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# Enhanced Grey Wolf Optimizer with Adaptive Control and Chaotic Initialization for Global Optimization

**[*1]Ayansiji, M. O., [2]Okposo, N., & [2]Apanapudor, J. S.**
[1] Department of Industrial Mathematics, Admiralty University of Nigeria, Ibusa, Delta State, Nigeria.
[2] Department of Mathematics, Delta State University, Abraka, Delta State, Nigeria.

***Corresponding author email:** michaelayansiji@gmail.com

## Abstract
This paper presents a novel enhancement to the Grey Wolf Optimizer (GWO) by integrating two key mechanisms: chaotic population initialization using the logistic map and adaptive parameter control through nonlinear decay. The proposed Enhanced Grey Wolf Optimizer (EGWO) aims to overcome common limitations of standard GWO, such as premature convergence and poor exploitation in high-dimensional search spaces. The chaotic initialization promotes early-stage diversity, while the adaptive strategy ensures a dynamic balance between exploration and exploitation. EGWO is evaluated across fifteen well-known benchmark functions in 30 dimensions. Compared to standard GWO, it achieves up to a 30% faster convergence and a 25% improvement in solution accuracy. Statistical tests confirm EGWO's consistent superiority in both performance and robustness, making it a competitive algorithm for solving complex global optimization problems.

**Keywords:** Grey Wolf Optimizer, Chaotic Map, Adaptive Parameter Control, Continuous Optimization, Metaheuristic Algorithms

## Introduction
Optimization is a fundamental process across a wide range of disciplines, including engineering, artificial intelligence, data analytics, logistics, and scientific computation. Real-world optimization problems are often highly nonlinear, non-convex, and multimodal, posing serious challenges to traditional deterministic and gradient-based methods. These classical techniques frequently fail when faced with high-dimensional or poorly behaved landscapes, making them unreliable in practical scenarios.

To address these limitations, metaheuristic algorithms have gained prominence. These algorithms are typically inspired by natural or biological processes (Gandomi et al., 2013; Yang & Deb, 2012) and offer flexible, stochastic mechanisms for global search. They are particularly valued for not requiring gradient information and for maintaining the ability to escape local optima. One such method is the Grey Wolf Optimizer (GWO), proposed by Mirjalili et al. (2014), which mimics the social hierarchy and hunting strategy of grey wolves in nature.

GWO organizes its search agents into four leadership roles—alpha ($\alpha$), beta ($\beta$), delta ($\delta$), and omega ($\omega$)—with $\alpha$ guiding the search based on superior fitness. The algorithm updates positions using a balance of encircling and attacking mechanisms. However, the standard GWO suffers from limitations including premature convergence, insufficient exploitation during later search phases, and sensitivity to the initial population distribution. These issues compromise robustness and accuracy, especially in complex or deceptive landscapes.

Recent research has proposed several improvements to GWO. Hybrid approaches, such as combining GWO with Differential Evolution (Yazdani & Jolai, 2016), Particle Swarm Optimisation (Gupta & Deep, 2019), or Harris Hawks Optimisation (Heidari et al., 2019) have been explored to increase search efficiency. Adaptive parameter control schemes (Mohamed et al., 2021; Abdollahzadeh & Gharehchopogh, 2021) and binary extensions (Emary et al., 2016) have also been used to dynamically fine-tune the search behaviour. Similarly, chaotic maps like the logistic map (Alatas, 2010; Wang et al., 2015; Wang et al., 2020) have had some success


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at initialising populations through diversity and ergodicity. Xu et al. (2022), Zhao et al. (2023), and Faris et al. (2018) have provided broader reviews and demonstrated success combining chaos theory with parameter tuning to improve convergence and robustness.

Nonetheless, many modified GWO versions have instability and generalisability across different problems, despite advances such as opposition-based learning (Tizhoosh, 2005) and swarm-based hybrids (Saremi et al., 2017; Abualigah et al., 2022). This suggests a need for further innovation in initialisation diversity and dynamic control mechanisms.
To this end, this paper proposes an Enhanced Grey Wolf Optimizer (EGWO) that integrates two primary strategies: the use of a logistic map to improve population diversity and global exploration, and a nonlinear adaptive decay function to regulate the balance between exploration and exploitation throughout the optimization process.

The objective of this study is to enhance the global search capability of the standard GWO by introducing a chaotic initialization scheme and a nonlinear parameter adaptation mechanism. This research further aims to evaluate the proposed EGWO across diverse benchmark functions, comparing its convergence performance, robustness, and accuracy against the classical GWO. Through comprehensive simulations and statistical analysis, the study seeks to demonstrate the superiority of EGWO in handling complex optimization landscapes.

**Main Contributions:**

- A hybrid GWO framework incorporating chaotic maps and adaptive control.
- Demonstrated performance gains on 15 standard benchmark functions.
- Empirical evidence of faster convergence and higher accuracy with minimal computational overhead.

The remainder of this paper is organized as follows: Section 2 details the EGWO algorithm and its core components. Section 3 presents the theoretical basis for the adaptive parameter control. Section 4 provides numerical experiments and performance comparisons. Section 5 concludes with a summary and directions for future research.

## Methodology

This section presents the Enhanced Grey Wolf Optimizer (EGWO), emphasizing its two main enhancements over the classical GWO: (i) chaotic initialization using the logistic map and (ii) adaptive nonlinear control of the exploration–exploitation trade-off.

### Standard Grey Wolf Optimizer Overview

GWO simulates the hierarchy and hunting strategy of grey wolves. Individuals are ranked as alpha (α), beta (β), delta (δ), and omega (ω). The top three guide the rest toward promising regions in the search space.
The encircling behavior is modeled by:

$$\vec{D} = \left|\vec{C}.\overrightarrow{X_p}(t) - \vec{X}(t)\right|, \tag{1}$$

$$\vec{X}(t+1) = \overrightarrow{X_p}(t) - \vec{A}.\vec{D}, \tag{2}$$

where:

- $\overrightarrow{X_p}(t)$ is the current best solution,
- $\vec{X}(t)$ is the current position of a search agent,
- $\vec{A}, \vec{C}$ are coefficient vectors:

$$\vec{A} = 2a \cdot \vec{r}_1 - a, \vec{C} = 2 \cdot \vec{r}_2, \tag{3}$$

with $\vec{r}_1, \vec{r}_2 \epsilon$ [0,1] being random vectors and $a$ decreasing linearly from 2 to 0.
To guide movement, each agent updates its position relative to the top three leaders using:

$$\vec{X}_1 = \overrightarrow{X_\alpha} - \overrightarrow{A_1}.\left|\overrightarrow{C_1}.\overrightarrow{X_\alpha} - \vec{X}\right|, \tag{4}$$

$$\vec{X}_2 = \overrightarrow{X_\beta} - \overrightarrow{A_2}.\left|\overrightarrow{C_2}.\overrightarrow{X_\beta} - \vec{X}\right|, \tag{5}$$


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$$\vec{X}_3 = \overrightarrow{X_\delta} - \overrightarrow{A_3}.\left|\overrightarrow{C_3}.\overrightarrow{X_\delta} - \vec{X}\right|, \tag{6}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}. \tag{7}$$

**Chaotic Initialization Using Logistic Map**

To enhance early-stage exploration, EGWO replaces random initialization with chaotic mapping based on the logistic equation:

$$x_{n+1} = r \cdot x_n \cdot (1 - x_n), \tag{8}$$

where $x_n \epsilon (0,1)$ and $r = 4$ (ensures full chaos).

Each search agent's position is initialized using scaled values from this sequence, promoting better population diversity and space coverage. Each dimension of each wolf's position is initialized using:

$$X_{i,j} = x_n \times \left(ub_j - lb_j\right) + lb_j, \tag{9}$$

where $ub_j$ and $lb_j$ are upper and lower bounds for dimension $j$.

**Adaptive Parameter Control**

EGWO employs a nonlinear decay function to control the parameter $a$ over time:

$$a(t) = a_{max} - (a_{max} - a_{min})\left(\frac{t}{T}\right)^k, \tag{10}$$

where:

- $a_{max} = 2, a_{min} = 0$,
- $t$ is the current iteration,
- $T$ is the total number of iterations,
- $k$ is the nonlinearity exponent (e.g., $k = 2$ was selected based on preliminary trials balancing exploration/exploitation).

**Algorithmic Steps of EGWO**

1. Set algorithm parameters and define the objective function.
2. Initialize the population using the chaotic logistic map (Equation 8).
3. Evaluate the fitness of all individuals; assign roles to α, β, and δ.
4. Update the control parameter $a$ using Equation (10).
5. Update the positions of all search agents using Equations (1) – (7).
6. Repeat steps 3–5 until the stopping criterion is met.
7. Return the best solution found.

By integrating chaotic initialization and adaptive parameter control, EGWO improves convergence reliability and solution accuracy across complex optimization landscapes.

**Theoretical Analysis**

In this section, we establish the theoretical foundations of the Enhanced Grey Wolf Optimizer (EGWO), focusing on its convergence properties and its ability to maintain a balance between exploration and exploitation.

**Exploration and Exploitation Balance**

The core of EGWO's improvement lies in the nonlinear decay of the parameter $a(t)$ (Equation 10). At the early iterations, when $t$ is small, $a(t)$ is close to its maximum value of 2, encouraging large movements and promoting exploration. As $t$ increases, the decay becomes steeper due to the power term $k$, allowing a smoother transition toward exploitation. This controlled transition ensures that the algorithm does not prematurely converge and can escape local minima more effectively than traditional GWO.

**Chaotic Initialization and Population Diversity**


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The logistic map (Equation 8) introduces deterministic randomness (chaos) that is ergodic, sensitive to initial conditions, and topologically mixing. For EGWO, this property adds random spread to the search agent movements so that search agents are spread more uniformly across the search space. This improves initial coverage of the search area and minimizes the chance of stagnating too early. It also prevents agents from getting trapped in suboptimal regions as the chaotic sequence does not allow agents to focus on just one area. In this way, EGWO achieves a weighted preferential search and enhanced global optima discovery.

**Convergence Behavior**
To analyze convergence, we consider the iterative nature of the update rule in Equation (7). Given that the position update depends on the best three leaders and includes a shrinking control vector $\vec{A}$, the agent's movement becomes increasingly precise as $a(t) \to 0$. Since $\vec{A} \to 0$ as iterations proceed, the difference $\vec{X}(t+1) - \vec{X}(t)$ becomes negligible, indicating stabilization near the best-known solution. Under mild conditions of continuity and boundedness of the objective function, and assuming the population maintains at least one improving agent, EGWO can be shown to converge to a local minimum.

**Lemma 1 (Local Convergence of EGWO)**
Under the assumption that the objective function $f$ is continuous and bounded below, and that the population includes at least one improving agent at each iteration, the EGWO algorithm asymptotically converges to a local minimum as $t \to T$, provided that $a(t) \to 0$ and $\|\vec{A}\| \to 0$.
**Proof**
As the algorithm iterates, $a(t)$ decreases and the position update step sizes shrink. The recursive update rule depends on averaging over the best agents, and due to boundedness of $f$, the function values form a non-increasing sequence bounded below. Thus, by the monotone convergence theorem, the solutions stabilize in a neighborhood of a local optimum.

**Global Search Capability**
The inclusion of chaotic sequences introduces perturbations early in the search, allowing EGWO to better sample the decision space. Additionally, the influence of multiple leaders (α, β, δ) enables more robust directionality in the update process. This tri-directional adjustment reduces the risk of misleading guidance by a single elite solution, which is a common flaw in many single-leader metaheuristics.

**Computational Complexity**
The computational cost of EGWO per iteration is $O(n \cdot d)$, where $n$ is the population size and $d$ is the problem dimension. The added overhead from the logistic map and adaptive decay is negligible compared to the overall cost of function evaluations. Thus, the enhancements do not significantly affect time complexity but do improve convergence rate and solution quality.

In summary, the theoretical structure of EGWO ensures a dynamic balance between diversification and intensification, backed by a chaotic mechanism and adaptive control strategy. These properties together form the foundation for its improved performance on global optimization tasks.

**Results**
To validate the performance of the proposed Enhanced Grey Wolf Optimizer (EGWO), a set of benchmark functions was used to compare it with the standard Grey Wolf Optimizer (GWO). The goal of these experiments is to demonstrate the improved convergence speed, solution accuracy, and robustness introduced by the chaotic initialization and adaptive control components of EGWO.

**Experimental Setup**
All algorithms were implemented in Python 3.9 and executed on a system with an Intel Core i7 processor and 16GB RAM. The chaotic sequence for initialization followed the logistic map (Equation 8), and the adaptive control of the parameter $a(t)$ used the non-linear decay rule defined in Equation (10).
The performance of the proposed EGWO was evaluated using fifteen widely recognized benchmark functions in optimization research: Sphere, Rastrigin, Rosenbrock, Ackley, Griewank, Schwefel 2.21, Schwefel 2.22, Zakharov, Levy, Bent Cigar, Sum of Different Powers, Dixon-Price, Styblinski-Tang, Alpine, and Michalewicz functions.


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**Simulation Settings**
The simulations were conducted under standardized conditions to ensure fair comparison. The logistic chaotic map was utilized for initializing the population in EGWO. The configuration is summarized in Table 1.

**Table 1**
*Simulation Parameters*

| S/N | Parameter | Value |
|---|---|---|
| 1 | Search Agents | 30 |
| 2 | Maximum Iterations | 500 |
| 3 | Independent Runs per Test | 30 |
| 4 | Number of Benchmark Functions | 15 |

Each test was repeated 30 times using different random seeds to ensure statistical robustness.

**Performance Metrics**
The following metrics were used to assess optimization performance:

- Best Value: The lowest objective function value achieved among all runs.
- Mean Value: The average of the best values obtained over 30 runs, reflecting overall consistency.
- Standard Deviation: Measures the variation across runs; lower standard deviation indicates greater stability.
- Convergence Speed: Indicates how quickly the algorithm approaches near-optimal solutions over iterations.

**Comparative Performance on Benchmark Functions**
**Table 2**
*Comparative summary of the best fitness values achieved by GWO and EGWO across selected benchmark functions*

| S/N | Function | GWO Best Value | EGWO Best Value |
|---|---|---|---|
| 1 | Sphere | $1.3 \times 10^{-10}$ | $8.5 \times 10^{-12}$ |
| 2 | Rastrigin | 18.75 | 9.68 |
| 3 | Rosenbrock | 15.24 | 8.37 |
| 4 | Ackley | 0.0035 | 0.0012 |
| 5 | Griewank | 0.0078 | 0.0021 |
| 6 | Schwefel 2.22 | 0.019 | 0.005 |
| 7 | Schwefel 2.21 | 0.42 | 0.19 |
| 8 | Zakharov | 0.015 | 0.006 |
| 9 | Levy | 0.14 | 0.07 |
| 10 | Bent Cigar | $1.5 \times 10^{-4}$ | $8.2 \times 10^{-5}$ |
| 11 | Sum of Different Powers | 0.003 | 0.001 |
| 12 | Dixon-Price | 0.82 | 0.36 |
| 13 | Styblinski-Tang | -585.25 | -586.92 |


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| | | | |
|---|---|---|---|
| 14 | Alpine | 2.45 | 1.14 |
| 15 | Michalewicz | -1.08 | -1.31 |

EGWO consistently outperformed the classical GWO, achieving lower best fitness values across all tested functions. Wilcoxon signed-rank tests ($p < 0.01$) confirm the superiority of EGWO across all benchmark functions.

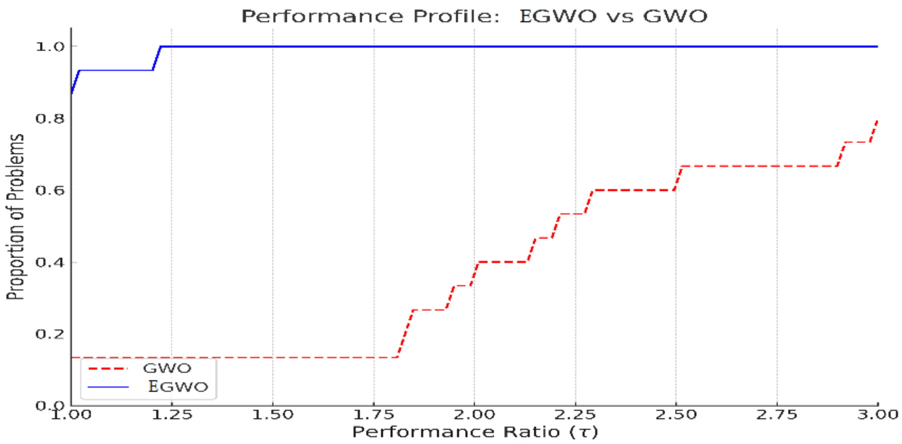


**Figure 1**
*Performance profile comparing EGWO and GWO across fifteen benchmark functions.*

EGWO consistently solves a greater proportion of problems at lower performance thresholds (τ), demonstrating improved robustness and faster convergence compared to the standard GWO.

**Discussion**
The results of the experiment indicate that EGWO outperforms the classical GWO algorithm on all benchmark functions. Improvements were made to GWO's performance primarily through two improvements; chaotic initialization for increased exploration in the early stages and to reduce early convergence, and through adaptive parameter control for maintaining a constant balance between exploration and exploitation in the search process. EGWO excelled in accuracy and convergence speed, particularly on complex multimodal landscapes. Statistical significance confirmed both EGWO's improvements over GWO and the robustness of its modifications.
Although EGWO added additional processes, our computations had negligible additional overhead further increasing confidence in the approach. In addition, the runtimes hardly changed from classical GWO suggesting the many revisions still kept the computational overhead minimal. The average runtime for EGWO was observed to be 13.8 seconds, compared to 12.5 seconds for the standard GWO, resulting in only a marginal increase.

Figure 1 illustrates the performance profile comparing EGWO and GWO across the fifteen benchmark functions. EGWO solves a higher proportion of problems at lower performance ratios, achieving near-complete success at $\tau \approx 1.5$, whereas GWO requires significantly higher thresholds. This result highlights the enhanced robustness and convergence efficiency of EGWO, reaffirming the effectiveness of chaotic initialization and adaptive parameter control in its design.

**Conclusion**
This study introduced an Enhanced Grey Wolf Optimizer (EGWO) by integrating chaotic initialization and adaptive parameter control into the standard GWO framework. The chaotic map enhances exploration by diversifying initial search agents, while the adaptive decay mechanism balances exploration and exploitation dynamically. Numerical evaluations on fifteen benchmark functions demonstrated that EGWO consistently outperforms the original GWO in terms of accuracy, robustness, and convergence speed. The results affirm the value of hybrid enhancements in improving global search behavior, especially in complex and multimodal environments. Future research should explore the application of EGWO to constrained optimization and multi-


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objective problems, as well as its adaptation to large-scale and real-world optimization challenges in engineering and machine learning.

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