



## Mathematical Modelling of Prey-Predator Poaching Dynamics Using a Lomax Function for Prey Species Growth

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### Abstract

In this paper, a Mathematical model of Prey and Predator involving Lomax distribution function is proposed and analyzed. The Lomax distribution function which is composed of exponential and gamma functions was applied on the prey growth in the absence of predator while the predator growth is logistically. The existence and the stability analysis of all possible equilibrium points are studied using the Jacobean matrix. Then, the optimal exertion is obtained by the minimization of both the prey species and predator species dynamics with respect to the exertions during the poaching process

**Keywords:** Lomax Distribution Function, Optimal Exertion, Refuge, Environment.

### Introduction

In Biological phenomenon, Mathematics have created tremendous impacts in major themes in ecological studies in the branch of Biomathematics. The hub of this area is the study of population dynamics which has been made more real through Mathematical models that are designed using nonlinear systems through the applications of ordinary and partial differential equations (Ejakpovi & Siloko, 2023; Yousef et al., 2018). The models range from single species, called the continuum to a varieties of species that live and interact in the same ecosystem or environment. This habitat of living organism populations is modelled with Mathematical models adopting differential equations to study their competitive relationship such as mutualism, symbiosis, parasitism and host of others in a prey-predator relationship. The interaction between species in predator-prey modeling is greatly affected by environmental factors such as the availability of nourishment, water, refuge, air (oxygen), sunlight amongst others (Shlyufman et al., 2018). The prey-predator Mathematical models in which predator and prey depend on the same source of nourishment, the predators are being affected by the climatic changes in the habitat (Charles et al., 2022; Aman & Dinku, 2021).

Thus, in population ecology, how prey and predator species are dispersed within the carrying capacity (environment) determines whether a particular species will flourish or go nonexistent. This is where the time delay has a considerable impact on the dynamics of prey-predator systems for in time delay Mathematical models, the time lags occurs as a result of a variety of Biological and environmental conditions, such as the time it takes the predator to seek and captures a prey after encountering it (Bezabih et al., 2021). The delays influenced the dynamics by considering the necessary reaction time, gestation period and feeding time of the prey-predator systems (Adama, 2018; Hoff & Fay, 2016; Arif et al., 2023).

The dynamics of the relationship between two species came into existence when Lotka model paired first order nonlinear differential equations showing the effects of a parasite on its prey and VioVolterra equated the relationship

between the two species independent to Lotka that led to the formulation of the Lotka-Volterra predator-prey model (Joshi et al., 2021). In the prey-predator dynamics, the effects of the predator's consumption rate are directly proportional to the prey density up to the point of the satiety and if the prey density rises, the predator consumption rate stays constant.

The population of the two kinds of animals is characterized by the declining and increasing trends of the prey-predator system (Abbas et al., 2024). The functional response that designates how the predator's feeding rate changes with regard to the prey density is a crucial component in the population dynamics of all nonlinear Mathematical models. This created the most critically considered in a realistic and plausible Mathematical model to include: carrying capacity which is the maximum number of prey that the ecosystem can sustain in absence of predator, interaction among prey and functional responses of the species.

### Aim and Objectives of the Study

This study explores the prey-predator model with a stochastic distribution function on species interactions which integrates environmental components and efforts of both species into the prey-predator framework. The main aim of this research is the application of the Lomax function which is a mixture of exponential and gamma function for prey species in the modelling of prey-predator dynamics. The specific objectives are to determine the growth rate of the prey population in the absence of predators, investigate the safety period of the prey species and determine the effort sustainability of the prey species during the poaching period.

### Materials and Methods

#### An Overview of Holling Type Functional Response Model.

Many Mathematical models incorporating diverse areas of interest such as Holling Type functional responses, ratio-dependent functional responses, bio-economic exploitation or harvesting, delayed harvesting and age-structured models (Ashine & Gebru, 2017). The Holling Type functional responses exists as Type I and II, with the Holling Type I functional response is prevalent and well-known type of functional response among the several functional response types. It brings two preys and one predator into the conflict and the Mathematical model with Holling type I functional response describes the connection between a predator's prey density and consumption rate. When the predator's consumption rate is directly proportional to the prey density up to a certain saturation point, it is one of the most fundamental functional response models. After this, even if the prey density rises further, the predator's consumption rate stays constant (Sharmila & Gunasundari, 2023). This functional response model incorporates the logistic growth function in the model which give the relevance of incorporating logistic growth in prey-predator models. In this response model, the rate of prey consumption by a predator rises as prey density increases, but eventually levels off at a plateau (or asymptote) at which the rate of consumption remains constant regardless of increases in prey density (Simon & Rabago, 2018; Kolpak et al., 2016).

The Mathematical analysis of Holling Type II functional response model include in each predator increased its consumption rate when exposed to a higher prey density and predator density increased with increasing prey density, which built the fulcrum of predator population to prey density as the functional response and numerical response (Adama, 2018; Ejaz et al., 2022). In this model the interaction of species affects both populations, it has two non-linear autonomous ordinary differential equations describing how the population densities of the two species would vary with time. It is assumed that a constant proportion  $m \in [0,1]$  of the prey can take refuge to avoid predation, this leaves  $(1 - m)X$  of the prey available for predation. Thus, the variables and parameters in the model are:  $X(t)$ -population of the prey at time,  $t$ ;  $Y(t)$ -population of the predator at time,  $t$ ;  $r$ - is the growth rate for the prey,  $s$ - is the maximum growth rate of the predator;  $k$ - is the carrying capacity of the environment,  $k_1$ -measures the level of security the prey derives from the environment,  $k_2$ - measures the level of security the predator derives from the environment,  $c_1$ -is the maximum value of the reduction rate of the prey and  $c_2$ -is crowding effect for the predator. Thus, the Mathematical model for prey-predator dynamics given by (Savitri, 2019; Ashine & Gebru, 2017) is:

$$\begin{cases} \frac{dX}{dt} = r \left(1 - \frac{X}{K}\right)X - \frac{C_1(1-m)XY}{k_1 + (1-m)X} \\ \frac{dY}{dt} = s - \frac{C_2Y}{k_2 + (1-m)X} \end{cases} \quad (1)$$

where all the parameters in the model assumes positive values and with initial value  $X(0) \geq 0$  and  $Y(0) \geq 0$ . Then, applying the non-dimensional state variables and parameters, the systems in Equation (1) takes the non-dimensional form as:

$$\begin{cases} \frac{dx}{dt} = (1-x)x - \left( \frac{\alpha(1-m)xy}{\beta + (1-m)m} \right) \cong F(x, y) \\ \frac{dy}{dt} = y \left( \gamma - \frac{\sigma y}{\omega + (1-m)x} \right) \cong G(x, y) \end{cases} \quad (2)$$

The model system in Equation (2) is solved in the region  $\{(x, y, z) \in R^3, (x(0) = x_0 \geq 0; y(0) = y_0 \geq 0)\}$ .

### Description of the Mathematical Model

The Mathematical framework of the model consists of the model description, formulation and analysis. Consider one prey and one predator system dynamics and let  $Q(t)$  and  $R(t)$  denote the population sizes of the prey and predator at time,  $t$  respectively. The main feature of the model is that the interaction of species affects both populations and the terms exponential gamma growth of the prey species either in the absence of the predator or otherwise. The study adopts the Holling Type II functional response to the consumption of the prey by the predator. The development of the model is based on the following assumptions:

- The prey species and predators live in stable ecosystem
- The predator is dependent on the prey as the source of survival with a factor  $c > 0$ . However, the population density of the predator grows logistically.
- The prey species have an unconstrained food supply.
- There is exponential-gamma growth rate for the prey species in absence of the predator or human poaching of the prey.
- There is no threat to the prey species besides the predator.

Thus, from the above assumptions, variables and parameters used in the model include:

- $Q(t)$  denotes the population sizes of the prey species at time,  $t$ .
- $R(t)$  denotes the population sizes of the predators species at time,  $t$ .
- $r$  denotes the growth rate of the prey species.
- $r_1$  denotes the growth rate of the predator species.
- $c$  denotes the effects of interactions between the population of the prey and predator.
- $C$  denotes the mortality rate of the prey species to the predator species.
- $n$  denotes the mortality rate of the predator species.

### Formulation of the Model

The Lomax distribution function model formulation is based on the above assumptions that modified the logistic growth function of the prey species with the Lomax distribution that consists of the exponential and gamma function. Therefore, the assumed growth for both the prey and predator species dynamics is given as:

$$\begin{cases} \frac{dQ}{dt} = rQ(Q(T \leq t)) - CQR \\ \frac{dR}{dt} = r_1R(Q(T \leq t)) + r_1QR \end{cases} \quad (3)$$

Then, incorporating the Lomax distribution function model into the Equation (3) with  $e_1$  and  $e_2$  representing the exertion the prey can take to go into refuge to avoid predation and the exertion the predator takes to poach the prey.

$$\begin{cases} \frac{dQ}{dt} = rQ \left( \int_0^\infty (1 - e^{-\lambda t}) \frac{Q^r \lambda^{r-1} e^{-Q\lambda}}{\Gamma r} d\lambda \right) - CQR - m_1 Q e_1 \\ \frac{dR}{dt} = r_1 R \left( 1 - \left( \frac{R}{R+t} \right)^{r_1} \right) + r_1 QR - nR - m_2 R e_2 \end{cases} \quad (4)$$

where all the parameters in the model assumes positive values and with initial value  $Q(0) \geq 0$  and  $Y(0) \geq 0$ . Then, considering the first component of Equation (4), where  $Q$  is characterized by the exponential distribution and  $\lambda$  are distributed across the prey population according to a gamma distribution, we have:

$$\frac{dQ}{dt} = rQ \left( \int_0^\infty \left( \frac{Q^r \lambda^{r-1} e^{-Q\lambda}}{\Gamma r} \right) d\lambda - \int_0^\infty \left( \frac{Q^r \lambda^{r-1} e^{-\lambda(Q+t)}}{\Gamma r} \right) d\lambda \right) - CQR - m_1 Q e_1 \quad (5)$$

$$\frac{dQ}{dt} = rQ \left( 1 - \frac{Q^r}{\Gamma r (Q+t)^r} \int_0^\infty \left( \frac{(Q+t)^r \lambda^{r-1} e^{-\lambda(Q+t)}}{\Gamma r} \right) d\lambda \right) - CQR - m_1 Q e_1 \quad (6)$$

where  $\Gamma r = \int_0^\infty e^{-\lambda} \lambda^{r-1} d\lambda$ ,  $R(r) > 0$  is called the Euler integral of the second kind and does not converge if  $r \leq 0$ . Then, solving the differential equation in Equation (6) gives the dynamical system of the prey species as:

$$\begin{cases} \frac{dQ}{dt} = rQ \left( 1 - \left( \frac{Q}{Q+t} \right)^r \right) - CQR - m_1 Q e_1 \\ \frac{dQ}{dt} = rQ - rQ \left( \frac{Q}{Q+t} \right)^r - CQR - m_1 Q e_1 \end{cases} \quad (7)$$

Then, substitute Equation (7) into Equation (4), gives the dynamical systems of both the prey and predator species as:

$$\begin{cases} \frac{dQ}{dt} = rQ - rQ \left( \frac{Q}{Q+t} \right)^r - CQR - m_1 Q e_1 \\ \frac{dR}{dt} = r_1 R - r_1 R \left( \frac{R}{R+t} \right)^{r_1} + r_1 QR - nR - m_2 R e_2 \end{cases} \quad (8)$$

Thus, applying the non-dimensional state variables and parameters of  $x_1 = \frac{Q}{Q+t}$  and  $x_2 = \frac{R}{R+t}$  into Equations (8) gives the non-dimensional form as:

$$\begin{cases} \frac{dx_1}{dt} = x_1 \{r(1 - x_1^r)\} - CR - m_1 e_1 \cong F(x_1, x_2) \\ \frac{dx_2}{dt} = x_2 \{r_1(1 - x_2^{r_1})\} + bQ - n - m_2 e_2 \cong G(x_1, x_2) \end{cases} \quad (9)$$

with initial values  $x_1(t) = x_0 \geq 0$ ;  $x_2(t) = y_0 \geq 0$  and from the non-dimensional state variable parameters  $Q = \frac{x_1 t}{(1-x_1)}$  and  $R = \frac{x_2 t}{(1-x_2)}$  respectively. The behaviour of the linearized system in Equation (8) can now be evaluated by determining the eigenvalues of the Jacobin matrix given by:

$$J(e_i) = \begin{bmatrix} \frac{\partial F(x_1, x_2)}{\partial x_1} & \frac{\partial F(x_1, x_2)}{\partial x_2} \\ \frac{\partial G(x_1, x_2)}{\partial x_1} & \frac{\partial G(x_1, x_2)}{\partial x_2} \end{bmatrix} \quad (10)$$

The Jacobean matrix presents the linearized system of the model and the asymptotic stability of each equilibrium point is determined by the Jacobean matrix to investigate whether the eigenvalues have negative or positive real parts. The eigenvalues computed at each of the equilibrium point through the Jacobean matrix using Equation (9) and substituting their values functions into the Jacobean matrix in Equation (10) and simplifying yields:

$$J(e_i) = \begin{bmatrix} m_1 e_1 \left( 1 - \frac{m_1 e_1}{r} \right)^{\frac{1}{r}} & 0 \\ 0 & m_2 e_2 \left( 1 - \left( \frac{n + m_2 e_2}{r_1} \right) \right)^{\frac{1}{r_1}} \end{bmatrix} \quad (11)$$

### Dynamical System Equilibrium Points

The equilibrium points in the analysis of prey-predator model involving differential equations is very crucial to study the solutions that do not change with time,  $t$ . These solutions are referred to as the equilibrium points which is obtained through the equilibrium steady-state Equations when the non-dimensional form of the model system in Equation (9) is equated to zero. This implies that by setting  $\frac{dx_1}{dt} = \frac{dx_2}{dt} = 0$ , for both the prey species and predator species dynamics gives:

$$\begin{cases} rx_1 - rx_1^{r+1} - \frac{Ct x_1 x_2}{1-x_2} - m_1 e_1 x_1 = 0 \\ r_1 x_2 - r_1 x_2^{r_1+1} - \frac{bx_1 x_2 t}{1-x_1} - nx_2 - m_2 e_2 x_2 = 0 \end{cases} \quad (12)$$

These points give the trivial equilibria cases, which implies equilibrium in the absence of the prey and equilibrium in the absence of the predator. They are obtained by solving simultaneously the Steady States equations in Equation (10) for  $x_1$  and  $x_2$  respectively. The trivial equilibrium case,  $e_0(x_1, x_2) = e_0(0,0)$ , in the absence of the prey,

$$(x_1 = 0) \quad e_1 \left( \left(1 - \frac{m_1 e_1}{r}\right)^{\frac{1}{r}}, 0 \right), \quad e_1(0,0) \quad \text{and the in the absence of the predator,} \quad (x_2 = 0) \quad e_2 \left( 0, \left(1 - \left(\frac{n+m_2 e_2}{r_1}\right)^{\frac{1}{r_1}}\right), e_2(0,0). \right.$$

These boundaries equilibria points indicate the extinction of both the prey and predator due to the hunting process by the predator nonexistence of the prey for the survival of the predator. Also, the system in Equation (12), has a positive equilibrium points,  $e(x_1^*, x_2^*)$  which will guarantee the maximum yield due to the exertion of the poaching process.

### Dynamical Stability Analysis

The stability analysis of the system is determined by the eigenvalues of the Jacobean matrix in Equation (11). If the eigenvalues have negatively real parts, the system is said to be asymptotically stable, whereas, if the eigenvalues of the Jacobean have positively real parts, the system is said to be asymptotically unstable and as such requires a different technique will be applied in determining the eigenvalues of the Jacobean. Thus, to analyze the stability, the equilibrium points  $(0,0)$  are substituted into the Jacobean to investigate the eigenvalues. Therefore, the asymptotic trivial equilibrium case,  $e_0(x_1, x_2) = e_0(0,0)$  is unstable point and the Jacobi matrix evaluation at  $e_0$  is:

$$J_0(e_0) = \begin{bmatrix} m_1 \left(1 - \frac{m_1 e_1}{r}\right)^{\frac{1}{r}} - \frac{m_1 e_1}{r} \left(\frac{rm_1}{r^2}\right) \left(1 - \left(\frac{m_1 e_1}{r}\right)^{\frac{1-r}{r}}\right) & 0 \\ 0 & m_2 \left(1 - \left(\frac{n+m_2 e_2}{r_1}\right)^{\frac{1}{r_1}}\right) - \frac{r_1 m_2^2 e_2}{r^3} \left(1 - \left(\frac{n+m_2 e_2}{r_1}\right)^{\frac{1-r}{r_1}}\right) \end{bmatrix} \quad (13)$$

Then, let the maximum yield due to the exertion of the poaching process by the predator on the prey achieved from the first component of Equation (11) be given as:

$$\begin{cases} y_1^* = m_1 e_1 x_1 \\ y_1^* = m_1 e_1 \left(1 - \frac{m_1 e_1}{r}\right)^{\frac{1}{r}} \end{cases} \quad (14)$$

$$\frac{\partial y_1^*}{\partial e_1} = m_1 \left( \left(1 - \frac{m_1 e_1}{r}\right)^{\frac{1}{r}} \right) - \frac{m_1 e_1}{r} \left( \frac{rm_1}{r^2} \right) \left(1 - \frac{m_1 e_1}{r}\right)^{\frac{1-r}{r}} \quad (15)$$

Therefore, the optimal exertion of the poaching process by the predator on the prey is achieved when  $\frac{\partial y_1^*}{\partial e_1} = 0$  in Equation (15) and solving yield the optimal exertion as:

$$e_1 = \left( \frac{r^{2r}}{m_1^r \left( 1 - \left( \frac{m_1}{r} \right) \right)^{1-2r}} \right)^{\frac{1}{1-r}} \quad (16)$$

Then, substituting the optimal exertion in Equation (16) into Equation (14), we have:

$$y_1^* = m_1 \left( \frac{r^{2r}}{m_1^r \left( 1 - \left( \frac{m_1}{r} \right) \right)^{1-2r}} \right)^{\frac{1}{1-r}} \left( 1 - \left( \frac{m_1}{r} \right) \right)^{\frac{1}{r}} \left( \frac{r^{2r}}{m_1^r \left( 1 - \left( \frac{m_1}{r} \right) \right)^{1-2r}} \right)^{\frac{1}{r}} \quad (17)$$

$$y_1^* = \left( \frac{r^2}{m_1^r \left( 1 - \left( \frac{m_1}{r} \right) \right)^{\frac{2 \pm \sqrt{2}}{2r}}} \right)^{\frac{1}{1-r}} \quad (18)$$

Next, is the maximum yield due to the exertion of the poaching process on the prey in the absence of the predator is achieved from the second component of Equation (11) expressed as:

$$\begin{cases} y_2^* = m_2 e_2 x_2 \\ y_2^* = m_2 e_2 \left( 1 - \left( \frac{n + m_2 e_2}{r_1} \right) \right)^{\frac{1}{r_1}} \end{cases} \quad (19)$$

$$\frac{\partial y_2^*}{\partial e_2} = \left( m_2 \left( 1 - \left( \frac{n + m_2 e_2}{r_1} \right) \right)^{\frac{1}{r_1}} \right) - \frac{r_1 m_2^2 e_2}{r_1^3} \left( 1 - \left( \frac{n + m_2 e_2}{r_1} \right) \right)^{\frac{1-r_1}{r_1}} \quad (20)$$

Therefore, the optimal effort of the poaching process on the prey in the absence of the predator is achieved when  $\frac{\partial y_2^*}{\partial e_2} = 0$  in Equation (20) and solving yield the optimal exertion as  $e_2 = 1$ . Then, substituting the optimal effort from Equation (20) into Equation (19), we have:

$$y_2^* = m_2 \left( 1 - \left( \frac{n + m_2}{r_1} \right) \right)^{\frac{1}{r_1}} \quad (21)$$

## Results Discussion.

The interaction between the prey and predator in the same ecosystem interact leading to a depletion of the prey and in the absence of the prey the predator goes into extinction. The study applied a probabilistic stochastic model using the Lomax function model which features both exponential and gamma growth on the prey species population. The prey and predator species population are poaching and the rate of the predator species getting a catch is proportional to the effort geared towards the poaching process. Now, when  $x_2(t) = 0$ , the prey population species grow exponential-gamma rate in the absence of the predator. The is the best period for the prey population because it is free from predation and the population is left without source of food. In general, the population of all the systems become extinct in the absence of  $x_1(t)$ . The Equation (18) is the yield that sustains the population of the prey species in the presence of poaching process as the population of the predator approaches zero. In Equation (21), ensured that the yield of the predator species population can be sustained during the poaching process while the prey species population goes to zero.

## Conclusion

In this paper, we have reviewed the Holling Type I and II function response models that involved the prey-predator dynamical interactions. The models showed dynamics that occurred in both prey and predator species populations when their stability analysis are considered. In this paper, we applied the Lomax distribution function on the prey



species population and assumed that the predator species prey upon the prey species accordingly in a logistical functional response. The results of the prey species dynamics with the Lomax function shows its exceptionality, novelty in obtaining the existence, uniqueness and boundedness of the solution of the systems. The standard linear stability analysis, was employed to analyze the sign of the real part of the eigenvalues of the Jacobian matrix, sometimes the Routh Hurwitz criterion could be applied. This helped examined the trivial equilibria points of both Lomax function of the prey species population and logistic function of the predator species population. The stability-state analysis of the model studied gives the dynamical behaviors of the system on analytical basis and in the future, a numerical simulation has to be done for the dynamical systems in Equation (3) for different sets of parameters and different set of initial points to confirm the obtained analytical results.

## References

- Abbas, I., Ejaz, A., & Shah, S.S. (2024). Mathematical Analysis of a Predator-Prey system with shared resource, climate effect and Neural Network weight. *Annals of Mathematics and Physics*, 7(2), 162-173.
- Adama, H. A. (2018). Mathematical Analysis of Predator-Prey model with two preys and one predator. *International Journal of Engineering and Applied Sciences*, 5(11), 17-23.
- Aman, A., & Dinku, T. (2021). Mathematical modelling of the dynamics of Prey-Predator with Scavenger in a closed Habitat. *IOSR, Journal of Mathematics*, 17(5), 8-21.
- Arif, M. S., Abodayeh, K., & Ejaz, A. (2023). Stability Analysis of Fractional-Order Predator-Prey System with Consuming Food Resource. *Axioms*, 12(1), 64-80.
- Ashine, A. B., & Gebru, D. M. (2017). Mathematical Modelling of Predator-Prey with modified Leslie-Gower and Holling Type II Scheme. *Global Journal of Science Frontiers Research: Mathematics and Decision Sciences*, 17(3), 21-40.
- Bezabih, A. F., Edessa, G. K., & Rao, K. P. (2021). Ecoepidemiological model and Analysis of Prey-Predator System. *Journal of Applied Mathematics*. <https://doi.org/10.1155/2021/6679686>.
- Charles, R., Makinde, O. D., & Kungaro, M. (2022). A Review of the Mathematical models for the Impact of Seasonal Weather variation and Infections on Prey-Predator Interaction in Serengeti Ecosystem. *Open Journal of Ecology*, 12(1), 718-732.
- Ejakpovi, S. U., & Siloko, I. U. (2023). A modified Logistic model for Population Growth of Fishes in an Environment. *Science World Journal*, 18(1), 43-47.
- Ejaz, A., Nawaz, Y., Arif, M. S., Mashat, D.S., & Abodayeh, K. (2022). Stability Analysis of Predator-Prey System with Consuming Resource and Disease in Predator Species. *CMES-Computer Modeling in Engineering & Sciences*, 132(2), 18-24.
- Hoff, Q., & Fay, T. H. (2016). A Predator-Prey Model with Predator Population Saturation. *Journal of Mathematics and Statistics*, 4(4), 101-107.
- Joshi, Y., Savesca, M., Syed, M., & Blackmore, D. (2021). Interesting Features of Three-Dimensional Discrete Lotka-Volterra Dynamics. *Applied Mathematics*, 12(3) 694-722.
- Kolpak, E. P., Stolbovaia, M.V., & Frantsuzova, I. S. (2016). A Predator-Prey Mathematical in a limited Area. *Global Journal of Pure and Applied Mathematics*, 12(5), 4443-4453.
- Savitri, D. (2019). Stability and Numerical Simulation of Prey-Predator System with Holling Type II Functional Responses for Adult Prey. *Journal of Physics, Conference Series*, 1417. <https://doi.org/10.1088/1742-6596/1417/1/012015>.
- Sharmila, N. B., & Gunasundari, C. (2023). Mathematical Analysis of Prey Predator Models with Holling Type I Functional Responses and Time Delay. *Communication Mathematical Biological Neuroscience*, <https://doi.org/10.28919/cmbn/8014>.
- Shlyufman, K. V., Neverova, G. P., & Frisman, E. Y. (2018). Phase Multistability of Oscillatory Regimes of the Dynamics of the Rikler Model with a Periodically Varying Malthusian Parameter. *Journal of Mathematical Bio information*, 13(1), 68–83.
- Simon, J. S. H., & Rabago, J. F.T. (2018). Optimal control for Predator-Prey model with disease in the prey population. *Malaysian Journal of Mathematical Science*, 12(2), 269-285.
- Yousef, A., Salmon, S., & Elsadany, A. (2018). Stability and Bifurcation of a Delayed Predator-Prey Model. *International Journal of Bifurcation and Chaos*, 28, 18501161850129. <https://doi.org/10.1142/S021812741850116>.