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New Piecewise Linear Approximation Techniques for Nonlinear Programming Problems

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Abstract
This study presented a novel piecewise linear approximation algorithm for efficiently solving large-scale separable optimisation and concave transportation problems. Six problem sets, three for separable optimisation problems and three for concave transportation problems, were extracted from published articles to demonstrate the effectiveness of the new algorithms. Comparative analysis with existing piecewise linear approximation PLA methods demonstrates that the new approaches achieve higher accuracy, reduced computational time, and improved stability in handling complex constraints. The findings suggest that the newly developed techniques can serve as a valuable tool for researchers and practitioners in optimisation, offering a balance between precision and computational efficiency. The results show significant reductions in the standard error of the mean for goodness-of-fit measures, indicating enhanced reliability and scalability. The study concludes that the techniques developed and employed contributed to the advancement of computational optimisation techniques, providing a robust and efficient solution for practitioners and researchers tackling large-scale separable optimisation and concave transportation problems.

Keywords: Piecewise Linear Approximation Techniques, Nonlinear Programming Problems

Introduction

Mathematical optimisation or mathematical programming problem is the main problem to be considered in the study. Mathematical programming, sometimes known as mathematical optimisation, is an area of mathematics that uses mathematical approaches to optimise a function within constraints. It is an effective tool for tackling complicated decision-making problems in a variety of disciplines, including economics, finance, engineering, logistics, and computer science. Mathematical programming problem seeks to minimise (or maximise) a function of many variables subject to a set of constraints (Ogbonna et al., 2019). If the objective and constraints of an optimisation problem are given as mathematical functions and functional relations, it is called mathematical programming. Linear Programming (LP, also called linear optimisation) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. The Linear Programming Problem can be represented mathematically as;

$$\left.\begin{array}{l} Maximize \quad f(z) \\ Subject \quad to \quad A(z) \le b \\ \qquad z \ge 0 \end{array}\right\}$$

The term 'nonlinear programming' usually refers to the problem in which the objective function in the equation above becomes non-linear, or one or more of the constraint inequalities have non-linear or both. If it $f(z)$ is a quadratic function with linear constraints, the problem is called a “quadratic programming problem”. For instance, in business, if a firm is seeking to maximise profit or minimise cost, but there are constraints on the amount of resources or monetary funds available, this type of problem comes up. Many works done in nonlinear programming problems, especially in transportation problems where discount was directly related to the unit commodity employed the Karush-Kuhn-Tucker (KKT) optimality algorithm, and also made use of the heuristic approaches of finding the initial basic feasible solution, which gave a large number of iterations before optimal solution. Again, past works suggested a new technique of solving nonlinear programming transportation problems that would produce an optimal solution to all problems, due to the inability of the existing techniques to tackle some real-life problems. However, an efficient method for the solution of a general nonlinear programming problem is still a subject of research. In this study, however, a new technique will be proposed to solve nonlinear programming problems more quickly and with high accuracy by first linearising the objective function. In particular, the study will consider the case when the objective function f(x) is a separable quadratic function and the constraints are linear. This study shall examine a non-linear separable quadratic function with many variables and constraints in the simulation case and using a special type of optimisation software, known as Wolfram Mathematica to solve the nonlinear programming test problems. It is as a result of the hiatus observed in the literature as stated above that brought about this present work to develop a new algorithm and different solution procedures to nonlinear programming problems; that would tackle all problems involving objective function being separable quadratic and the constraints being linear and as well reduce the time of execution via Wolfram Mathematica optimization software.

Duguet and Ngueveu (2022) worked on piecewise linearization of bivariate nonlinear functions: minimising the number of pieces under a bounded approximation error. The work focused on the approximation of bivariate functions into piecewise linear ones with a minimal number of pieces and under a bounded approximation error. Applications included the approximation of mixed integer nonlinear optimisation problems into mixed integer linear ones that were, in general easier to solve. A framework to build dedicated linearization algorithms was introduced, and a comparison to the state-of-the-art heuristics showed their efficiency. Alkhalifa and Mittelmann (2022) developed a novel piecewise linear approximation technique for solving mixed integer quadratically constrained quadratic programming (MIQCQP) issues. The twentieth century saw a substantial advancement in linear optimisation techniques and procedures, which paved the way for the creation of trustworthy mixed integer linear programming (MILP) solvers. The study provided a brief overview of the background and literature before introducing piecewise linear approximation (PLA), outlining its contribution and finally showing the outcomes of computing experiments and their conclusions. The work aimed to: (a) enhance PLA models through the use of non-uniform domain partitioning; and (b) provide a method for partially using PLA to solve more manageable mixed integer nonlinear programming (MINLP) problems. Quadratically constrained quadratic programming (QCQP) and MIQCQP were used in the computational tests, which demonstrated that PLA issues with non-uniform partition produced more accurate solutions in less time than PLA problems with uniform partition. Vielma, Ahmed, and Nemhauser (2010) conducted computational experiments in which they used CPLEX to solve the PLAs and 100 test examples to approximate functions with one variable. All models were found to function well for fewer than ten breakpoints, with the multiple choice model marginally outperforming the others. The logarithmic models began to prevail as the number of breakpoints increased. The logarithmic models were over 20 times faster than the incremental model when 33 breakpoints were utilised, while the incremental model was more than twice as fast as the multiple choice and CC models. The convex combination that has been broken down moves significantly more slowly than the others. Testing one hundred problems with two variables in the approximated functions produced similar results. In a less comprehensive study, Geißler, Martin, Morsi, and Schewe (2012) found that, despite the logarithmic model's reduced size, there are situations when the incremental model is preferable.

Using the piecewise linear approximation technique, Hou and Liu (2023) worked on a global algorithm for a class of multiplicative programmes. An effective approach for resolving a class of multiplicative programmes (MP) was provided in the paper. They converted the issue (MP) into the equivalent problem (EP) by adding auxiliary variables before finding the global optimal solution to the problem (MP). To determine the lower bound of the optimal value for the problem (EP), the problem (EP) was then methodically transformed into a sequence of linear relaxation programming problems (RP) using the recently developed piecewise linear approximation approach. Then, certain space-accelerating strategies were developed to speed up the algorithm's convergence, taking into account the features of the problem (EP) and the branch-and-bound framework's structure. Additionally, they provided an estimate for the algorithm's maximum iterations by examining the algorithm's computational complexity. Lastly, numerical data were presented to show the algorithm's robustness and efficacy. Rao et al. (2019) researched on effect of volume discounts on optimal transportation cost. In the current investigation, various optimal solutions have been worked out under different conditions of discount and the total transportation cost, which helped in decision making. The linear transportation solution obtained was highest while the solution from the all quantity discount scheme was lowest; it was due to the discount not being apportioned in the former case while being allowed to each item transported in the latter case. The incremental cost stood between these two because the differential discount implemented based on batches led to a higher cost than the all-quantity discount scheme. The solution obtained by the genetic algorithm could be improved further by having more iterations and also by selecting rational mathematical functions and by other mathematical techniques like response surface methodology. The linking of unitising the load based on the shape and size was assumed to be constant while transporting in various modes; otherwise, the cost would be subjected to further perturbation. The study concluded that the cost of transportation would be computed by Vogel's trial solution and improved iteratively by a modified distribution method without taking any discount factor on the unit transportation cost. The discounts offered in the transportation based on the quantity transported from source to destination eventually reduced the optimal cost.

In a work written by Juman and Nawarathne (2019) on an efficient alternative approach to solve a transportation problem, an efficient technique named JHM (Juman and Hoque's Method) was proposed to compute a better initial feasible solution (IFS) to a transportation problem. In JHM, only column penalties were considered. In their study, a new approach was proposed with row penalties to obtain an IFS to a transportation problem. The researchers demonstrated a numerical technique to illustrate the new technique. A comparative study on a set of benchmark instances showed that the new technique provided the same or better initial feasible solution to all the problems except one. The researchers, therefore, concluded that their new technique could be considered as an alternative technique for obtaining an initial feasible solution to a transportation problem. A study was carried out by Nasrollahi et al. (2022) on a developed nonlinear model for the location-allocation and transportation problems in a cross-docking distribution network. They proposed a developed mixed-integer nonlinear model, which was used for a post-distribution cross-docking strategy with multiple cross-docks and products that could be connected. The total cost, which consists of the cost of established cross-docks and transportation costs, was minimized using the objective function. The researchers introduced two models, which were compared, and the models were nonlinear models 1 and 2 with the possibility of connections between cross-docks. The results of the study showed that since the connection between the cross-docks existed, the total cost reduced. The result further showed that model 2 outperformed model 1, which warranted the consolidation of plant orders being added to model 2 and the developed model was formed. The software GAMS was employed to solve some numerical problems of different sizes generated at random in order to evaluate the accuracy of the model.

Aim and Objectives of the Study

This study aimed to develop an algorithm that would provide a solution to piecewise linear approximation for nonlinear programming problems with linear constraints. The specific objectives of the study will be to:

i. Formulate a new algorithmic approach for solving separable optimization problems using piecewise linear approximation and develop a Wolfram Mathematica script for its implementation;
ii. Evaluate the accuracy, precision, and reliability of the new algorithm for solving large-scale separable optimization problems, using goodness-of-fit measures in comparison to the traditional computational approach;

Materials and Methods

Illustration of New Algorithmic Approach for Solving Separable Optimization Problems using Piecewise Linear Approximation. This section explained the developed approach of piecewise linear approximation for solving separable optimization problems and creating the corresponding Wolfram Mathematica script for its implementation in which the objective one of this study was achieved. However, it would be ideal for a proper understanding of piecewise linear function and description for simplex method for a maximum (minimum) linear programming problem, before the description of the new approach.

Piecewise Linear Function

Linear mathematical programming is employed even when nonlinear terms are present. Consider the programming model in its algebraic state.

$$\left.\begin{aligned} &Min.\ f(z)=\sum_{j=1}^{n} f_j(x_j) \\ &Subject\ to: \sum_{j=1}^{n} a_{ij}x_j \le b_i, i=1,2,\cdots,m \\ &\qquad\qquad x_j \ge 0, j=1,2,\cdots,n \end{aligned}\right\} \tag{1}$$

In the model, the m constraints are linear, while the objective function involves n nonlinear, separable terms, each a function of a single variable only. The above maximization problem can only be solved by linear programming algorithm if approximated with a linear model on a condition that each $f_j(x_j)$ is concave in the objective function. A function that has a continuous first derivative is concave if the second derivative is everywhere non-positive.

The approximation identifies r break points along the x_j axis: $d_1, d_2, ..., d_r$, and r corresponding points along the f_j axis: $c_1, c_2, ..., c_r$. Define new variables $x_{j1}, x_{j2}, ..., x_{jr}$ to denote the pieces when the piecewise linear approximation is employed in a linear programming model. The kth segment has its slope as

$$s_{jk} = (c_k - c_{k-1})/(d_k - d_{k-1})$$

The jth term in the objective function for the piecewise linear approximation is

$$\sum_{k=1}^{r} s_{jk} x_{jk} \to f_j(x_j)$$

The variable x_j is replaced in each constraint by

$$\sum_{k=1}^{r} x_{jk} \to x_j$$

and the new variables should satisfy the following bounds

$$0 \le x_{j2} \le d_k - d_{k-1}, k = 1,2,...,r$$

However, if $f_j(x_j) = c_j x_j$, there will not be need for substitution; then each original variable x_j must be replaced with r_j new variables. The number of break points along the x_j-axis is denoted by r_j and may change from one variable to the next. The approximate model is formed if and only if the appropriate substitutions are carried out and the model is given by

$$\left.\begin{aligned} &Min.\ z = \sum_{j=1}^{n}\sum_{k=1}^{r_j} s_{jk} x_{jk} \\ &Subject\ to: \sum_{j=1}^{n}\sum_{k=1}^{r_j} a_{ij} x_{jk} \le b_i, i = 1,2,\cdots,m \\ &\qquad 0 \le x_{jk} \le d_k - d_{k-1}, k = 1,2,\cdots,r_j, j = \cdots, n \end{aligned}\right\} \tag{2}$$

By defining enough break points, with a corresponding increase in dimensionality, the approximation is achieved as accurate as required.

It should be noted that minimizing a function is the same with maximizing the same function with its sign reserved. If for minimization problem, the $f_j(x_j)$ is convex, separable terms, then it can be approximated in the same way as a concave function when the goal is to maximize.

Description of the Developed Approach of Piecewise Linear Approximation for Solving Separable Optimization Problems

In describing the developed approach, consider the problem of the form:

$$\left.\begin{aligned} &Min.\ f(z) = \sum_{i=1}^{n}\left(c_i z_i + d_i z_i^2\right) \\ &Subject\ to: Cz \ge \underline{d} \\ &\qquad B_l \le z \le B_u \end{aligned}\right\} \tag{3}$$

where $z = (z_1, z_2, \cdots, z_n) \in \mathbb{R}^n$ and $f(z)$ is a nonlinear separable function such that:

$$f(z) = f_1(z_1) + f_2(z_2) + \cdots + f_n(z_n) = \sum_{i=1}^{n} f_i(z_i) \tag{4}$$

Here we say that a function $f(z_1, z_2, \cdots, z_n)$ is separable if it can be expressed as the sum of n single-

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variable functions $f_1(z_1)+f_2(z_2)+\cdots+f_n(z_n)$ as stated in Equation (4)

The first step of this algorithm is to introduce the quadratic approximation formula as:

$$f(z)\approx f(k)+f'(k)+f'(k)(z-k)+\frac{f''(k)}{2}(z-k)^2,\ for\ z\approx k \tag{5}$$

Let the function $f(z)$ be given as;

$$f(z)=\cos z \tag{6}$$

The next step is to obtain the quadratic approximation of the function in Equation (6) valid for $z\approx 0$

$$f(0)=1 \tag{7}$$

$$f'(z)=-\sin z \tag{8}$$

$$f'(0)=0 \tag{9}$$

$$f''(z)=-\cos z \tag{10}$$

$$f''(0)=-1 \tag{11}$$

From Equation (5) for $z\approx 0$, we have:

$$f(z)\approx f(0)+f'(0)+f'(0)(z)+\frac{f''(0)}{2}z^2 \tag{12}$$

Substitute Equations (7), (9) and (11) into Equation (12) to obtain Equation (13)

$$\cos z\approx 1-\frac{z^2}{2} \tag{13}$$

Making z^2 the subject of the relation in Equation (13), we get Equation (14)

$$z^2=2(1-\cos z) \tag{14}$$

For multiple z^2, Equation (14) can be written as stated in Equation (15)

$$z_i^2=2(1-\cos z_i) \tag{15}$$

For this algorithm, we shall employ 4 uniform breakpoints (1/4, 1/2, 3/4 & 1) and evaluate z_i^2 in Equation (15) given $z=\frac{1}{4},\frac{1}{2},\frac{3}{4}\ and\ 1$.

Data Collection

Secondary data (problem sets) to illustrate the authenticity of the self-developed algorithms were collected from Gupta and Hira (2011), and also from published works of Ojekudo and Opara (2022) as presented in this study.

Results

Example I: A manufacturing unit produces two products, the radios and TV sets. The production cost of each product depends upon the number of units being produced. If z_1 and z_2 are the number of radios and TV sets produced, then the production costs are $200z_1+0.2z_1^2$ and $300z_2+0.2z_2^2$ respectively. There is restriction on the production capacity of the radios and TV sets to 100 and 80 units respectively. Similarly, there is restriction on the manpower available. A total 520 man-days are available. The production of one piece of radio requires 2 man-days and one TV set requires 3 man-days. The sale price is dependent upon the quantity to be produced and the sale relationships are given in Table 1.

Table 1: Information about Example 1

Product	Quantity demanded	Unit price
Radios	2,000 – 5p	P
TV sets	4,000 – 10q	Q

The problem is to determine the number of radios and TV sets which should be produced to maximize the

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profits (Gupta & Hira, 2011).

Formulation of Mathematical Model

Since z_1 and z_2 are the quantities of radios and TV sets to be produced by the firm,

$$z_1 = 2000 - 5p \quad or \quad p = 400 - 0.2z_1, \; and$$
$$z_2 = 4000 - 10q \quad or \quad q = 400 - 0.1z_2$$

If the total production cost of z_1 unit of radios and z_2 units of TV sets is denoted by K_1 and K_2 respectively, then it is also given that

$$K_1 = 200z_1 + 0.2z_1^2,$$
$$K_2 = 300z_2 + 0.2z_2^2.$$

The total revenue = Revenue of radios + Revenue of TV sets.

i.e.,
$$\begin{aligned} R &= pz_1 + qz_2 \\ &= (400 - 0.2z_1)z_1 + (400 - 0.1z_2)z_2 \\ &= 400z_1 - 0.2z_1^2 + 400z_2 - 0.1z_2^2 \end{aligned}$$

Therefore, total profit P is,

$$\begin{aligned} P &= R - (K_1 + K_2) \\ &= 400z_1 - 0.2z_1^2 + 400z_2 - 0.1z_2^2 - 200z_1 - 0.2z_1^2 - 300z_2 - 0.2z_2^2 \\ &= 200z_1 - 0.4z_1^2 + 100z_2 - 0.3z_2^2 \end{aligned}$$

There are constraints on the production capacity:

$$z_1 \le 100$$
$$z_2 \le 80$$

Similarly, man-days available are also limited to 520. Since one unit of radio requires 2 man-days and one unit of TV requires 3 man-days,

$$2z_1 + 3z_2 \le 520$$

Since z_1 and z_2 cannot take negative values,

$$z_1, z_2 \ge 0$$

Thus, the model is

$$\left.\begin{aligned} &\text{Max. f} = 200z_1 - 0.4z_1^2 + 100z_2 - 0.3z_2^2 \\ &Subject\ to: 2z_1 + 3z_2 \le 520 \\ &\qquad 0 \le z_1 \le 100 \\ &\qquad 0 \le z_2 \le 80 \end{aligned}\right\} \tag{4.1}$$

Existing Technique

Equation (1) was written in a Wolfram Mathematica script as shown Wolfram Mathematica Script 1:

Wolfram Mathematica Script 1

```
Clear[Maxf,constraints,z1,z2,tim];
Maxf=200 z1-0.4 z1^2+100 z2-0.3 z2^2;
constraints={2 z1+3 z2<=520,0<=z1<=100,0<=z2<=80};
tim=Timing[NMaximize[{Maxf,constraints},{z1,z2},StepMonitor:>Print["Step : z1, z2 = ",z1,", ",z2]]]
sol=Timing[NMaximize[{Maxf,constraints},{z1,z2}]];
Print["Optimal solution: ",sol[[2]]];
Print["Maximum value: ",sol[[1]]];
Print["Time of execution: ",tim[[1]]," seconds"];
```

Solving the problem in Mathematica Script 1 yielded the optimal solution (See Appendix I):

$z_1 = 100, z_2 = 80$ with time of execution as 0.328125 seconds.

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Example 2

Unilever Nigeria Plc located in Apapa Ikeja, Lagos manufactures and markets consumer products primarily in the home, personal care and foods categories. The Company sells products such as Omo washing powder, Key soap, Royco bouillon, Lipton tea, Blue Band margarine; Pears baby care goods, Vaseline petroleum jelly, Lux soap, and Close Up toothpaste. The products are supplied to various market segments in Nigeria. For the purpose of this study, only 3 of these demand points, and 3 products will be considered. The estimated supply capacities of the three products, the demand requirements at the three sites (market centres) and the transportation cost per carton of each product are given in Table 2.

Table 2: Information about Example 2 on Supply and Demand of Products

	MARKETS SEGMENTS			SUPPLY
	P	Q	R	
Omo washing powder	5	4	6	13,000
Blue Band margarine	7	6	5	13,000
Vaseline	9	11	8	15,000
DEMAND	11,000	18,000	12,000	

The policy of the company allows discounts on each transported product from source to each of the destinations and the percentage discounts are shown in Table 4.3.

Table 3: Information about Example 2 on Percentage Volume Discounts

	P	Q	R
Omo washing powder	0.03	0.015	0.04
Blue Band margarine	0.02	0.03	0.05
Vaseline	0.035	0.05	0.03

Source: Opara and Ojekudo (2022)

The nonlinear transportation programming problem is:

Existing Technique

Equation (9) was written in a Wolfram Mathematica script as shown Wolfram Mathematica Script 9:

$$\left.\begin{aligned}
&\text{Min. f} = 5z_{11} - 0.03z_{11}^2 + 4z_{12} - 0.015z_{12}^2 + 6z_{13} - 0.04z_{13}^2 + 7z_{21} - 0.02z_{21}^2 + 6z_{22} - 0.03z_{22}^2 + \\
&\qquad 5z_{23} - 0.05z_{23}^2 + 9z_{31} - 0.035z_{31}^2 + 11z_{32} - 0.05z_{32}^2 + 8z_{33} - 0.03z_{33}^2 \\
&\textit{Subject to}: z_{11} + z_{12} + z_{13} \le 13 \\
&\qquad z_{21} + z_{22} + z_{23} \le 13 \\
&\qquad z_{31} + z_{32} + z_{33} \le 15 \\
&\qquad z_{11} + z_{21} + z_{31} \ge 11 \\
&\qquad z_{12} + z_{22} + z_{32} \ge 18 \\
&\qquad z_{13} + z_{23} + z_{33} \ge 12 \\
&\qquad z_{ij} \ge 0, \forall i, j
\end{aligned}\right\} \quad (9)$$

Wolfram Mathematica Script 9

```
Clear[f2,z,L,K,V,a];
f2[z_]:=5*z[1,1]-0.03*z[1,1]^2+4*z[1,2]-0.015*z[1,2]^2+6*z[1,3]-0.04*z[1,3]^2+7*z[2,1]-
0.02*z[2,1]^2+6*z[2,2]-0.03*z[2,2]^2+5*z[2,3]-0.05*z[2,3]^2+9*z[3,1]-0.035*z[3,1]^2+11*z[3,2]-
0.05*z[3,2]^2+8*z[3,3]-0.03*z[3,3]^2;
L1:=z[1,1]+z[1,2]+z[1,3]==13;
L2:=z[2,1]+z[2,2]+z[2,3]==13;
L3:=z[3,1]+z[3,2]+z[3,3]==15;
L4:=z[1,1]+z[2,1]+z[3,1]==11;
L5:=z[1,2]+z[2,2]+z[3,2]==18;
```

```
L6:=z[1,3]+z[2,3]+z[3,3]==12;
L=Join[{L1},{L2},{L3},{L4},{L5},{L6}];
K1:=z[1,1]>=0;
K2:=z[1,2]>=0;
K3:=z[1,3]>=0;
K4:=z[2,1]>=0;
K5:=z[2,2]>=0;
K6:=z[2,3]>=0;
K7:=z[3,1]>=0;
K8:=z[3,2]>=0;
K9:=z[3,3]>=0;
K=Join[{K1},{K2},{K3},{K4},{K5},{K6},{K7},{K8},{K9}];
V=Join[L,K];
a=Variables[f2[z]];
tim=Timing[NMinimize[{f2[z],V},a,StepMonitor:>Print["Step : z[1,1],z[1,2],z[1,3],z[3,3] =
",z[1,1]",",z[1,2]",",z[1,3]",",z[3,3]]]]];
sol=Timing[NMinimize[{f2[z],V},a]];
Print["Optimal solution: ",sol[[2]]];
Print["Time of execution: ",sol[[1]]," seconds"];
```

Solving the problem in Wolfram Mathematica Script 9 yielded the optimal solution (See Appendix II): $z_{11}=0,\ z_{12}=13,\ z_{13}=0,\ z_{21}=0,\ z_{22}=5,\ z_{23}=8,\ z_{31}=11,\ z_{32}=0,\ z_{33}=4$ with time of execution as 1.10938 seconds.

Conclusion

Objective one formulated a new algorithmic approach for solving separable optimization problems using piecewise linear approximation and developed a Wolfram Mathematica script for its implementation, and the results gotten from the new algorithm via the Wolfram Mathematica script showed that the new technique is accurate, more precise, and reliable than the traditional/conventional approach especially for solving large-scale separable optimization problems. The finding of this study is related to the work of Bazaraa et al. (2011) on linear programming and network flows whose findings demonstrated the effectiveness of piecewise linear approximation in solving large-scale optimization problems. The result of this study also aligns with the findings of Sherali and Adams (1999) on a reformulation-linearization technique for solving discrete and continuous non-convex problems whose conclusion was that piecewise linear approximation could improve the accuracy and efficiency of optimization algorithms. The findings revealed that the developed piecewise linear approximation technique: achieved identical optimal solutions as traditional methods, but with significantly reduced execution times; exhibited higher accuracy, precision, and reliability for large-scale problems, as evidenced by lower standard errors of the mean for goodness-of-fit measures.

Recommendations

Based on the results obtained from this study, the following are recommended:

1. The written Wolfram Mathematica programming language scripts in this study should be employed by future researchers.
2. Practitioners and researchers should consider the developed piecewise linear approximation algorithm for solving large-scale separable optimization and concave transportation problems.

References

Alkhalifa, L. & Mittelmann, H. (2022). New algorithm to solve mixed integer quadratically constrained quadratic programming problems using piecewise linear approximation. *Mathematics*, *10(2)*, 198-207. https://doi.org/10.3390/math10020198

Bazaraa, M. S., Jarvis, J. J., & Sherali, H. D. (2011). Linear programming and network flows. Wiley.

Duguet, A., & Ngueveu, S. U. (2022, May). Piecewise linearization of bivariate nonlinear functions: minimizing the number of pieces under a bounded approximation error. In *International Symposium on Combinatorial Optimization* (pp. 117-129). Cham: Springer International Publishing.

Geißler, B., Martin, A., Morsi, A. & Schewe, L. (2012). Using piecewise linear functions for solving MINLPs. *In Mixed integer nonlinear programming*, 287-314. New York, NY: Springer New York.

Gupta, P.K. and Hira, D.S. (2011). Operations Research. S. Chand & Company Ltd.,7361, Ram Nagar, New Delhi.

Hou, Z. & Liu, S. (2023). Global algorithm for a class of multiplicative programs using piecewise linear approximation technique. *Numerical Algorithms*, 92, 1063–1082.

Juman, Z. A. M. S. & Nawarathne, N. G. S. A. (2019). An efficient alternative approach to solve a transportation problem. *Ceylon Journal of Science*, *48(1)*, 19–29.

Nasrollahi, S., Hosseini-Nasab, H., Fakhrzad, M. B. & Honarvar, M. (2022). A developed nonlinear model for the location-allocation and transportation problems in a cross-docking distribution network. *Operations Research and Decisions*, *32(1)*, 127–148.

Ogbonna, C., J., Opara, J., Iheagwara, A. I. & Ihekuna, S. O. (2019). Proposed Piecewise Linear Approximation for Tackling Nonlinear Programming Problems with Separable Objective Function. *International Journal of Engineering Applied Sciences and Technology*, *4(3)*, 338-341.

Ojekudo, A. N., & Opara, J. (2022). An explicit and efficient self-developed algorithm for IBFS of a transportation problem. *Asian Journal of Multidimensional Research*, *11*(11), 58–72. https://doi.org/10.5958/2278-4853.2022.00325.1

Rao, D. R. S. N., Reddy, G. S., Satyanarayana, V. V. & Kumar, J. J. (2019). Effect of volume discounts on optimal transportation cost. *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*, *8(8)*, 472–474.

Sherali, H. D., & Adams, W. P. (1999). A reformulation-linearization technique for solving discrete and continuous nonconvex problems. Springer.

Vielma, J. P. Ahmed, S. & Nemhauser, G. (2010). Mixed-integer models for non-separable piecewise-linear optimization: unifying framework and extensions. *Operations Research*, 58, 303–315.

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