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Queue Modelling and Customer Management in Bank Automated Teller Machines: A Case Study of Zenith Bank Plc, Ozoro

***Joel, E.I., & Augustine, C.D.**
*[1]Department of Statistics, Faculty of Science, Dennis Osadebay University, Asaba, Nigeria
[2]Department of Mathematics, Ignatius Ajuru University of Education, Port Harcourt, Nigeria

***Corresponding author email:** israel.joel@dou.edu.ng

Abstract

This research work was carried out to investigate queue modelling and customer management in bank automated teller machines: a case study of Zenith Bank Plc, Ozoro. Purposive sampling as a non-probability sampling technique, was utilized in selecting the Zenith Bank ATM. To enable us to achieve our goal, the study considered a Single-server queuing model. The one-sample Kolmogorov-Smirnov test was used to determine the distribution that fits the data, i.e. to test the hypothesis of a theoretical distribution. The test was carried out using SPSS software version 20.0, and the arrival rate and service rate of the system for the period of study were computed. Result from the study showed that customer service times had an average of 1.14 minutes per customer and a standard deviation of 0.43 and with the highest duration of service lasting 3.56 minutes, while the shortest lasted for 0.35 minutes. It was concluded that the distribution for the ATM-customer relationship exists and was obtained by the convolution of the two independent service processes (the customer and the machine time). The study recommended, amongst others, that ATMIA should encourage the purchase of ATMs which user user-friendly or which have a short guide around them.

Keywords: Queue Modelling, Automated Teller Machines,

Introduction

Queuing theory, a branch of mathematical study, delves into the intricacies of waiting lines. Its applications have proven highly effective in analyzing and designing systems related to traffic, services, and more. Everyday scenarios like waiting at ATMs, post offices, ticket counters, public transportation, or getting stuck in traffic, as well as technical domains such as manufacturing, computer networks, and telecommunications, frequently involve delays and queuing challenges. Queuing theory offers insights into understanding and alleviating congestion and delays within these waiting lines. Its primary aim is to enhance the efficiency of queuing systems, reducing customer waiting times while increasing the number of customers served. Customer wait times are influenced by factors like the queue size, the number of service providers, the service process, and the time each customer takes to be served. Delays and queuing problems are most commonly seen in banks as well as banks' ATM points (Adeniran, 2018).

A financial institution offers a range of financial services, whether through electronic means or traditional methods. These services encompass tasks such as receiving deposits, managing cash for clients, tracking and reporting account transactions, as well as extending loan facilities. It's important to note that there are also entities outside the traditional banking sector that offer similar financial services, albeit without meeting the formal legal criteria of a full-fledged bank (Joseph, 2016).

An Automated Teller Machine (ATM) is a sophisticated electronic communication device that empowers customers of financial establishments, like banks, to carry out a range of financial activities independently. These activities include withdrawing and depositing cash, transferring funds, and checking account details, all accessible around the clock without the need for direct interaction with bank personnel (Wikipedia, 2019). These machines go by different names depending on the region, for instance, "automated teller machine" in the United States, the UK, Australia, Malaysia, South Africa, Singapore, India, and more. In British English, commonly used terms include "cash point," "cash machine," "mini bank" (official term for Yorkshire Bank ATMs), and the informal "hole in the wall".

According to Mike (2018), the global count of ATMs in operation has now reached nearly 3.5 million. In modern ATMs, customers are typically identified by inserting a plastic ATM card or another valid payment card into the machine. Authentication is accomplished by the customer entering a Personal Identification Number (PIN), which must match the PIN stored in the card's chip (if equipped) or in the database of the issuing financial institution. Through an ATM, customers gain access to their bank accounts, enabling a wide array of financial transactions like cash withdrawals, balance inquiries, and even mobile phone credit top-ups.

It's a common sight to witness lengthy queues of customers waiting for service, whether at Automated Teller Machines (ATMs) or inside bank branches. While we encounter similar waiting lines at places like bus stops, fast food joints, clinics, traffic lights, and supermarkets, the prolonged queues within the banking sector are particularly concerning (Sharma, 2007). The concept of waiting in line is a ubiquitous phenomenon in our daily lives. Queues form when customers, be they humans or other entities, require service and must wait because their number exceeds the available service providers or the system's efficiency falls short, resulting in service times that exceed expectations. Some customers have to wait when the total number of service seekers surpasses the available service facilities, while, conversely, some service facilities remain underutilized when they outnumber the customers requiring their services.

Queuing at an ATM especially during the salary or festive period, has become a usual scene these days. The queue formed at the ATM might be caused either by the customer's slowness and illiteracy (that is, inability to use the ATM) or due to Machine faults and network problems. Hence queue will form at the ATM because the customer performs self-service, in if he or she wastes time doing that service will result in a queue. The queue due to the machine may also be in two ways, whether due to the network or due to the age of the machines because indicating that the machines are old and slow. Queue due to customer: This also may be due to the level of literacy of the customer (ability to operate the machine) or a single customer with multiple cards or performing different types of transactions with a single card; these are all factors that lead to queuing at the ATM service point.

Statement of the Problem

Even in this age of advanced technology, developing nations often contend with longer lines at banks and ATMs compared to more developed regions. The introduction of ATMs was intended to alleviate this issue, offering significant convenience to both financial institutions and their customers. ATMs enable customers to conduct financial transactions at their convenience, even outside regular banking hours and at various locations. Their primary goal is to deliver efficient and expedited services, prioritizing a swift customer experience. Businesses, particularly banks, are committed to enhancing their service quality, striving to reduce wait times and provide customers with an improved overall experience. It's worth noting that service inefficiencies have been identified as a key factor contributing to queue formation at ATM locations. This service inefficiency can largely be due to inability of the customer to use the machine appropriately or due to machine faults (e.g. network problem). Hence, in this situation where a queue arises in the system, it is appropriate to study the relationship between the ATM and the Customer service processes, in that the customer performs a self-service (in giving Commands to the ATM); the ATM performs a service of dispensing cash or information output. In this regard, this project is an attempt to describe or model the ATM-Customer relationship.

Aim and Objectives

This project aims to study and analyze queuing at the Zenith Bank ATM point in Ozoro, Delta State. The specific objectives are to:

1. Identify a distribution that describes the system of the ATM-Customer relationship
2. Find the validity, CDF and the memoryless property of the distribution.
3. Compare the relationship between the arrival rate (λ) and service rate of the ATM-customer (μ)

Methodology

The choice of a case study design was fitting for this project as it allowed us to delve deeply into the real-world context of the queuing phenomenon. The case study design was specifically selected because our focus was on queue modelling and customer management, and this topic could be best understood within the framework of the banking sector, with a particular emphasis on the dynamics of ATM-customer interactions. In this study, we adopted a single-case approach, examining the Zenith Bank Plc ATM in Ozoro. The selection of this specific ATM was done purposefully, using a non-probability sampling technique. We chose this approach not only for its convenience but also because it aligned with the research's objectives. The participants, who were ATM users included in our time studies, were individuals who visited the ATM terminal during specific time windows:

between 9:00 AM and 12:00 PM on Saturdays and 2:00 PM to 5:00 PM on other days.

Stochastic Queuing System

We consider a Single-server queuing model. The arrival process is Poisson with average arrival rate λ, and the service process is considered to have two processes. One involves the time elapsed for the self-service of the customer, and the other service process for the time elapsed by the ATM in dispensing cash. We let X_1 be the variable for time of self-service by the customer and X_2 is the timeof service by the machine such that $X_1 \sim Exp(\mu_1)$, and $X_2 \sim Exp(\mu_2)$ in this research single server was adopted because the Zenith Bank Plc have only one ATM located at Ozoro, Delta State. A waiting line forms when there are more customers than the service capacity available. When a customer joins this line, they can expect to wait for a specific duration before their service commences.

Service Distribution of ATM-Customer Relationship

Let X_1 ad X_2, be two independent and identically distributed exponential random variables with parametes μ_1 *and* $\mu_{2,}$ respectively,

Then $f(x_1) = \mu_1 e^{-\mu_1 x_1}$ and $\mu_1 > 0, x_1 > 0$ (1)

$f(x_2) = \mu_2 e^{-\mu_2 x_2}$ for $\mu_2 > 0, x_2 > 0$ (2)

Let $Z = x_1 + x_2$ (The goal is to find the distribution of Z).

Implies $x_2 = Z - x_1$

$$f(x_1, x_2) = f(x_1, Z - x_1)$$

$$= \mu_1 e^{-\mu_1 x_1} . \mu_2 e^{-\mu_2 (Z - x_1)}$$

$$f(x_1, Z - x_1) = \mu_1 \mu_2 e^{(\mu_2 - \mu_1) x_1} . e^{-\mu_2 Z} \quad (3)$$

By the concept of convolution;

$$f(z) = \int_0^z f(x_1) . f(z - x_1) dx_1 \quad (4)$$

$$= \int_0^z \mu_1 \mu_2 e^{(\mu_2 - \mu_1) x_1} . e^{-\mu_2 Z} dx_1$$

$$= \frac{\mu_1 \mu_2}{\mu_2 - \mu_1} e^{\mu_2 z} \int_0^z e^{(\mu_2 - \mu_1) x_1 dx_1}$$

$$= \frac{\mu_1 \mu_2}{\mu_2 - \mu_1} e^{-\mu_2 z} \left[e^{(\mu_2 - \mu_1) x_1} \right]_0^z$$

$$= \frac{\mu_1 \mu_2}{\mu_2 - \mu_1} \left[e^{\mu_1 z} - e^{\mu_2 z} \right] \quad (5)$$

$\mu_2 \neq \mu_1$ z>0

The distribution, *f(z),* above represents the probability distribution for the sum of two identically independently distributed exponential random variables (Oguntunde et al. 2013). Hence, we proceed to prove the validity of the distribution.

Validity of model *f (z)*

For the model, *f (z)*, to be a valid model, it should suffice that $\int_0^\infty f(z)\, dz = 1$

$$\int_0^\infty f(z)\, dz = \frac{\mu_1 \mu_2}{\mu_2 - \mu_1} \int_0^\infty (e^{-\mu_1 z} - e^{-\mu_2 z}) dz$$

$$= \frac{\mu_1 \mu_2}{\mu_2 - \mu_1} \left[\frac{1}{\mu_2} e^{-\mu_2 z} - \frac{1}{\mu_1} e^{-\mu_1 z} \right]_0^\infty$$

$$= \frac{\mu_1\mu_2}{\mu_2-\mu_1}\left[\frac{\mu_2-\mu_1}{\mu_1\mu_2}\right] = 1$$

Cumulative Density Function (CDF)

By definition, the cdf is derived by; $F(z) = P(Z \le z)$

$$= \int_0^z f(t)dt$$

$$= \frac{\mu_1\mu_2}{\mu_2-\mu_1}\int_0^z (e^{-\mu_1 t} - e^{-\mu_2 t})dt$$

$$F(z) = 1 + \frac{\mu_1}{\mu_2-\mu_1}e^{-\mu_1 z} - \frac{\mu_2}{\mu_2-\mu_1}e^{-\mu_2 z} \quad (6)$$

We can deduct from Equation (6) that $lim_{z\to\infty} F(z) = 1$

Moment Generating Function

The moment generating function (m.g.f) of a random variable Z is denoted by

$M_z(t) = E(e^{tz})$ where, $Z = X_1 + X_{2\,,}$

From the properties of m.g.f,

$M_{X_1+X_2}(t) = E(e^{tx_1}).E(e^{tx_2})$ where

$E(e^{tx_1}) = \frac{\mu_1}{\mu_1-t}$ and $E(e^{tx_2}) = \frac{\mu_2}{\mu_2-t}$

$E(e^{tx_1})$ and $E(e^{tx_2})$ are the moment generating functions for a convoluted exponential distribution with parameters, $\mu_1$and μ_2, respectively. Hence,

$$M_{X_1+X_2}(t) = \frac{\mu_1\mu_2}{(\mu_1-t)\,(\mu_2-t)} \quad (7)$$

Equation (7) can be re-written as $M_z(t) = \mu_1\mu_2[(\mu_1-t)\,(\mu_2-t)]^{-1}$

From the result in Equation (7), we can confidently generalize that if $X_1, X_2 \ldots\ldots, X_{n\,,}$are independently and identically distributed random variables, each having exponential distribution with parameter, μ_i, the moment generating function of the sum $X_1 + X_2 + \cdots. + X_{n\,,}$can be expressed as

$$M_z(t) = \prod_{i=1}^{n} \mu_i\,[\mu_i - t]^{-1} \quad (8)$$

Moments

The r^th^ raw moment of a random variable, say Z is given by;

As derived in Equation (7), $M_z(t) = \mu_1\mu_2[(\mu_1-t)\,(\mu_2-t)]^{-1}$Therefore, the first two moments are derived below as;

$$E(Z) = M'_z(0)$$

$$= \frac{1}{\mu_1} + \frac{1}{\mu_2}$$

$$E(z^2) = M''_Z(0) = 2[\mu_1^{-2} + \mu_1^{-1}\mu_2^{-1} + \mu_2^{-2}]$$

Hence, we can take the following generalizations;

1. The mean of the sum of ‘n’ independent Exponential distribution is the sum of individual means.

That is, if $X_1 + X_2 + \cdots + X_n$, then

$$E(Z) = \sum_{t=1}^{n} \frac{1}{\mu_i} \quad (9)$$

2. The rth moment of Z can be expressed as;

$$E(Z^r) = r^i \sum_{t=0}^{r} \mu_1^{-i} \mu_2^{-(r-i)} \quad (10)$$

Memory-less Property of the Distribution

The memory-less property of an exponential distribution is evident when the following condition is met: If a customer has already received service for a certain amount of time (s), the likelihood of them being served for an additional period of time (t) is identical to that of a completely new customer being served for the same duration (t).

That is,

$$P(X > s + t | X > s) = P(X > t)$$

Where X is a random variable

$$P(X > s + t | X > s) = \frac{P(X>s+t, X>s)}{P(X>s)}$$

$$= \frac{P(X>s+t)}{P(X>s)} \quad (11)$$

$$= \frac{\int_{(s+t)}^{\infty} \mu_e^{-\mu x} dx}{\int_{s}^{\infty} \mu_e^{-\mu x} dx}$$

$$= \frac{\mu_e^{-\mu(s+t)}}{e^{-\mu s}}$$

$$= e^{-\mu t} = P(X > t)$$

That is,

$$P(X > s + t | X > s) = P(X > t)$$

Let us now consider the distribution of the sum of two independent Exponential distributions given in Equation (5);

$$P(Z > s + t | Z > s) = \frac{P(Z>s+t)}{P(Z>s)}$$

From Equation (11),

$$\frac{\int_{s+t}^{\infty} \frac{\mu_1\mu_2}{\mu_2-\mu_1}(e^{-\mu_1 z} - e^{-\mu_2 z})dz}{\int_{s}^{\infty} \frac{\mu_2\mu_2}{\mu_2-\mu_2}(e^{-\mu_1 z} - e^{-\mu_2 z})dz} \neq P(Z > t)$$

Indicating that the equation cannot be the same or be in form $P(Z > t)$

Where $P(Z > t) = \frac{1}{\mu_2 - \mu_1}[\mu_2 e^{-\mu_1 t} - \mu_1 e^{-\mu_2 t}]$.

Therefore, we can infer that the memory-less property does not hold for the distribution F(z). Hence the need for the M/G/1 model to get the measures of the system. In the subsequent section we consider the M/G/1 model and its assumptions as well as to get the parameters of the model.

The Coefficient of Variation

The Coefficient of variation for the self-service process was computed by finding the quotient between standard deviation values of the service process and the mean of the service process. For example, the CV of the customer service process is CV = $\frac{STD}{Mean}$ *100 which is similarly done for the machine service process.

Methodology

The One sample Kolmogorov-Smirnov test was used to determine the distribution that fits the data (i.e. to test the hypothesis of a theoretical distribution). This test was carried out using SPSS software version 20.0 for windows. The arrival rate and service rate of the system for the period of study was computed. The Excel Queuing calculator for windows was employed to evaluate the system characteristics, and the results analyzed. The relationship were determined for the deviation of service time, and that of the waiting time was determined technically using the queuing calculator for Windows.

Results

Here, the descriptive statistics for the data, which includes the Minimum and Maximum values as well as the mean and its standard deviation values, as shown in Table 4.1.

Table 1: Descriptive statistics of the service time data

Service Processes	N	Minimum	Maximum	Mean	Std. Deviation
Customer Service	3185	0.35	3.56	1.14	0.43
Machine Service	3185	0.15	1.23	0.24	0.14
Turnaround time	3185	0.45	5.22	2.26	0.52

Let X_1 and X_2 be the observations of service time (in minutes), such that X_1 is the Customer service times and X_2 is the Machine service times. A total of 3185 customers were observed at the Zenith Bank ATM point. Customer service times had an average of 1.14 minutes per customer and a standard deviation of 0.43 and with the highest duration of service lasting 3.56 minutes while the shortest lasted for 0.35 minutes. The machine service times had a mean service time of 0.24 minutes with a deviation of 0.14 and the highest duration of service is 1.23 minutes while the shortest duration of service is 0.15 minutes. Similarly, the turnaround time which is the total time taken for the whole service process have an average mean of 2.26 minutes with a deviation of 0.52, also having the highest duration of 5.22 minutes and the shortest duration of 0.45 minutes, respectively.

Fitting the Service Time Distributions

One of the underlying assumptions in our selected models was that the service times at the ATM followed an exponential distribution. Therefore, to verify whether our service time data indeed adhered to this exponential distribution, we employed the Kolmogorov-Smirnov test. This test was conducted using SPSS software version 20 on a Windows platform. We opted for the Kolmogorov-Smirnov test because the specific distributional form of the random variable in question was uncertain.

Statement of Hypothesis for the distributions of service times

The following null and alternative hypotheses is used for the customer service times.

Ho: The distribution of the service times (customer, machine and turnaround times) is exponential

H_1: The distribution of the customer services times (customer, machine and turnaround times) is not exponential

The test analysis of the service time distributions is shown in the Table 2.

Table 2: Test analysis of the service time distributions.

One-Sample Kolmogorov-Smirnov Test				
		Customer service time	Machine service time	Turnaround times
N		90	90	90
Exponential parameter.[a,b]	Mean	36.98889	9.03333	56.55556
Most Extreme Differences	Absolute	.335	.441	.409
	Positive	.470	.420	.405
	Negative	-.75	-.241	-.209
Kolmogorov-Smirnov Z		3.179	4.182	3.877
Asymp. Sig. (2-tailed)		.057	.061	.071

a. Test Distribution is Exponential.
b. Calculated from data.

Decision rule: Reject Ho if the P-value is less than 0.05, accept otherwise.

The p-values associated with the customer service times exceed the 0.05 threshold, leading us to the conclusion that the observed service time data aligns with an exponential distribution. Given that the p-value for the service time exceeds 0.05, we confirm the null hypothesis, establishing that the service time follows an exponential distribution.

Fitting a Poisson distribution

One of the foundational assumptions in our selected models was that customer arrivals at the ATM followed a Poisson distribution. To assess whether our arrival time data conforms to this Poisson distribution, we conducted a Kolmogorov-Smirnov test.

Statement of Hypothesis for the Arrival distribution

The following null and alternative hypotheses were used for the arrival rates.

Ho: The distribution of the Customer arrival rate is Poisson

H_1: The distribution of the Customer arrival rate is not Poisson.

The test analysis of the service time distributions is shown in the Table 3.

Table 3: One-Sample Kolmogorov-Smirnov test for the Customer arrivals

One-Sample Kolmogorov-Smirnov Test		
		Arrival Rates
N		90
Poisson Parameter[a,b]	Mean	33.47
Most Extreme Differences	Absolute	.193
	Positive	.114
	Negative	-.193
Kolmogorov-Smirnov Z		2.830
Asymp. Sig. (2-tailed)		.006

a. Test distribution is Poisson.
b. Calculated from data.

Decision rule: Reject Ho if the P-value is less than 0.05, accept otherwise.

The P values for the customer arrivals is larger than 0.05, hence we conclude that the random observations of the customer arrivals come from the Poisson distribution.

Computations for the Arrival Rate of Customers (λ) at the ATM point

The arrivals of customers at the ATM point were counted for each day throughout the working period of the system and for the span of thirteen weeks of the data collection tabulation of the arrivals is shown in Table 4.4.

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Table 4: Arrivals of Customers at ATM Point

week	Queue at Zenith Bank ATM Point-Arrival rates								
	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total	λ $/min$
1	-	39	35	38	10	40	30	192	0.219
2	29	40	38	35	37	39	15	233	0.201
3	32	35	37	30	40	45	33	252	0.228
4	35	33	31	38	39	38	37	251	0.216
5	20	30	39	28	38	37	34	232	0.227
6	40	38	34	31	39	40	35	257	0.204
7	38	35	37	35	35	36	38	254	0.223
8	36	34	36	36	17	35	-	194	0.225
9	39	37	36	38	39	39	35	263	0.214
10	30	40	37	39	41	42	36	265	0.217
11	31	33	38	39	30	40	30	241	0.23
12	35	38	42	37	38	45	35	270	0.228
13	36	37	39	42	41	50	36	281	0.30
Total	401	475	479	466	444	526	394	3185	
$\boldsymbol{\lambda}$ $\boldsymbol{/min}$	0.208	0.219	0.213	0.216	0.221	0.251	0.212		0.225

A total of 3185 customers arrived to obtained service at Zenith Bank ATM point, for which a total of 248 hours dedicated for the observation.

$$\text{Hence, the arrival rate } (\lambda) = \frac{Total\ number\ of\ customers\ attended\ to}{total\ time\ of\ operation} \quad (12)$$

which will give 13 customers per hour, and 0.22 customers per minute, respectively. These results for the period of study were calculated and tabulated in Table 4

From the Table 4, on customers count for each day of the period under study. It can be seen that 3185 customers were observed in the period of 248 hours of the period studied. The average arrival of 13 customers per hours and 0.21 customers per minutes. It can also be seen that the ATM witnessed a moderately high rate of arrival on Saturdays, Fridays and Tuesday, while low turnout was observed on Mondays, Wednesdays and Sundays, from the results on weekly basis, it is clear that week 13, 8, 11, 12, 5 and 6 witnessed a moderately high turnout of customers, while week 2, 1, 7, 4, 10 witnessed a very low turnout, thus; we can refer that customers are usually seen on Saturdays, Fridays and Tuesdays than any other days of the week.

From the Appendix, showing the observation of 10 customers taken on day one of week one, we can see that the first customers arrive the ATM point in 3:03:18PM and depart the ATM point by 3:04:36PM; the customers wasted 1:00 minutes in imputing his particulars/details into the machine, while the machine takes about 0.18 minutes in dispensing the cash to the customer, which cumulatively sum to 1.18 minutes of the turnaround time. The next customer arrives point at 3:04:45 and have to wait for about 1.18. minutes or more before performing his services; this simply implies that the next customers must wait for 2.24 minutes, this definitely must have reduced his waiting time if the first and second customer only spent 1 minutes. This goes for all the customers in the system. We deduce that variation in the customer's service time and the machine service time actually impact on the other customers waiting time. We can also have observed that this help explain the memory-less property absent in our service distributions, that is our service distribution have a memory in the system.

Computation of the Mean Service Rate (μ)

The mean service rate was computed by finding the quotient between one and the sum of the two service processes; i.e. customer service time and the machine service times (in minutes). Referring to the equation 3.9 in chapter three, where we derived the mean of two exponential service process. The mean of the sum of 'n' independent Exponential distribution is the sum of individual means. That is, if $X_1 + X_2 + \cdots + X_n$, then $E(Z) = \sum_{t=1}^{n} \frac{1}{\mu_i}$. Let K= $\mu_1 + \mu_2$

$$\text{Mean service rate } (\mu) = \sum_{i=1}^{2} \frac{1}{\mu_i} \quad (13)$$

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$= \frac{1}{K} \quad = \frac{1}{2.2}$

$= 0.4$

The results for the period of study were calculated and the total time (in minutes) taken to serve the customers; this was computed for each service process per day and the results for the period of study are shown in Appendix VI. Thus, the arrival rates and service rates for the system were compared in Fig. 1 below

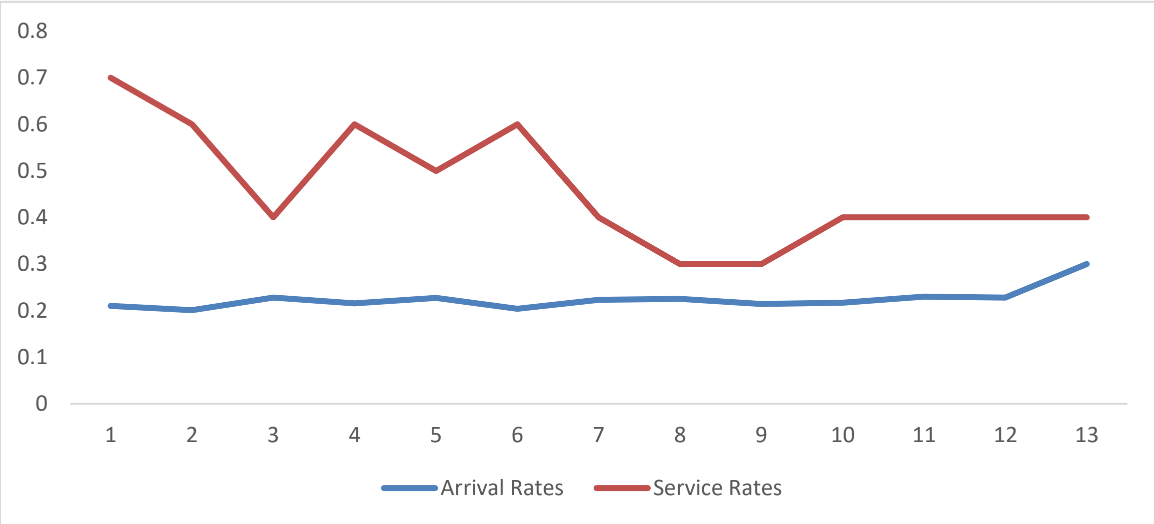


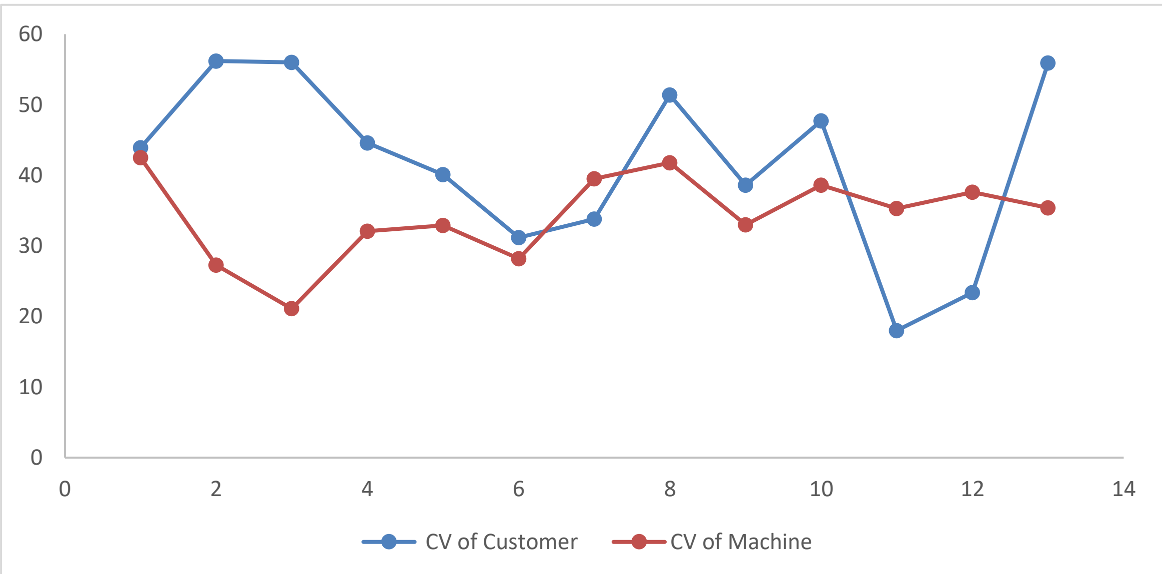
Fig. 1: Comparing the arrival rates and service rates

From the Figure 1, comparing the arrival rate of customers and their service rates across the weeks, it can be seen that service rate is higher in week one, two and three and keeps decreasing across the remaining weeks, while the arrival rates continue with rise and fall across the weeks with a steady movement. Thus, generally, queue was present on some weeks while not seen in some the weeks.

Computation of the Coefficient of Variation

The Coefficient of variation for the self-service process is computed by finding the quotient between standard deviation values of the service process and the mean of the service process. For example, the CV of the customer service process is CV = $\frac{STD}{Mean}$ *100 which is similarly done for the machine service process. The result for the period of the study were calculated as shown in Tables 4.9 and 4.10.

Fig. 2: Coefficient of variation for the two-service process across the month



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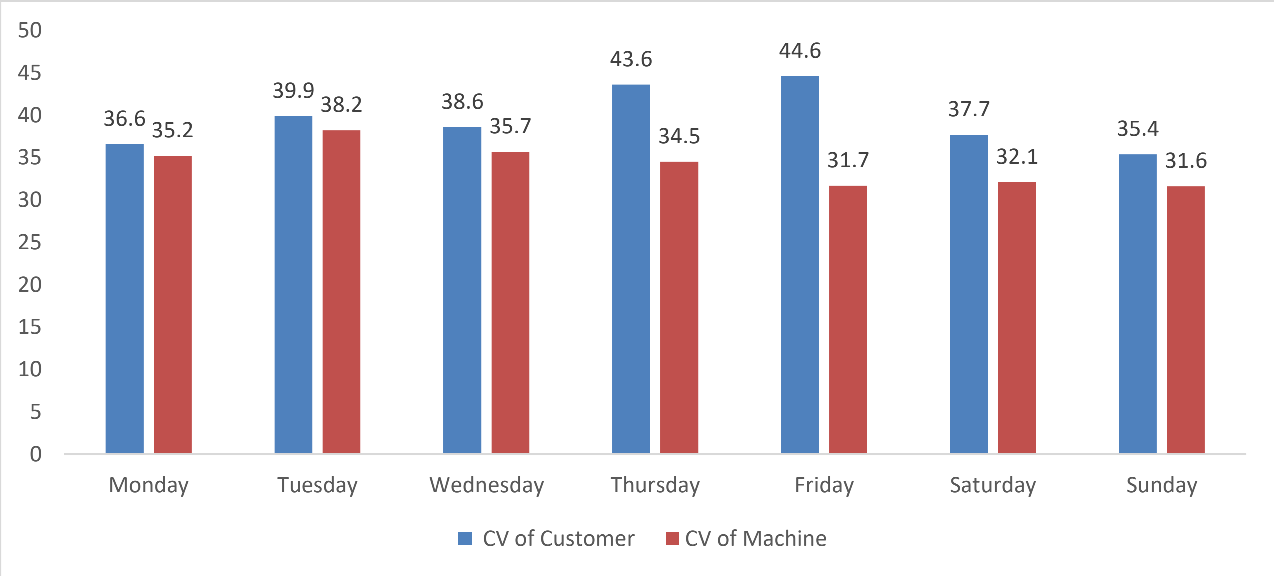
From Figure 2 above, showing the variation in the service process, it indicates that the average variation caused due to the customer is 40.15% and the average variation cause due to insufficiencies of the machine is 34.2%, and when these two service processes are pooled together, the average variation is 64.1%. This simply means that of the total variation in the system, the variation in the service time are majorly caused by the customers themselves. While on some few occasion, the variation is caused by the machine. Considering the result on weekly basis for the variation in the customer service process we can see, that, week 13 records the highest variation of 58.7% followed by week two, three and so on, and week eleven records the lowest variation of 18% whereas for the machine, week one had the highest variation of 42.5% followed by week seven, ten and week three had lowest of 21.1%. When 50% is taken as the cut–off point for the variation in the system, only week 2, 3, 8 and 13 had its variation unstable, i.e. above 50%, while the remaining weeks are stable below 50% cut-off point. whereas for the machine the whole weeks are considered stable under the 50% cut-off point, as is presented in Fig. 3 below.

Fig. 3: The Coefficient of variation of the service processes across the days of the week



From the Figure 3 above showing variation in the service process across the day of the week, we can see that the customer coefficient of variation is higher in all the days than the machine variation and again we see that all the days' variation is considered stable under the 50% cut-off point.

Discussion

The findings of this study highlights key trends and relationships within the investigated variables. The analysis reveals that the ATM-customer service process at Zenith Bank Plc, Ozoro, is best described by the convolution of two exponential distributions – one for the customer's self-service time and the other for the machine's dispensing time. The resulting distribution is valid and possesses a well-defined cumulative density function; however, it notably lacks the memory-less property. This deviation suggests that past service durations impact subsequent waiting times, potentially contributing to cumulative delays during peak periods.

The Kulmogorov-Smirnov test confirmed that the individual service times for both the customer and the machine adhere to an exponential distribution, supporting the theoretical assumptions of queuing models. Specifically, customer service times averaged 1.14 minutes (with a standard deviation of 0.43 minutes), while machine service times averaged 0.24 minutes (with a standard deviation of 0.14 minutes). The total turnaround time averaged 2.26 minutes, indicating that the convolution of the two independent processes effectively captures the overall service experience at the ATM.

Analysis of the arrival data – 3185 customers over 248 hours, shows an average arrival rate of approximately 13 customers per hour (or 0.22 customers per minute). The study further highlights significant daily and weekly variations; Saturdays, Fridays, and Tuesdays consistently recorded higher arrival rates compared to the other days. This fluctuation points to the potential for congestion during specific periods, suggesting that operational strategies could be optimized by reallocating resources or adjusting service processes during peak times.

The coefficient of variation (CV) analysis provides additional insights into the system's variability. The CV for customer service times was found to be approximately 40.15%, while that for machine service times was 34.2%. When combined, the overall variation reached 64.1%, indicating that most of the service time variability is driven by the human element – namely, the customer's interaction with the ATM. This finding aligns with Adeniran (2018) and Joseph (2016) that have underscored the significant impact of customer behaviour on service efficiency.

Weekly trend analysis further illustrates that while some weeks experienced stable service processes, others – specifically weeks 2, 3, 8, and 13 exhibited instability likely due to higher customer-related delays. Such variability suggests that addressing customer familiarity with ATM operations could mitigate some of the observed delays, thereby enhancing the overall efficiency of the system.

Conclusion

Based on the findings of the study, it was concluded that the distribution for the ATM-customer relationship exist. It is obtained by convolution of the two independent service processes (the customer and the machine time). It is confirmed that the distribution is valid, has a cumulative density function but does not possess the memory- less property of service distribution. It is observed during the period of the study that queue is prevalent at some point in time but shows more customers come on Saturday and Friday, respectively. We also conclude that the inefficiencies or delay in the system (ATM) are usually due to the customer that is partly because of the customer inability to operate the machine appropriately.

Recommendations

The recommendations contained in this report may be useful for the bank management, ATM industry association (ATMIA) and other stake holders in facilitating the efficient service delivery and enhancing the customer satisfaction and ensuring that hindrances to the customer satisfaction and profit maximization of the bank are adequately addressed.

i. The bank should provide adequate, well placed piece of instruction or guide on how to operate the machine, which should be placed conspicuously around the ATM point, to cater for customers who doesn't know how to operate the machine as is revealed in the study that there is high variation in the customer service process and that the customer service time have a significant impacts on the turnaround time.
ii. ATMIA should encourage the purchase of ATMs which user friendly or which have short guide around it.
iii. ATM card manufacturers or the Bank should provide a comprehensive or detailed manual that would educate customers on how to use the card in the machine.
iv. The bank should encourage their customers to use the bank counter in the withdrawal of a huge amount of cash as was revealed in the study that multiple withdrawals by customers impacts or increases the waiting time of customers in the queue.

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