



Application of Linear Programming for Profit Maximization at Toprano Bakery, Benin City, Edo State, Nigeria

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Abstract

Managerial decision-making problems with the distribution of scarce resources to maximize profit and minimize cost are commonly resolved by the application of an operation research technique known as linear programming. In order to calculate the profit of Toprano Bakery in Benin City, Edo State, Nigeria, using the linear programming technique, this paper focuses on the application of the optimization principle. Specifically, it analyzes the quantity of various raw materials used in production, analyzes production costs, and determines the selling price of raw materials that are available in stock. The study took into account the seven types of bread that the bakery produces: small bread, sardine bread, coconut bread, sliced bread, medium bread, and large bread. A model for the problem was established and optimum results were determined using Tora software and the Maple program that applied the simplex approach. The result suggests that medium bread contributes more to the profit made and hence should be produced more by the bakery.

Keywords: Constraints, Linear programming problem, Objective function, Optimization

Introduction

Linear programming problems deal with defining the optimal allocations of limited resources to attain the objectives. A linear programming problem deals with optimizing (maximizing or minimizing) a function. It consists of three parts objective functions, decision variables and constraints. In linear programming problems the term linear refers to the fact that all variables involved in the objective function and constraints are linear, that is of degree 1 in the problems under consideration. Every organization aims to make a profit from its continuous existence and growth. In present times, the challenge is more critical and prominent for manufacturing industries at all levels of creating goods of the right quality and quantity. The aim of every manufacturing industry is to make a profit that will guarantee its continuous existence and productivity. In this industrial era, especially after the Industrial Revolution, bakery industries are faced with more and more challenges. The challenges are producing goods of the right quality and quantity at the right time and at minimum cost and maximum profit for their survival and growth (Chanda et al. 2022).

Akpan et al. (2016) utilized the Simplex method, a component of linear programming, to allocate raw materials to the bakery's finishing variables (big loaf, little loaf, and huge bread) with the goal of maximizing profit. After the assessment was completed, the results showed that in order to make N20385 in profit, little loaves (962), huge loaves (38), and giant loaves (0 units) need to be created, correspondingly. According to the evaluation, the profit is fairly contributed by small and large loaves, respectively. Thus, in order to maximize profit, more tiny and large loaves had to be made.

Kalwar et al. (2022) proposed a linear programming model that provides analytical support to the decision for the selection of footwear articles to be produced, which is aimed to suggest the analytically supported decision-making for deciding the number of pairs of different articles to be produced in a way that the cost and profit could be minimized and maximized, respectively. The linear programming (LP) model was then formulated and implemented in Microsoft Excel by using the solver add-in. When the outputs of the traditional approach and the LP model were compared, it was discovered that with 22% less production, 39% more profit could be produced if the article selection and production amount were calculated appropriately.

Mohammed *et al.* (2020) used a slack-starting solution approach of the Simplex Optimization method to build a linear programming (LP) model in which the optimal solution for the LP was obtained at the fifth iteration. The result shows that the daily estimated value of the objective function was obtained to be 9498.4802 (approximately 9,500), and the daily contributions of each of the four decision variables to the objective function are 0, 0, 0, and 379.94 (approximately 380), which will A cursory examination of this daily realization shows a unit profit of 25 (41.67%) per saloon bread.

Mathematical formulation

A Linear Programming Problem (LPP) is of this form (Ekoko, 2016)

$$\text{Max/min } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n \tag{1}$$

Subject to the constraints:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n (\leq, =, \geq) b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n (\leq, =, \geq) b_2$$

$$a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}x_n (\leq, =, \geq) b_3$$

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$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n (\leq, =, \geq) b_m$$

And the non-negative restrictions $x_j \geq 0, j = 1, 2, \dots, n$ where $a_{ij}, b_i,$ and c_i are constants and x_j are variables. Eqn (1) is called the objective function.

The data used for this research work was obtained from Toprano Bakery in Benin City. The collected data was based on the different varieties of bread produced by the bakery which are small bread, medium bread, large bread, sardine bread, coconut bread, butter bread and sliced bread.

Table 1 shows the cost analysis for the various raw materials which is the constraints needed for the production of bread.

Table 2 shows the production cost per loaf which is obtained from table 1, the selling price and the profit made per loaf of the various varieties of bread.

Table 1: Cost Analysis of variable items

Constraints	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	Remark
Flour	$\frac{43,000}{302} = 142$	$\frac{43,000}{200} = 215$	$\frac{43,000}{103} = 417.5$	$\frac{43,000}{103} = 417.5$	$\frac{43,000}{103} = 417.5$	$\frac{43,000}{103} = 417.5$	$\frac{43,000}{103} = 417.5$	1 bag of salt divide by 72 mix divide by number of loaves it can produce.
Sugar	$\frac{3176}{302} = 10.5$	$\frac{3176}{200} = 15.9$	$\frac{3176}{103} = 30.8$	$\frac{3176}{103} = 30.8$	$\frac{3176}{103} = 30.8$	$\frac{3176}{103} = 30.8$	$\frac{3176}{103} = 30.8$	1 bag of sugar at ₦54,000 divide by 17 mix, divide by the number of loaves
Butter	$\frac{1,352}{302} = 4.5$	$\frac{1,352}{200} = 6.8$	$\frac{1,352}{103} = 13.1$	$\frac{1,352}{103} = 13.1$	$\frac{1,352}{103} = 13.1$	$\frac{1,352}{103} = 13.1$	$\frac{1,352}{103} = 13.1$	1 carton of butter at ₦23,000 divided by 17 mixes, divided by a number of loaves it can produce.
Vegetable oil	$\frac{1824}{302} = 6.0$	$\frac{1824}{200} = 9.1$	$\frac{1824}{103} = 17.7$	$\frac{1824}{103} = 17.7$	$\frac{1824}{103} = 17.7$	$\frac{1824}{103} = 17.7$	$\frac{1824}{103} = 17.7$	25kg of vegetable oil at ₦31,000 divided by 17 mix and divide by the number of the loaves.
Eggs	$\frac{4500}{302} = 14.9$	$\frac{4500}{200} = 22.5$	$\frac{4500}{103} = 43.7$	$\frac{4500}{103} = 43.7$	$\frac{4500}{103} = 43.7$	$\frac{4500}{103} = 43.7$	$\frac{4500}{103} = 43.7$	1½ crates of eggs at ₦4500 divided by the number of loaves it can produce.
Salt	$\frac{166}{302} = 0.5$	$\frac{166}{200} = 0.8$	$\frac{166}{103} = 1.6$	$\frac{166}{103} = 1.6$	$\frac{166}{103} = 1.6$	$\frac{166}{103} = 1.6$	$\frac{166}{103} = 1.6$	1 bag of salt divided by 72 mixes divided by the number of loaves it can produce.
Yeast	$\frac{1000}{302} = 3.5$	$\frac{1000}{200} = 5.0$	$\frac{1000}{103} = 9.7$	$\frac{1000}{103} = 9.7$	$\frac{1000}{103} = 9.7$	$\frac{1000}{103} = 9.7$	$\frac{1000}{103} = 9.7$	1 carton of yeast at ₦32,000 divided by 32 mix divide by the number of loaves It can produce
Milk	$\frac{2669}{302} = 8.0$	$\frac{2669}{200} = 13.3$	$\frac{2669}{103} = 25.9$	$\frac{2669}{103} = 25.9$	$\frac{2669}{103} = 25.9$	$\frac{2669}{103} = 25.9$	$\frac{2669}{103} = 25.9$	25kg bag of milk at ₦77,500 divided by 32 mixes, divided by the numbers of loaves it can produce

Flavor	$\frac{247}{302} = 0.8$	$\frac{247}{200} = 1.2$	$\frac{247}{103} = 2.4$	$\frac{247}{103} = 2.4$	$\frac{247}{103} = 2.4$	$\frac{247}{103} = 2.4$	$\frac{247}{103} = 2.4$	1 litre of flavour at ₦4,200 divided by 17 mixes, divided by the number of loaves it can produce.
Preservative	$\frac{437}{302} = 1.2$	$\frac{437}{200} = 2.2$	$\frac{437}{103} = 4.2$	$\frac{437}{103} = 4.2$	$\frac{437}{103} = 4.2$	$\frac{437}{103} = 4.2$	$\frac{437}{103} = 4.2$	1 sachet of preservative at ₦3500 divided by 8 mixes, divided by the number of loaves it can produce.
Coconut	0	0	0	0	$\frac{7500}{103} = 72.8$	0	0	1 bag of coconut at ₦7500 divide by number of loaves it can produce.
Water	$\frac{500}{302} = 1.7$	$\frac{500}{200} = 2.5$	$\frac{500}{103} = 4.9$	$\frac{500}{103} = 4.9$	$\frac{500}{103} = 4.9$	$\frac{500}{103} = 4.9$	$\frac{500}{103} = 4.9$	2 bags of sachet water at ₦500, divide by number of loaves it can produce.
Sardine	0	0	0	$\frac{2500}{103} = 24.2$	0	0	0	1/2 carton of tin sardine ₦2500, divide by number of loaves it can produce
Total	194.9	298.8	580.3	822.3	653.1	580.3	580.3	

Table 2: Varieties of Bread Produce by Toprano Bakery

Name of product (Varieties of bread)	Production Cost per Loaf (₦)	Selling price (₦)	Profit (₦)
Small bread (x_1)	194.9	350	155.1
Medium bread (x_2)	298.8	600	301.2
Large bread (x_3)	580.3	1500	919.7
Sardine bread (x_4)	822.3	2000	1177.7
Coconut bread (x_5)	653.1	1800	1146.9
Butter bread (x_6)	580.3	1300	719.7
Sliced bread (x_7)	580.3	1400	819.7

Formulation of the linear programming problem

Let $x_1, x_2, x_3, x_4, x_5, x_6$ and x_7 be the decision variables which represent the non-negative number of small bread, medium bread, large bread, sardine bread, coconut bread, butter bread and sliced bread to be produced by the factory. a_{ij} 's represent the quantity of each raw material used in the production of the seven types of bread. The objective is to maximize the profit Z of the bakery and thus a linear programming model for the maximization of the objective function Z can be stated mathematically as follows:

Maximize the objective function

$$Z = 155.1x_1 + 301.1x_2 + 919.7x_3 + 1177.7x_4 + 1146.9x_5 + 719.7x_6 + 819.7x_7$$

Subject to the constraints

$$142.4x_1 + 215x_2 + 417.5x_3 + 417.5x_4 + 417.5x_5 + 417.5x_6 + 417.5x_7 \leq 43000$$

$$10.5x_1 + 15.9x_2 + 30.8x_3 + 30.8x_4 + 30.8x_5 + 30.8x_6 + 30.8x_7 \leq 3176$$

$$4.5x_1 + 6.8x_2 + 13.1x_3 + 13.1x_4 + 13.1x_5 + 13.1x_6 + 13.1x_7 \leq 1352$$

$$6.0x_1 + 9.1x_2 + 17.7x_3 + 17.7x_4 + 17.7x_5 + 17.7x_6 + 17.7x_7 \leq 1824$$

$$14.9x_1 + 22.5x_2 + 43.7x_3 + 43.7x_4 + 43.7x_5 + 43.7x_6 + 43.7x_7 \leq 4500$$

$$0.5x_1 + 0.8x_2 + 1.6x_3 + 1.6x_4 + 1.6x_5 + 1.6x_6 + 1.6x_7 \leq 166$$

$$3.3x_1 + 5.0x_2 + 25.9x_3 + 25.9x_4 + 25.9x_5 + 25.9x_6 + 25.9x_7 \leq 1000$$

$$8.9x_1 + 13.3x_2 + 25.9x_3 + 25.9x_4 + 25.9x_5 + 25.9x_6 + 25.9x_7 \leq 2669$$

$$0.8x_1 + 1.2x_2 + 2.4x_3 + 2.4x_4 + 2.4x_5 + 2.4x_6 + 2.4x_7 \leq 247$$

$$1.4x_1 + 2.2x_2 + 4.2x_3 + 4.2x_4 + 4.2x_5 + 4.2x_6 + 4.2x_7 \leq 437$$

$$72.8x_5 \leq 7000$$

$$3.0x_1 + 4.5x_2 + 8.8x_3 + 8.8x_4 + 8.8x_5 + 8.8x_6 + 8.8x_7 \leq 906$$

$$1.7x_1 + 2.5x_2 + 4.9x_3 + 4.9x_4 + 4.9x_5 + 4.9x_6 + 4.9x_7 \leq 500$$

$$24.2x_4 \leq 25000$$

By introducing slack variables which are x_8, x_9, \dots, x_{21} converting inequalities to equations, we have the standard form as follows:

$$\text{Maximize } Z=55.1x_1+301.1x_2+919.7x_3+1177.7x_4+1146.9x_5+719.7x_6+819.7x_7$$

Subject to the constraints

$$142.4x_1 + 215x_2 + 417.5x_3 + 417.5x_4 + 417.5x_5 + 417.5x_6 + 417.5x_7 + x_8 = 43000$$

$$10.5x_1 + 15.9x_2 + 30.8x_3 + 30.8x_4 + 30.8x_5 + 30.8x_6 + 30.8x_7 + x_9 = 3176$$

$$4.5x_1 + 6.8x_2 + 13.1x_3 + 13.1x_4 + 13.1x_5 + 13.1x_6 + 13.1x_7 + x_{10} = 1352$$

$$6.0x_1 + 9.1x_2 + 17.7x_3 + 17.7x_4 + 17.7x_5 + 17.7x_6 + 17.7x_7 + x_{11} = 1824$$

$$14.9x_1 + 22.5x_2 + 43.7x_3 + 43.7x_4 + 43.7x_5 + 43.7x_6 + 43.7x_7 + x_{12} = 4500$$

$$0.5x_1 + 0.8x_2 + 1.6x_3 + 1.6x_4 + 1.6x_5 + 1.6x_6 + 1.6x_7 + x_{13} = 166$$

$$3.3x_1 + 5.0x_2 + 25.9x_3 + 25.9x_4 + 25.9x_5 + 25.9x_6 + 25.9x_7 + x_{14} = 1000$$

$$8.9x_1 + 13.3x_2 + 25.9x_3 + 25.9x_4 + 25.9x_5 + 25.9x_6 + 25.9x_7 + x_{15} = 2669$$

$$0.8x_1 + 1.2x_2 + 2.4x_3 + 2.4x_4 + 2.4x_5 + 2.4x_6 + 2.4x_7 + x_{16} = 247$$

$$1.4x_1 + 2.2x_2 + 4.2x_3 + 4.2x_4 + 4.2x_5 + 4.2x_6 + 4.2x_7 + x_{17} = 437$$

$$72.8x_5 + x_{18} = 7000$$

$$3.0x_1 + 4.5x_2 + 8.8x_3 + 8.8x_4 + 8.8x_5 + 8.8x_6 + 8.8x_7 + x_{19} = 906$$

$$1.7x_1 + 2.5x_2 + 4.9x_3 + 4.9x_4 + 4.9x_5 + 4.9x_6 + 4.9x_7 + x_{20} = 500$$

$$242x_4 + x_{21} = 25000$$

Optimal solution obtained after using simplex method on Tora software and Maple software as follows

Table 3: Result From The Analysis using Simplex Method

Variables	Value	Explanation	Obj Value Contrib.
No Of iteration	3		
Objective value	60047.91		
Small bread(x_1)	0.00	55.10	0.00
Medium bread(x_2)	197.84	301.10	59569.76
Large bread(x_3)	0.00	919.70	0.00
Sardine bread(x_4)	0.00	1120.4	0.00
Coconut bread(x_5)	0.42	1146.90	478.14
Butter bread (x_6)	0.00	719.70	0.00
Sliced bread(x_7)	0.00	819.70	0.00
Constraint	RHS	Slack- /Surplus	
x_8	43000.00	290.24-	
x_9	3176.00	17.50-	
x_{10}	1352	17.50-	
x_{11}	1824	16.27-	
x_{12}	4500.00	30.37-	
x_{13}	166.00	7.06-	

x_{14}	1000.00	0.00
x_{15}	2669.00	26.92-
x_{16}	247.00	8.59-
x_{17}	437.00	0.00
x_{18}	7000.00	6969.65-
x_{19}	906	12.05-
x_{20}	500.00	3.36-
x_{21}	25000.00	25000-

Discussion

From the result obtained in Table 3 reveals that small bread(x_1)=0, medium breads(x_2) = N 59,569.76 , large bread (x_3)=0, sardine bread(x_4)=0, coconut bread(x_5)=~~N~~478.14, butter bread(x_6)=0 and sliced bread(x_7)=0. This solution simply shows that x_2 and x_5 are the only products that contribute meaningfully to improving the value of the objective function. It entails that for the bakery to make a profit of ₦60,048 from the available resources based on the cost of raw materials then the bakery should produce more medium bread(x_2)

Conclusion

The various varieties of bread, the amount of raw materials used and the production cost in Toprano Bakery have been considered in this work. Using the data obtained from the bakery on the seven varieties of bread produced, the optimal solution was obtained after a formulation of the linear programming problem. The result obtained shows that the management of Toprano Bakery should focus more on the production application of Linear programming for profit maximization of the medium bread and produce less of the other products since they contribute less so as to realize a profit of ₦60,048.

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