symmetry transformation procedure components of the data framework. The flux conservation principle and pipeline network measurements are taken into consideration when establishing the equation for pipeline system network traffic. For the purpose of developing a sophisticated multi- and mobile e-commerce application model and building it for telecom businesses and operators, the relationship governing the transition of flow states within the pipeline system's solution framework is determined.

Keywords: Group, Symmetry, Pipeline And Pipeline Systems, Matrix, Networking And Networks

Introduction

Ever since the first nomadic people started to settle approximately 7000 years ago, humans have tried to regulate the flow of water. Water was transported by the Romans via pipes and channels, but they were unaware of this connection, which took Leonard da Vinci and Castelli to rediscover. 1500 years or so later. It is acknowledged that Leonardo da Vinci (1452-1519) was the first to comprehend and articulate the law of continuity. Castelli is credited with formulating continuity mathematically between 1577 and 1644. Resistance, as shown by Isaac Newton (1542–1727), is proportional to the square of the velocity. Numerous mathematical techniques have been employed to address issues pertaining to the networking and transmission (distribution) of commodities, services, and information, as applicable. In this study, we aim to use group theory to the solution of pipeline communication difficulties. In mathematical research, group theory is fundamental and essential to the analysis of algebraic structures. Group theory is used to one-dimensional pipeline content in this research work, which models and analyzes pipeline entities into group elements and uses them to distribute or offer high-quality services (such as water, network, communication, and transportation).

Group

A group can be thought of as a collection of individuals with distinct identities or as a collection of elements having noteworthy qualities. However, under the framework of algebra, Think of a binary operation * on a non-empty finite or infinite set of items, G. If the algebraic system meets the requirements of closure, associativity, identity, and inverse characteristics, it is considered a group.

Mathematically as $\forall a, b, \ldots .. \in G, a * b \in G.$

By associative, $\forall a, b, \ldots .. \in G, a*(b*c) = (a*b)*c \in G.$ whereas for identity property, an element say

$\ell \in G, such\ that\ \forall a, b, \ldots .. \in G, a*\ell = \ell * a = a.$s (1)

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Finally, by inverse property, $\forall a,b,.....\in G,$ there exists an inverse, $a^{-1},b^{-1},...\in G,$ such that $a^{-1}*a=\ell\in G.$
Cancellation law $ay*za=a\Rightarrow z=y=a^{-1}\in G.$ (2)
also holds in a group G. (Gupta, 2019).
In addition to these properties of the group, when a group $(G,*)$ satisfies the commutative property in addition to all other group qualities, it is said to be abelian. Mathematically by commutative law if: $a*b=b*a$ holds for $\forall$ a, b $\in$ G. (Rotman, 2003).
Example 1.1
The group $H=\{a^n: n\in Z\}$ is an abelian group because for every element *a,..v, u...z in H*, $v*u=u*v.$
By identity, let $u=x^s$ *and* $v=x^t$ *then* $uv=x^s x^t=x^{s+t}=vu$ therefore *uv=vu* (3)
Also $u^{-1}v^{-1}=(x^s)^{-1}(x^t)^{-1}=x^{-(s+t)}=(uv)^{-1}$ (4)
That is $(uv)^{-1}=u^{-1}v^{-1}$ a group in which the group operation is not commutative is said to be "non-abelian group" or non-commutative.

Subgroup
A set of non-empty elements G that meet the closure, associative, identity, and inverse qualities can be subjected to an algebraic operation *, which is defined by $(G,*)$. Also, let $H\subseteq G$ such that $H\neq 0,$ and $\bullet$ be the restriction of * to *H* that is $\forall a,b,c\in G, a*b=a\bullet b\in H.$ We said $(H,\bullet)$ is a sub-group of G if it satisfies the following properties

1. . the operation is closed: $\forall$ a, b, c.. $\in$ G, a $\bullet$ b $\in$ H
2. the operation is associative: $\forall a,b,.....\in G, a\bullet(b\bullet c)=(a\bullet b)\bullet c\in H.$
3. $\forall a,b,.....\in G, \exists$ an identity element $\ell\in$ H $\ni$ a $\bullet\ \ell$ = a w.r.t the operarion $\bullet$.
4. $\forall a,b,.....\in G, \exists$ an element $a^{-1}, b^{-1}\in H$ $\ni\ a^{-1}\bullet a=a\bullet a^{-1}=\ell=b^{-1}\bullet b$

The concept of Subgroup could be linked to the basic knowledge of subset G. (Asibong-Ibe, 1992).
More so, property (1), and (4) are important conditions necessary and sufficient for a subset of group G to be a subgroup, (Gupta, 2019).

Further analysis shows that the symmetry of a group of object consists of all transformations that leave the object unchanged, with the composition of these transformations serving as the group operation. (Rose, 1978).
This is group symmetry of a regular shape thus includes rotation and reflections with respect to the group order (Gardiner, 1980).
The order of a group G is simply the number of elements in group G (Asibong-Ibe, 1992). Each of these elements can generate a cyclic of the group in a definite operator. Thereby defining a cyclic group to be a group that be generated by a single element (Asibong-Ibe, 1992).

Properties associated with group

i. Homomorphism
The mapping $f:G\rightarrow H$ from a group G to H satisfying $f(x\,o\,y)=f(x)*f(y)\forall, y\in G$ is said to be group homomorphism (Hell, & Nesetril, 2006). This is characteristic as shown in this research analysis and also helps to determine the efficiency of quality service delivery.
Example 1.3
The mapping $f:R\rightarrow R^+$ defined by $f(x)=e^x$ is a group homomorphism that transforms the semigroup of positive reals into the abelian group of reals. We note that

$$f(x+y)=e^{x+y}=e^x e^y \text{ and } f(x)f(y)=e^x e^y \text{ therefore, } f(x+y)=f(x)*f(y) \quad (5)$$

ii. Isomorphism

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A homomorphism from G to H is called isomorphism if it is bijective (one to one and unto). Asibong-Ibe (2019).

Pipeline, pipeline group and pipeline group stages

Pipeline group refers to the collection of symmetry transformations applied to a pipeline entity. In general terms, long pipes are believed to be included in the definition of the term "pipeline." for the purpose of transporting liquids over short, medium, or long distances using a large pipe that is usually buried. An additional type of pipeline used in computing is a linear sequence with a specific modulus. It's the void left behind when a massive wave breaks, especially when surfing.

The examination of an entity in an abstract group form to transport a substance by means of structuring the channel (pipeline) flow or pipelining is the only topic of this research activity, and the action word "pipeline" is located there (Antaki, 2003).

The stages involved in a Pipeline system include

a) Instruction Fetch aka (Undertaking pipeline).
b) Instruction Decode or (Service Pipeline).
c) Instruction Execute.
d) Memory Access or (Logistics Pipeline).
e) Write Back or (Data Pipeline)

Concepts involved in the pipeline group

i. Pipeline Entity Element

This is an abstract operation $(*,+,o,\text{etc.})$ unit that changed the state of the pipelines functioning in the logical and estimative communication pipeline (Jianqing et al., 2018).

ii. Pipeline Object Status

Treatment of the state of quantities is equivalent to object minus (OM) defined in the information model of pipeline objects $R_1, R_2, B_1.....$ The pipeline object (O) used in this work to express the pipeline state model is defined to be superpositioned state vector quantities of $R_1, R_2, B_1.....$ is said to be $OM = o\{R_1, R_2, B_1.....\}$abbreviated as O and the condition of A at various points through the transformation $A^1 = gA,\ g \in G$ is is A^1 where G is pipeline entity group.

Network and Network System

This is an operating system designed specifically for a network device, like a firewall, switch, or router (Dalamu, 2019).

Statement of the problem

A pipeline network is a certain type of communication of a group defined in terms of a fixed element of the group. This research focused on the transformation of group elements which worked on matrix groups and permutation groups on a given vector space while maintaining their intrinsic structure.

Numerous works done on group theory failed to define the pipeline matrix group system of flux transmission, and without simplified mathematical formulation of relevant network communication problems, major works on pipeline group failed to illustrate and examine the symmetry conversion of matrix pipeline components.

Motivation

The primary aim of this research is to use Group theory in the analysis of pipeline networking. The key objectives of the study are to;

a) Resolve pipeline crisis by analysis of pipeline entity group model for Telecommunication Companies and operators alongside other distribution companies.
b) Establish the equation for the analysis of pipeline flux traffic based on the flux law of conservation principle and also the pipeline network matrix.
c) Obtain the Solutions framework for the transition relationships of flow states within a pipeline system.

Mathematical Formulation

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The use of group theory and other techniques to elevate abstract problems to the status of analytical problems has become evident with the current technological advancements and electronic computation of mathematical problems. In certain respects, digital computing has caused the majority of students who could not afford such technology to avoid abstract mathematics, depriving them of the fundamental understanding, significance, and, to some extent, applications of the subject.

Nonetheless, given the prevalence of algebra in practically all facets of life, this study tackles a real-world communication problem in a pipeline system that is modelled, examined, and evaluated in a group setting. Several important theorems and mathematical formulations related to group theory and networking are presented and validated. Furthermore, the approaches utilized in the analysis of the network problem for telecommunication operations are provided with an understanding of these theorems.

Theorem 2.1

Let G be a group. Then for any element $a, x, y \in G,$ thus $ax = ay \Rightarrow x = y$

Proof:

$a \in G,$ the inverse a^{-1} also belongs to $G.$ then we have $a^{-1}(ax) = a^{-1}(ay)$

Suppose $but\ \ a^{-1}(ax) = (a^{-1}a)x = ex = x$

and $a^{-1}(ay) = (a^{-1}a)y = ey = y$

Therefore $x = a^{-1}(ax) = a^{-1}(ay) = y$ Hence $but\ \ ax = ay \Rightarrow x = y$

This demonstrates that both the left and right cancellation properties hold, and during the course of the investigation, it fulfils the quality of service delivery with an identical outcome regardless of formation or origin. Page 31 provides an illustration of this.

Theorem 2.2:

Let $f : G \rightarrow H$ function as group homomorphism with kernel k. Then k constitute a normal subgroup within $kerf = \{x \in G | f(x) = e_H \in H\}$

Proof:

$Let\ k = \ker f$ and suppose that $xy \in k$ then $f(x) = f(y) = e_H$ and

$f(x).f(y) = e_H\ but\ f(xoy) = f(x) * f(y)$ therefore $f(x * y) = e_H$

and also for each $x \in k,\ f(x^{-1}) = f(x)^{-1} = e_{H^{-1}} = e_H, hence\ \ x^{-1} \in k$

$evidently\ \ k\ is\ a\ subgroup\ of\ G.$

secondly to show that k is normal subgroup of $G.$

$Let\ u \in G, v \in G.$ then $f(u^{-1}vu) = f(u^{-1})f(v)f(u)$ but $f(v) = e$ and $f(u^{-1}) = f(u)^{-1}$

certainly $f(u^{-1}vu) = f(u^{-1})f(v)f(u) = f(u^{-1})ef(u) = e_{H,}$

showing that $v \in H \Rightarrow u^{-1}vu \in k\ \ \forall\ u \in G.$

$therefore$

$u^{-1}ku \subseteq k$ and k must be a normal subgroup in the group $G.$

Lemma 2.1:

Let G be a group and $g \in G$. The map $f_y : G \rightarrow G$ given by $f_y(x) = y^{-1}xy,$ for all $x \in G$ is an inner automorphism.

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Data Analysis

This work presents the analysis of "secondary data of telecommunication network problem ranging from the e-commerce to architectural designs judged by group of order 12. The application of the Distribution network model and equations, using graphs as a case study for efficient delivery of services and discrete value generation as an example. A proper model simulation between the methods stated above are properly defined and stated as used in this work. The application of pipeline state play an important role here. The state of pipeline object which can be modelled into six (6)

but three (3) basic states (Jianqing et al., 2018) namely; The Ready State, The Running state and The Blocked State. With a pipeline object information model, $OM = o\{R_1, R_2, B, ...\}$ abbreviated as 0. The element 0 is a vector operation and superposition of $R_1, R_2, B, ...$ expresses amounts. It is an operation vector based on the pipeline states' Stages such that the pipeline entity's operation is equal to the state quantities' treatment $R_1, R_2, B, ...$ depicts the pipeline state model for the pipeline system, with ***A*** serving as the state limitation point and the stages of the pipeline entities being a group G that satisfies all recommended properties of its kind. The state of A at different times of the transformation is defined by $A^I = gA, \ g \in G,$ where G is a pipeline entity group. O. Blocked
The flow chart (flow in-flow out) diagrammatically represents the states used in this modelling simulation.

Group of order 12.
Let G be a group of order 12. Then there are up to isomorphism, exactly five group of order 12: $C_{12}, \ C_2 \times C_6, \ C_3 \times C_3,$ Alternating group $A_4, \ C_4 \times S_3$. There are two and three Abelian and non-Abelian groups respectively.

Table 1: Table of C_{12} the cyclic group of order 12 described via the generator a with relation $a^{12} = 1$

*	e	**a**	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}
e	e	a	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}
A	a	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	e
a^2	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	e	a
a^3	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	e	a	a^2
a^4	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	e	a	a^2	a^3
a^5	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	e	a	a^2	a^3	a^4

The group of order 12 can be permutated or have dihedral D6 group given as.

$$\left\{ \begin{array}{l} \mathrm{D}_6 = (1), \quad (123456), \quad (165432), \\ (135)(246), (153)(264), \quad (26)(35), \\ (13)(46), \quad (15)(24), \quad (14)(23)(56), \\ (14)(25)(36), \quad (12)(36)(45), \quad (16)(25)(34) \end{array} \right\}$$

which is generated by the following permutations of the vertices of hexagon, $a = (1,2,3,4,5,6,)$, $b = (2,6)(3,5)$
Now assuming G is a non-Abelian group of order 12 then G has an element of order 6 and no element of order 12.
Let $a \in \mathrm{G}$ be an element of order 6 and we have put $H = gp(a) = \{e, a, a^2, a^3, a^4, a^5, a^6,\}$. Then
$G = H \cup Hb \ \ for \ \ some \ \ b \in G$ that is where
Since the symmetry difference $H \otimes G$ and $b^2 \in H,$ we have the possibility that $b^2 = e, \ if \ \ b^2 = e,$ Let .
$K = gp(b) \Rightarrow K = \{e, b^2, b^3, b^4, b^5\} \ then \ \ H \cap K = \{e, a^2, a^3, a^4, a^5\} \cap \{e, b^2, b^3, b^4, b^5\} = \{e\}$
And $HK = G$

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Now the symmetry inverse of $H \otimes G$ is such that $b^{-1}ab \in H$ and since a is of order 6, we have that $b^{-1}ab = a,$ or a^3 if $b^{-1}ab = a,$ then $ab = ba \Rightarrow G$ satisfies Abelian property which is contrary to our assumption so, $b^{-1}ab \neq a,$ hence $b^{-1}ab = a^5$

$\Rightarrow (b^{-1}ab)b = a^5b$ where $b^2 = e$ and $a = a^5$

thus $(b^{-1}ab)b = a^5b \Rightarrow ba(b^{-1}b) = a^5$

$\therefore ba = a^5b$ Hence

$G = \{e, a, a^2, a^3, a^4, a^5, b, ab, ab^2, ab^3, ab^4, ab^5\}$

where $ba = a^5b, \quad b^2 = a^6 = e, \quad a^2b = a^{10}b = a^4b, \quad a^3b = a^{15}b = a^3b$

With this transformation, we therefore obtain the equality of the table 3.1 below

Table 2: Symmetry Table of *D₆*

*	e	a	a^2	a^3	a^4	a^5	b	ab	a^2b	a^3b	a^4b	a^5b
e	e	a	a^2	a^3	a^4	a^5	b	ab	a^2b	a^3b	a^4b	a^5b
a	A	a^2	a^3	a^4	a^5	e	ab	a^2b	a^3b	a^4b	a^5b	b
a^2	a^2	a^3	a^4	a^5	e	A	a^2b	a^3b	a^4b	a^5b	b	ab
a^3	a^3	a^4	a^5	e	A	a^2	a^3b	a^4b	a^5b	b	ab	a^2b
a^4	a^4	a^5	e	a	a^2	a^3	a^4b	a^5b	b	ab	a^2b	a^3b
a^5	a^5	e	a	a^2	a^3	a^4	a^5b	b	ab	a^2b	a^3b	a^4b
b	b	a^5b	a^4b	a^3b	a^2b	ab	e	a^5	a^4	a^3	a^2	a
ab	ab	b	a^5b	a^4b	a^3b	a^2b	a	e	a^5	a^4	a^3	a^2
a^2b	a^2b	ab	b	a^5b	a^4b	a^3b	a^2	a	e	a^5	a^4	a^3
a^3b	a^3b	a^2b	ab	b	a^5b	a^4b	a^3	a^2	a	e	a^5	a^4
a^4b	a^4b	a^3b	a^2b	ab	b	a^5b	a^4	a^3	a^2	a	e	a^5
a^5b	a^5b	a^4b	a^3b	a^2b	ab	b	a^5	a^4	a^3	a^2	a	e

From the table, the following axioms (Identity, inverse, associativity, symetricity) are satisfied.

Transformation of Group of Order-12 To Matrix in Four Equal Dimension

The table 3.0 from left to right and down-upward can be divided equally as

Table 3. A: Top left Transformation

*	e	a	a^2	a^3	a^4	a^5
e	e	a	a^2	a^3	a^4	a^5
a	a	a^2	a^3	a^4	a^5	e
a^2	a^2	a^3	a^4	a^5	e	a
a^3	a^3	a^4	a^5	e	a	a^2
a^4	a^4	a^5	e	a	a^2	a^3
a^5	a^5	e	a	a^2	a^3	a^4

Table 3. B: Top right transformation

b	ab	a^2b	a^3b	a^4b	a^5b
b	ab	a^2b	a^3b	a^4b	a^5b
ab	a^2b	a^3b	a^4b	a^5b	b
a^2b	a^3b	a^4b	a^5b	b	ab
a^3b	a^4b	a^5b	b	ab	a^2b
a^4b	a^5b	b	ab	a^2b	a^3b
a^5b	b	ab	a^2b	a^3b	a^4b

Table 3. C: Downleft transformation

b	b	a^5b	a^4b	a^3b	a^2b	ab
ab	ab	B	a^5b	a^4b	a^3b	a^2b
a^2b	a^2b	ab	B	a^5b	a^4b	a^3b
a^3b	a^3b	a^2b	ab	b	a^5b	a^4b
a^4b	a^4b	a^3b	a^2b	ab	B	a^5b
a^5b	a^5b	a^4b	a^3b	a^2b	ab	b

Table 3. D: Downright transformation

e	a^5	a^4	a^3	a^2	a
a	e	a^5	a^4	a^3	a^2
a^2	a	e	a^5	a^4	a^3
a^3	a^2	a	e	a^5	a^4
a^4	a^3	a^2		e	a^5
a^5	a^4	a^3	a^2	a	E

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Matrix Representation

Let ***A*** be a matrix with an independent vector representation ***o*** of every element of the Table 3 such that at every point of significance of interest, ***o=1***.

From table 3.3 A

$$a^5 = \begin{pmatrix} 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \end{pmatrix} \quad a^4 = \begin{pmatrix} 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \end{pmatrix}, \quad a^3 = \begin{pmatrix} 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \end{pmatrix}, \quad a^2 = \begin{pmatrix} 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \end{pmatrix}, \quad a = \begin{pmatrix} 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \end{pmatrix}, \quad e = \begin{pmatrix} 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \end{pmatrix}$$

From table 3.3 B

$$a^5b = \begin{pmatrix} 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \end{pmatrix}, \quad a^4b = \begin{pmatrix} 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \end{pmatrix}, \quad a^3b = \begin{pmatrix} 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \end{pmatrix}, \quad a^2b = \begin{pmatrix} 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \end{pmatrix}, \quad ab = \begin{pmatrix} 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \end{pmatrix}, \quad b = \begin{pmatrix} 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \end{pmatrix}$$

From table 3.3 C

$$a^5b = \begin{pmatrix} 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \end{pmatrix}, \quad a^4b = \begin{pmatrix} 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \end{pmatrix}, \quad a^3b = \begin{pmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \end{pmatrix}, \quad b = \begin{pmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}, \quad ab = \begin{pmatrix} 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \end{pmatrix}, \quad a^2b = \begin{pmatrix} 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \end{pmatrix}$$

From table 3.3 D

$$a^5 = \begin{pmatrix} 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \end{pmatrix}, \quad a^4 = \begin{pmatrix} 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \end{pmatrix}, \quad a^3 = \begin{pmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \end{pmatrix}, \quad e = \begin{pmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix} \quad a = \begin{pmatrix} 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \end{pmatrix}, \quad a^2 = \begin{pmatrix} 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \end{pmatrix}$$

Comparing matrices, **A** and **B**

$$a^5 = a^5b, \quad a^4 = a^4b, \; a^3 = a^3b, \quad a^2 = a^2b, \quad a = ab, \quad e = b,$$

Comparing matrices, **C** and **D**

$$a^5 = a^5b, \quad a^4 = a^4b, \; a^3 = a^3b, \quad a^2 = a^2b, \quad a = ab, \quad e = b,$$

Model Representation With Respect To Pipeline Network Flow.

According to the pipeline state model, of this section, it is illustrated that the condition of the pipeline entity can be is defined into ready state, running state, and blocking state. Within this system, the pipeline object in the running state operates with its own distinct state. Let us consider the Jianqing et al. (2018), telecommunication model plane for improving telecommunication network services below.

Table 4: telecommunication model plane table

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S/N	Pipeline object	State of an object	Status Symbol	Related to traffic movement	Explanation of the function of the pipeline Configuration
1	Undertaking pipeline (**UP**)	* Retail Facility *Goods bearing *Business interface	C: construct L: load U: union	*Generate information flow. *Generate information flow. *Pass information flow.	Router, linker, 2G/3G/4G/WiFi
2	Service pipeline (**SP**)	*Advisory Services *Marketing of Service	C: consult S: sale	*Generate information flow. *Push information flow.	Mobile phone app, mall application software
3	Payment channel (**PC**)	*Merchandise quotation. *Integral payment	O: offer P: pay	*Generate information flow. *Generate information flow.	Mobile shopping mall, Mobile phone customers, rewarding system
4	Logistics pipeline	* Merchandise logistics access *Logistics integration	A: add L: link	*Pass information flow. *Pass information flow.	Logistics business Commodity storage
5	Data pipeline (**DP**)	*Data acquisition *Data summary *Data mining and Transportation	G: gather T: total D: dig	*Generate information flow. *Pass information flow. *Pass information flow.	Customers, order, delivery Mall business, chart Excavating tools
6	**Dis/Normal(DN)**	Negative	Negative	Negative	Negative

Source of table: Jianqing et al. (2018).

Let the component elements of Table 4
Above be categorized into six (6) groups defined by

$$G_1 = UPC = \begin{Bmatrix} \text{C : Construct} \\ \text{L : load} \\ \text{U : union} \end{Bmatrix} \qquad G_2 = \text{SP} = \begin{Bmatrix} \text{C : Consult} \\ \text{S : Sale} \end{Bmatrix}, \qquad G_3 = \text{PC} = \begin{Bmatrix} \text{O : Offer} \\ \text{P : Pay} \end{Bmatrix}$$

$$G_4 = \text{LP} = \begin{Bmatrix} \text{A : Add} \\ \text{L : Link} \end{Bmatrix} \qquad G_5 = \text{DP} = \begin{Bmatrix} \text{G : Gather} \\ \text{T : Total} \\ \text{D : Dig} \end{Bmatrix} \text{ and } \qquad G_6 = \text{DN} = (\text{Identity}).$$

which are in symetric equality with D6 of order6 that is $\text{C, L, U, C, S, O, P, A, L, G, T, D.} = 12$

The components are 12 of six groups, generated from Table 3.2. We comfortably will proceed to the judgment of our analysis by group of order 12. From the symmetry group property of order 12, since it can be seen in two dimensions, we can say that.

$$\text{Let } e=b=\begin{pmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix},\ a^5b=a^5=\begin{pmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&0&0&0&0\end{pmatrix},\ a^4b=a^4=\begin{pmatrix}0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&0&0&0&0\\0&1&0&0&0&0\end{pmatrix}$$

Pipeline Part Keep (PK), Pipeline Part Big (PB), Pipeline Uniform conflict (UC),

$$a^3b=a^3=\begin{pmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{pmatrix},\ a^2b=a^2=\begin{pmatrix}0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\end{pmatrix},\ ab=a=\begin{pmatrix}0&0&0&0&0&1\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\end{pmatrix}$$

Pipeline Entity Abstract (EA), Pipeline Object Plus (OP), Pipeline Part Small (PS),

$$\text{Let } a^5b=a^5=\begin{pmatrix}0&0&0&0&0&1\\0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&1&0&0&0\\0&1&0&0&&0\\1&0&0&0&0&0\end{pmatrix},\ a^4b=a^4=\begin{pmatrix}0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\end{pmatrix}\ a^4b=a^4=\begin{pmatrix}0&0&0&1&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\end{pmatrix}$$

Pipeline Trend Positive (TP), Pipeline Uniform Double (UD), Pipeline Uniform Measure (UM),

$$a^3=a^3b=\begin{pmatrix}0&0&1&0&0&0\\0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\\0&0&0&1&0&0\end{pmatrix}\ a^2b=a^2=\begin{pmatrix}0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&1&0&0&0\end{pmatrix}\ a=ab=\begin{pmatrix}1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\end{pmatrix}$$

Pipeline Trend Negative (TN), Pipeline Uniform Double (UD) Pipeline Object Minus (OM)

We may verify that the transformation meets all group properties with regard to linear mapping and homomorphism from the group to the converted network table by looking at the trace of a specific element. By applying the pipeline state model with M nodes, as indicated in the above table, we may use the following formulas to get solutions for every given system, which are then presented.

The pipeline entity encompasses various components, including the operational pipeline, service pipeline, payment channel, logistics pipeline, and data pipeline, is a platform system of business support services at the software application level of the pipeline system. A pipeline entity can connect, convey, and package data at the physics and system levels. All types pertaining to the connections between objects, including payment and logistics systems, as well as application and physical connection nodes, can be completed by the pipeline entity. The pipeline system's one-dimensional pipe has connection, support, and transmission effects. It is an entity object that is connected and a conduit for information transmission. The pipeline system will evolve into a platform for increased operating output when the pipeline entity is built to grow.

Within the pipeline group G, the elements G_i or G_j must ensure that the outcome of the calculation $G_i \bullet G_j$ is contained within the group G ; the symbol "$\bullet$" represents a particular type of calculation, akin to integer addition. The formula for the characteristic matrix of a group element can be expressed as follows:

$$Let\ \ Z = a_1\{G_1\} + a_2\{G_2\} + a_3\{G_3, G_7, G_9\} + a_4\{G_4, G_8, G_{10}\} + a_5\{G_5, G_{11}\}\ldots\ldots + a_6\{G_6, G_{12}\} = 0. \qquad (6)$$

If only $a_i \Rightarrow a_1 = a_2 = a_3 = a_4 = a_5 = a_6$ will Z be achieved.

$Let\ G_1^{\ 1} = \{G_1\}, G_2^{\ 1} = \{G_2\}, G_3^{\ 1} = \{G_3, G_7, G_9\}, G_4^{\ 1} = \{G_4, G_8, G_{10}\}, G_5^{\ 1} = \{G_5, G_{11}\}, and$

$G_6^{\ I} = \{G_6, G_{12}\} = 0$

Pipeline System Model Based on Group Theory

Case of Independence of Pipeline Elements

Let the component elements of the table 3.1 above be of six (6) categories defined by

$$UPC = \begin{Bmatrix} \text{C : Construct} \\ \text{L : load} \\ \text{U : union} \end{Bmatrix} = G_1 \qquad \text{SP} = \begin{Bmatrix} \text{C : Consult} \\ \text{S : Sale} \end{Bmatrix} = G_2$$

$$\text{PC} \ = \begin{Bmatrix} \text{O : Offer} \\ \text{P : Pay} \end{Bmatrix} = G_3 \qquad \text{LP} = \begin{Bmatrix} \text{A : Add} \\ \text{L : Link} \end{Bmatrix} = G_4$$

$$\text{DP} \ = \begin{Bmatrix} \text{G : Gather} \\ \text{T : Total} \\ \text{D : Dig} \end{Bmatrix} = G_5 \qquad \text{DN} = \text{(Identity)}. = G_6.$$

$$\text{which are } \ \text{C, L, U, C, S, O, P, A, L, G, T, D.} = 12$$

The components are 12 of six groups, generated from table 3.1.we comfortably will proceed to the judgment of our analysis by group of order 12.

Pipeline Entity Object State Transition and Simulation Model Equation.

The pipeline object's state, which is separated into three states: ready, running, and blocking, in the system where the running state's pipeline object has its own state. Table 3.1 illustrates that g is a group element of G within the pipeline entity group.

The base vector can represent g as follows:

$$g = \sum_{i=1}^{6} G_i^{'}\lambda_i \ = (G_i^{'}, G_2^{'}, G_3^{'}, G_4^{'}, G_5^{'}, G_6^{'})\begin{pmatrix} \lambda_1 \\ . \\ . \\ \lambda_6 \end{pmatrix} \qquad (7)$$

According to the definition of pipeline Entity Object status and state, pipeline object in Hilbert Space is measured by the state and is a space vector, respectively, *R1, R2, R3,………* which complies with principle of superposition in Hilbert space.

According to $A^I = gA, \ \ g \in G$ The base vector representation approach, as outlined, for the pipeline group within a 6-dimensional space can be defined as follows:

$$A^I \ = \sum_{i=1}^{6} G_i^{'}, G_2^{'}, G_3^{'}, G_4^{'}, G_5^{'}, G_6^{'})\begin{pmatrix} \lambda_1 \\ . \\ . \\ \lambda_6 \end{pmatrix} A_i \ldots \qquad (8)$$

Consequently, by extending this approach to the pipeline object *O*, it conforms to the following equation:

$$O' = G * O \qquad (9)$$

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For the pipeline entity, O denotes the initial state of the pipeline object, while $\boldsymbol{O}^{\mathbf{I}}$ signifies the inherent vector of the pipeline object, representing the state vector of the pipeline entity subjected to the transformation process. Referencing equation (8) and formula (9), the entity state transition formula can be obtained:

$$G * O_{Start} = E * O_{End} \quad (10)$$

At this stage, the formula for the state of the pipeline object has been defined. O_{Start} represents the expression for the initial state space of the pipeline system, while O_{end} denotes the expression for the terminal state space. In the equation for state transitions (10)

There are a number of pipeline sections connected at each stage; the sections and node flow equation can be obtained as

$$\sum_{i \in S_j} (\pm q_i) + Q_j = 0, \quad j = 1,2,3,4,5,\ldots\ldots N \quad (11)$$

Where q_i = is the flow of section$i$;

Q_j= represents the flow through the node j;

S_j= refers to the sets of node correlations j; and

N= represents the total count of nodes within the network model.

$\sum_{i \in Sj}(\pm qi)$ = the total of the nodes connected to the focal section; a minus sign indicates that the section is heading toward the node, while a plus sign indicates that it is leaving the node. The traffic section receives a plus sign when it flows from the node. Traffic enters a minus sign when it enters a node. The equations governing node flow within the network model can be composed of simultaneous equations for every N node. The pipeline network model's node flow equations can be represented using (11) as:

$$A\bar{q} + \bar{Q}. = \bar{0} \quad (12)$$

Where A= the incidence matrix of network graph;

$$\bar{q} = |q_1 \;\; q_2 \;\; q_3 \ldots\ldots q_n|^T \text{ is column vector of section flow and}$$

$$\bar{\text{Q}} = |Q_1 \;\; Q_2 \;\; Q_3 \ldots\ldots Q_n|^T \text{ is column vector of node flow}$$

The object "state space explosion" is caused by the vast majority of Communication pipeline system components and the intricate interconnections between complex variables that influence each other. The state variable cannot be addressed directly due to the size of the correlation matrix A. As a result, symmetrically simplified equations are used to achieve stat status solutions. It is assumed that traffic flow can be represented as a vector of pipeline objects. When the flow conservation equations (12) and the state equation of the object (10) are applied to the pipeline system, they demonstrate that the overall conservation of flow equates to the aggregate of traffic inflows and outflows. During the process of transmitting and receiving information, the rate of flow entering the system matches the rate of flow exiting the pipeline system This is define by

$$\sum_N q + \sum_M Q = Flow = \lambda * Flow \quad (13)$$

where q is the section's output flow rate (takes a negative value if it flows against the pipeline's direction); Any node's input flow rate, represented by Q (which takes a negative value if it is flowing against the pipeline's direction); The total number of sections is N, and the total number of nodes is M.

Results

Correlation matrix A The pipeline network flow formula outlines the N×M configuration of pipeline network connections, and it can be generally represented as follows::

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$$A=\begin{bmatrix} 1 & \ldots K_{1j} \ldots\ldots K_{1M} \\ K_{11} \ldots K_{2j} \ldots\ldots K_{1M} \\ . \quad \ldots\ldots \quad \ldots\ldots \quad . \\ K_{ij} \ldots K_{ij} \ldots\ldots K_{iM} \\ \ldots\ldots \quad \ldots\ldots \quad . \\ \ldots\ldots \quad \ldots\ldots \quad . \\ K_{N1} \ldots K_{NJ} \ldots\ldots 1 \end{bmatrix} \tag{14}$$

Element k_{ij} indicates whether nodes i and j are connected or not;
When k_{ij} =0, nodes i and j are connected;
When k_{ij}=1, nodes i and j are not connected. Particularly when $i=j$, k_{ij}=0.

Results

When evaluating a multipoint crossover pipeline system, the parameters of the pipeline section and node differ, making it challenging to describe the symmetrical feature of the network in two dimensions. Large volume pipeline network system status problems are typically hard to solve, hence the following steps in the Group theory-solving process are designed to help.

Step 1.

Under pipeline entity G group symmetry transformation operation, the flow of the pipeline network where Mnodes are connected to each other is transformed for a pipeline network composed of nodes and pipeline sections. The pipeline network traffic incidence matrix is constructed using the demand response model of the pipeline network, as shown in (3.8), and the pipeline sections and nodes flow expressions regarding a single pipeline segment are as follows:

$$Flow_l = \sum_{3} t_l^i Bps * Flow = tBps * (vector)Flow$$

For Npipeline sections, (15)

$$\sum_N q = \sum_N Flow_l = \sum_N \sum_S t_l^i Bps^i * (vector)Flow_l \tag{16}$$

For One Node:

$$Flow_n = \frac{P_n}{I_n} * Flow \tag{17}$$

$$\sum_M Q = \sum_M Floew_n = \sum_M \frac{P_n}{I_n} * Flow \tag{18}$$

According to (4.2), (4.3), (4.4), and (4.5),

$$(initial)Flow = \sum_M \frac{P_n}{I_n} * Flow_n \tag{19}$$

$$where \ t_l^i = 0$$

And the end Flow state of the pipeline grid is as follows: $$(final)Flow = \sum_N t_l^i Bps^i * Flow_l + \sum_M \frac{P_n}{I_n} * Flow_n \tag{20}$$

When the pipeline network system is in both its initial and final states, the traffic balance equation can be derived using equations (4.6) and (4.7)

$$\sum_N t_l^i Bps^i * Flow_l + \sum_M \frac{\left(P_{_n n}^{li} - P_n^{oi}\right)}{I_{_n n}^{i}} * Flow_n = 0 \tag{21}$$

Where: Bps' intensity $_n$ = are the characteristics of the pipeline segment and the constant node parameters within the pipeline network, and $t_l^i, P_n^{1i}, and\ P_n^{0i}$ The variables associated with pipeline network transmission typically represent only a portion of the overall informational potential within the pipeline network P_n^{1i}, P_n^{0i} .

Step 2.

Based on equation (16), the matrix representation of the traffic demand response model is determined as follows

$$\begin{pmatrix} 0 & t_1^i Bps_1^i,..,t_1^j Bps_1^j \ t_1^m Bps_1^m \\ . & . \quad . \quad . \\ . & . \quad . \\ . & . \quad . \quad . \\ t_1^m Bps_1^m & t_1^j Bps_1^j ..t_1^i Bps_1^i,.0 \end{pmatrix}_{M*M} \begin{pmatrix} Flow_{Pipelinesection} \\ . \\ . \\ . \\ Flow_{Pipelinesection} \end{pmatrix}_M + \begin{pmatrix} \sum_M \frac{\left(P_1^1 - P_1^0\right)}{I_1} & 0...0 & 0 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0 & 0....0 & \sum_M \frac{\left(P_M^1 - P_M^0\right)}{I_M} \end{pmatrix}_{M*M} \begin{pmatrix} Flow_{node} \\ . \\ . \\ . \\ Flow_{node} \end{pmatrix}_M = 0.$$

……..(22)

Equation (17) can be simplified as follows:

$$\begin{pmatrix} 0 & t_1^i Bps_1^i,..,t_1^j Bps_1^j \ t_1^m Bps_1^m \\ . & . \quad . \quad . \\ . & . \quad . \\ . & . \quad . \quad . \\ t_1^m Bps_1^m & t_1^j Bps_1^j ..t_1^i Bps_1^i,.0 \end{pmatrix}_{M*M} * \text{Flow pipeline section} = \begin{pmatrix} P_1 * K & 0.....0 & 0 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0 & 0....0 & P_M * K \end{pmatrix} * Flow_{node}. \quad ...(23)$$

Where t_1^i = the pipeline section's effective transmission time;

Bps_1^i = the pipeline section's rate of information transmission;

I_1 = the first node capacity;

P_1= the first throughput traffic;

K = the ratio of pipeline section to node flow direction; and

$Flow_{\text{pipeline section}}$ and $Flow_{\text{node}}$ are pipeline sections and node vector units.

Step 3.

The Transformation Matrix of Pipeline network G_M is as follows:

$$G_M = \begin{pmatrix} 0 & t_1^i Bps_1^i,..,t_1^j Bps_1^j \ t_1^m Bps_1^m \\ . & . \quad . \quad . \\ . & . \quad . \\ . & . \quad . \quad . \\ t_1^m Bps_1^m & t_1^j Bps_1^j ...t_1^i Bps_1^i,.0 \end{pmatrix}_{M*M} \quad (24)$$

$$M - 1 \leq N \leq M^{(M-1)/2}$$

Where N represents the number of pipeline segments connected to a single node. The traffic balance equation indicates that, as transmission time and node potential differences vary within pipeline systems, the pipeline network establishes a G_M matrix based on the known group G, transfer rate of information, and the informational capacity linked to the pipeline's structural features. Assuming this setup, the pipeline network is made up of M Nodes and Npipeline sections. Pipeline sections and nodes link to a network under G group operation, and the pipeline network is represented by the GM matrix. The aforementioned matrix represents M nodes that are connected to a pipeline network. In general, (24) states that GM can be used to characterize the pipeline network system in its whole.

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$$G_{M \text{ general}} = \begin{pmatrix} 0 & t_1^i Bps_1^i ,..,t_1^j Bps_1^j & t_1^m Bps_1^m \\ . & . & . & . \\ . & . & . \\ . & . & . & . \\ t_1^m Bps_1^m & t_1^j Bps_1^j ..t_1^i Bps_1^i ,.0 \end{pmatrix}_{M*M} \quad (25)$$

Group theory irreducible representation technique states that $\{GM\}$ group homomorphism in pipeline entity group G group:$\{gi\}$,$\{GM\}\sim$ follows G group, provided that pipeline network flow conservation is met. Therefore, the number of nodes has no bearing on the network structure. Three intrinsic properties determine network structure: transfer duty cycle time (t), potential strength (information capacity) intensity (I), and information transformation rate (Bps). Hence, the expression of GM can be primarily stated as a linear integration of the 6-dimensional field. Node flows, Flow1, Flow2,..., Flow$M$. Where M is 6 and it define the GM matrix, whereas group transformations, $f_{1,2}$,f_3,f_4,f_5,f_6, define the homomorphism of matrix of transformations within the pipeline entity category: G group: $\{gi\}$.

Step 4. Six base vectors of pipeline entity G group are as follows:

$$G_1^{'} = \{G_1\}, G_2^{'} = \{G_2\}$$
$$G_3^{'} = \{G_3, G_7, G_9\}, G_4^{'} = \{G_4, G_8, G_{10}\} \quad (26)$$
$$G_5^{'} = \{G_5, G_{11}\}, G_6^{'} = \{G_6, G_{12}\}$$

Within the pipeline entity category, G_{ii} are linearly independent. Based on the characterization of the group elements matrix, there exists a collection of real numbers, a_1, a_2, a_3, a_4, a_5, a_6 that satisfy the following equation:,

$$G_M^{'} = \{a_1 G_1^{\,i} + a_2 G_2^{\,i} + a_3 G_3^{\,i} + a_4 G_4^{\,i} + a_5 G_5^{\,i} + a_6 G_6^{\,i} \quad (27)$$

From **Step 1**, a_1, a_2, a_3, a_4, a_5, a_6, can be calculated according to G_1^i, $G_2^{\,i}$, G_3^i, G_4^i, G_5^i, G_6^i, t_1, t_2, t_3,……, t_n ,of G_M.
The above describes the method of utilizing Group theory in the context of the information pipeline system.
Regardless of the quantity of nodes and pipeline sections, the GM matrix can be reduced to a 6-dimensional matrix.
Consequently, the issue of 'state space explosion' in the pipeline network can be addressed, providing a universal solution.

Conclusion

In this work, the state elements of the pipeline system were modelled and judged using group order 12 which was partitioned into six dimensional order and 3 basic flow model.s. Using the principles and properties of Group, the pipeline complex state nodes system is generated with respect to flux state and mass conservation. The cyclic group of order 12 was generated with regards to the flow state with two generators, while the Alternating group has three generators, with the relationships $a^2=b^2=e$,$a^{-1}b=ab$, $b^2=e$. The multiplication table was obtained for group order 12. Also, transformation matrices of flow trend and the transformation equations are defined.

The inner product automorphism (f_1,f_2,f_3,f_4,f_5,f_6) which represents the vector associated with the pipeline entity flow state at any given point and time can be computed using $f_y(x) = y^{-1}xy$ *for all x in G.*

Recommendations

Though the method needs commitment and broad in nature, it is recommended that;

1. Software application of this work should be considered using Math Lab, Python and other computer.
2. Application of group automorphism in solving technical engineering problems should as well be considered.

3. The symmetry generators should be applied in modelling of various physical systems, such as crystals, and hydrogen atom.
4. The application of group should be considered alongside with graph theory in marriage and matching.
5. Finally we recommend review of this work for numerical solutions

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