FNAS Journal of Mathematical Modeling and Numerical Simulation
Print ISSN: 3027-1282
www.fnasjournals.com
Volume 2; Issue 3; May 2025; Page No. 96-112.
DOI: https://doi.org/10.63561/jmns.v2i3.868

A Mathematical Model of Competition between Cassava and Yam

***[1]Abanum, G. C., [1]Iweobodo, D. C., [1]Ogoegbulem, O., Obaremi, L.M., & [2]Imoni, S. O.**
[1]Department of Mathematics, Dennis Osadebay University, Asaba, Delta State, Nigeria.
[2]Department of Mathematics, Federal University, Lokoja, Kogi State, Nigeria.

***Corresponding author email:** abanum.godspower@dou.edu.ng

Abstract
In this paper, we propose a mathematical model for the interaction between Cassava (Manihot esculenta) and Yam (Dioscorea spp.) biomass using a system of non-linear ordinary differential equations (ODEs). The mathematical analysis of the uniqueness, local and global stability of the model was investigated. Additionally, a computationally efficient numerical method for solving ordinary differential equations of order 45 (ODE45) was used to simulate the biomass of Cassava and Yam, considering variations in the growth rate parameters. It was discovered that decreasing the growth rates of Cassava from 10 percent to 99 percent results in biodiversity loss. In contrast, when the growth rates were increased from 101 percent to 120 percent, the results changed from biodiversity loss to biodiversity gain.

Keywords: Cassava, Yam, Biodiversity, Stability, ODE45, Simulation, Biomass, Mathematical model.

Introduction

In recent years, some mathematical concepts have been extensively applied by researchers to model problems arising in different fields such as Biology, Agriculture, control theory, and fluid dynamics; and this is often achieved by transforming these problems into a system of ordinary differential equations. The study of population dynamics has become of great interest lately, this is due to the fact that it provides solutions to visible problems affecting life and existence. Different mathematical models have been developed by researchers (Iweobodo et al., 2024; Li et al., 2017; Abanum & Oke, 2019). Among all the models, Ecological resources play a vital role in population dynamics. Hence, many researchers in Mathematical Biology and Ecological Modelling propose different models in ecological resources interaction (Agyemang & Freedman, 2009; Agyemang et al., 2007; Abanum et al., 2020; Abanum & Eli, 2022; Donatelli et al., 2017; Mittelbach, 2012; Morin, 2002, Vandermeer & Goldberg, 2013; Imoni et al., 2019; Lanlege & Imoni, 2018; Eli & Abanum, 2020). Others are- the impact of diseases and pests on Agricultural systems (Donatelli et al., 2017; Wang. 2008), and the interaction between predator-prey (Liu, 2010).

Quinn et al. (2013) studied the biodiversity and ecosystem on a farm using the farm scale method, and the results exhibited a convenient and good evaluation capable of helping farmers in making good decisions relating to biodiversity management and ecological reasoning. Antle and Valdivia (2006) worked on the modeling of agricultural ecosystem services, a minimum-data approach was used to analyze the supply of ecosystem services. The method was applied to simulate the supply of carbon that could be segregated in soils and grain-producing regions in dry. Eleki and Ekaka-a (2019) studied the biodiversity scenario on ecospheric assets, they considered a method of semi-stochastic analysis in checking the effect of variation of the per-asset degradation rate coefficient of ecospheric assets, and they were able to predict the level of biodiversity. In a similar research, Eleki and Ekakaa (2019) applied an ordinary differential equation of order 45 (ODE45) numerical simulation to predict the biomass of the ecosphere. The study predicts that the smaller the per-asset degradation coefficient of the ecospheric asset, the bigger the opportunity of getting a good biodiversity gain. Abanum et al. (2024) worked on the computational and mathematical modeling of agricultural assets, a method of ordinary differential equation of order 45 was considered to predict the extent of biodiversity loss and gain due to the variation of the inter-competitive growth rate coefficients of normal agriculture. In this paper, we proposed a model for the interaction between cassava biomass, yam biomass, and ecospheric assets. The aim is to investigate the effect of varying the growth rate coefficients of cassava and yam, and then investigate the conditions for local stability, global stability, and its uniqueness.

Materials and Methods

Model Assumptions

We consider the following assumptions in modelling the interaction between cassava and yam ecosystems.

(i) The growth of cassava biomass over time depends on the interaction between cassava and yam, providing and impacting factors called the inter-competitive coefficient of cassava.

(ii) The growth of cassava depends on self-interaction and the ecosystem.

(iii) The growth of cassava biomass over time depends on the interaction between cassava and yam and itself, and the interaction with the ecosphere.

(iv) The growth of cassava and yam hurts the ecosphere and it may cause cassava and yam a lot to replenish the ecosphere.

(v) The growth of the ecospheric assets depends mostly on the interaction between the ecospheric asset and itself; it depends between cassava and yam and the rate of effort input to restore the ecosphere by cassava and yam.

(vi) The growth of cassava and yam wholly depends on the growth rates of cassava and yam.

Mathematical Formulation

In this section, we consider the following

$$\frac{dx}{dt} = \alpha_1 x - \beta_1 x^2 - \gamma_1 xy - \rho xz \quad (1)$$

$$\frac{dy}{dt} = \alpha_2 y - \beta_2 y^2 - \gamma_2 xy - \rho yz \quad (2)$$

$$\frac{dz}{dt} = r(1-z)z - \gamma_3 (x+y)z - \theta(x+y)z \quad (3)$$

With initial condition $x_0 = x(0) \geq 0, y_0 = y(0) \geq 0 \; and \; z_0 = z(0) \geq 0, 0 \leq z \leq 1$, where all the parameters are assumed to be positive constant.

$x(t)$ denotes the biomass of cassava.

$y(t)$ denotes the biomass of yam.

$z(t)$ denotes the ecosphere.

α_1 and α_2 denotes the growth rate of cassava and yam

β_1 and β_2 denotes the intra-competitive coefficients of cassava and yam.

γ_1 and γ_2 denotes the inter-competitive coefficients of caType equation here.ssava and yam.

ρ represents the net cost rate of cassava and yam to restore the ecosphere.

θ is the input rate of effort to restore the ecosphere by cassava and yam.

γ_3 is the per asset degradation rate coefficient of the ecosphere due to cassava and yam production activities.

r is the natural restoration rate coefficient for the ecosphere.

Equilibrium Analysis

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$

Therefore,

$$\alpha_1 x - \beta_1 x^2 - \gamma_1 xy - \rho xz = 0$$
$$\alpha_2 y - \beta_2 y^2 - \gamma_2 xy - \rho yz = 0 \quad (4)$$
$$r(1 - z)z - \gamma_3(x + y)z + \theta xz + \theta yz = 0$$

Let $G(x, y, z)$ be the interacting equilibrium point function. Given $x = y = z = 0$, substitute (4)-(6), we have $G(0,0,0)$ which is an equilibrium and (0,0,0) is a trival steady state.

When $\frac{dx}{dt} = 0, x \neq 0, y = 0, z = 0$

Substitute into equation (4)

$$\alpha_1 x - \beta_1 x^2 - \gamma_1 x(0) - \rho x(0) = 0$$
$$\alpha_1 x - \beta_1 x^2 = 0 \quad (5)$$
$$x(\alpha_1 - \beta_1 x) = 0$$

Since $x \neq 0$, we have $x = \frac{\alpha_1}{\beta_1 c}$ (6)

Then $G_1(x, 0,0)$ is a steady-state solution in which y and z are extinct.

When

$\frac{dy}{dt} = 0, y \neq 0, x = z = 0$, Substituting in (5),

$$y = \frac{\alpha_2}{\beta_2} \quad (7)$$

Then $G_2(0, y, z)$ is a steady state in which $x\ and\ z$ are extinct

Consider

$$\frac{dz}{dt} = 0, z \neq 0, x = y = 0, \ z = 1. \quad (8)$$

Therefore, $G_3(0,0, z)$ is also a steady-state solution where only z survives.

Consider the non-negative dimensional equilibrium

$G_{xz}(x, 0,1), x \neq 0, z \neq 0, y = 0.$

Substituting into (4),

$$x = \frac{\alpha_1 - \rho_1}{\beta_1} \quad (9)$$

$G_{xy}(x, y, 0), x \neq 0, y \neq 0, z = 0.$

Substituting into (4),

$$\alpha_1 x - \beta_1 x^2 - \gamma_1 xy = 0$$
$$x(\alpha_1 - \beta_1 x - \gamma_1 y) = 0$$

Since $x \neq 0$, we have

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

$\alpha_1 - \beta_1 - \gamma_1 y = 0$

$x = \frac{\alpha_1 - \gamma_1 y}{\beta_1}$ (10)

Substituting (10) into (5) gives

$(\gamma_1 - \beta_1\beta_2)y^2 + (\alpha_2\beta_1 - \gamma_2 - \gamma_1\beta_1)y - \rho\alpha_1\beta_1 = 0$

Considering the standard form of the equation,

$a = \gamma_1 - \beta_1\beta_2$

$b = \alpha_2\beta_1 - \gamma_2 - \gamma_1\beta_1$

$c = \rho\alpha_1\beta_1$

Substituting into

$-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ (11)

Uniqueness of Steady-State Solution

Theorem 2.1

A function $f(t, y)$ is Lipschitz in x on a set $\mathfrak{R}$ with Lipschitz constant k if

$|f(t, y_1) - f(t, y_2)| \leq k|y_1 - y_2|$

For all $f(t, y_1)$ and $f(t, y_2)$ in $\mathfrak{R}$. A unique continuous vector solution $y(t)$ of the model in the interval $t - t_0 \leq \delta$ exists such that $\delta > 0$.

Proof:

The set $\mathfrak{R}$ denote the region $0 \leq \delta \leq \mathfrak{R}$, to prove the uniqueness of the steady-state solution of (1)-(3), we have to show that the partial derivatives of (1)-(3) are continuous and bounded in the region $\mathfrak{R}$. Assume

$A_1 = \alpha_1 x - \beta_1 x^2 - \gamma_1 xy - \rho xz$ (12)

$A_2 = \alpha_2 y - \beta_2 y^2 - \gamma_2 xy - \rho yz$ (13)

$A_3 = r(1 - z)z - \gamma_3(x + y)z + \theta xz + \theta yz$ (14)

Taking the partial derivatives of (12)-(14) yield

$\left|\frac{\partial A_1}{\partial x}\right| = |\alpha - 2\beta_1 - \gamma_1 y - \rho z| < \infty$

$\left|\frac{\partial A_1}{\partial y}\right| = |-\gamma_1 x| < \infty$

$\left|\frac{\partial A_1}{\partial z}\right| = |-\rho x| < \infty$

$\left|\frac{\partial A_2}{\partial x}\right| = |-\gamma_2 y| < \infty$

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

$$\left|\frac{\partial A_2}{\partial y}\right| = |\alpha_2 - 2\beta_2 y - \gamma_2 x - \rho z| < \infty$$

$$\left|\frac{\partial A_2}{\partial x}\right| = |-\rho_2 y| < \infty$$

$$\left|\frac{\partial A_3}{\partial x}\right| = |-\gamma_3 z + \theta z| < \infty$$

$$\left|\frac{\partial A_3}{\partial y}\right| = |-\gamma_3 z + \theta z| < \infty$$

$$\left|\frac{\partial A_3}{\partial z}\right| = |r - 2rz - \gamma_3 x - \gamma y + \theta x + \theta y| < \infty$$

It shows clearly that the partial derivatives of (1)-(3) are finite, exist and bounded.

Therefore, the model (1)-(3) has a unique solution.

Optimal Stabilisation

To check the various steady states of (1)-(3), we have to obtain the variational matrix by differentiating each function concerning x, y, z. we have

$$V = \begin{pmatrix} J_1 1 & J_1 2 & J_1 3 \\ J_2 1 & J_2 2 & J_2 3 \\ J_3 1 & J_3 2 & J_3 3 \end{pmatrix}$$

$$= \begin{pmatrix} J_1 1 & -\gamma_1 x & -\rho x \\ -\gamma_2 y & J_2 2 & -\rho y \\ -\gamma_3 z & -\gamma_3 z + \theta z & J_3 3 \end{pmatrix}$$

Where

$J_1 1 = \alpha_1 - 2\beta_1 x - \gamma_1 y - \rho z$

$J_2 2 = \alpha_2 - 2\beta_2 y - \gamma_2 x - \rho z$

$J_3 3 = r - 2rz - \gamma_3 x - \gamma_3 y + \theta x + \theta y$

Given the characteristics equation

$$|J - \lambda I| \tag{15}$$

Applying the ODE 45 numerical method where I is the identity matrix of order 3 x 3, and d is the scalar. We obtain the eigen values using Matlab ODE 45.

$\lambda_1 = \lambda_2 \; and \; \lambda_3.$

Where the possible solid state is the point $(x, y, z) = (0,0,0)$

$$V = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$\lambda_1 = 3, \; \lambda_2 = 3, and \; \lambda_3 = 2.$

Local and Global Stability Analysis of (1) – (3)

Local Stability Analysis of $G_0(0,0,0)$

The variational matrix v about the equilibrium G_0 is given by

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

$$V = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 1 \\ 0 & 0 & r \end{pmatrix} \tag{16}$$

The eigenvalues of G_0 are given by α_1, α_2 and r. This equilibrium is unstable since the eigenvalues are all positive.

Lemma 1

The equilibrium G_0 is non-hyperbolic and locally unstable.

Local stability Analysis of $G_z(0,0,1)$

The variational matrix v about the equilibrium G_z is given by

$$VG_z = \begin{pmatrix} \alpha_1 - \rho & 0 & 0 \\ 0 & \alpha_2 - \rho & 0 \\ -\gamma_3 + \theta & -\gamma_3 + \theta & r - 2r \end{pmatrix}$$

The eigenvalues of VG_z are given by $\alpha_1 - \rho$, $\alpha_2 - rho \ \ and\ r - 2r.$

Lemma 2

The equilibrium F_z is locally asymptotically stable if

$\lambda_1 = \alpha_1 - \rho < 0\ and\ \ \lambda_2 = \alpha_2 - \rho < 0$.

Lemma 3

The equilibrium G_z is a hyperbolic saddle point if either

$\alpha_1 - \rho > 0$ or $\alpha_2 - \rho > 0.$

Local Stability Analysis of $G_z(\frac{\alpha_1}{\beta_1}, 0,1)$

The variational matrix about G_{xz} is given by

$$VF_{xz} = \begin{pmatrix} -\alpha_1 - \rho & \dfrac{-\gamma_1\alpha_1}{\beta_1} & \dfrac{-\gamma_1\alpha}{\beta_1} \\ 0 & \dfrac{\alpha_2 - \gamma_1\alpha_1 - \rho}{\beta_1} & 0 \\ -\gamma_3 + \theta & -\gamma_3 + \theta & r - 2r - \dfrac{\gamma_3\alpha_1}{\beta_1} + \dfrac{\theta\alpha_1}{\beta_1} \end{pmatrix}$$

The eigenvalues of VG_{xz} are $-\alpha_1 - \rho$, $\frac{\alpha_2 - \gamma_1\alpha_1 - \rho}{\beta_1}$ and $r - 2r - \frac{\gamma_3\alpha_1}{\beta_1} + \frac{\theta\alpha_1}{\beta_1}$

Lemma 4

The equilibrium G_{xz} is locally asymptotically stable if $\alpha_2\beta_1 - \gamma_2\beta_1 - \beta_1\rho < 0$ and a hyperbolic saddle point if

$\alpha_2\beta_1 - \gamma_2\beta_1 - \beta_1\rho < 0$

And a hyperbolic saddle point if

$\alpha_2\beta_1 - \gamma_2\beta_1 - \beta_1\rho > 0$

Repelling in the direction of y.

Local Stability Analysis of $G_{yz}\left(0, \frac{\alpha_2}{\beta_2}\right)$

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

$$VG_{yz} = \begin{pmatrix} \alpha_1 - \frac{\gamma_1\alpha_2}{\beta_2} - \rho & 0 & -\rho \\ \frac{-\gamma_2\alpha_2}{\beta_2} & \alpha_2 - 2\frac{\gamma_2\alpha_2}{\beta_2} - \rho & -\rho \\ -\gamma_3 + \theta & \gamma_3 + \theta & r - 2r - \frac{\gamma_2\alpha_2}{\beta_2} + \frac{\theta\alpha_2}{\beta_2} \end{pmatrix}$$

Lemma 5

The equilibrium G_{yz} is locally asymptotically stable if

$\alpha_1\beta_2 - \gamma_1\beta_2 - \beta_2\rho < 0$

and a hyperbolic saddle point if

$\alpha_1\beta_2 - \gamma_1\alpha_2 - \beta_2\rho > 0$

Repelling in the direction of x.

Results and Discussion

Model parameters and Meanings

Table 1: Parameters in Model (1) –(3), values and their meanings

Parameter	Meaning	Value	
α_1	Growth rate of cassava	3	assumed
α_2	Growth rate of the yam	3	assumed
β_1	Intra-competitive coefficient of cassava	0.2	assumed
β_2	Intra-competitive coefficient of yam	0.2	assumed
γ_1	Inter-competitive coefficient of cassava	0.1	assumed
γ_2	Inter-competitive coefficient of yam	0.1	assumed
ρ	Net cost rate of cassava and yam to restore the ecosphere	0.25	assumed
θ	Input rate of effort to restore the ecosphere by cassava and yam	1	assumed
γ_3	Per asset degradation rate coefficient of the ecosphere due to cassava and yam production	0.1	assumed
r	Natural restoration rate coefficient for the ecosphere.	2	assumed

Numerical Simulation

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

Table 2: Quantifying the effect of decreasing α_1 and α_2 by 20% on the yield of cassava and yam using the ODE45 numerical method.

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	1.6934	81.6340	9.2293	1.8835	79.5922
9.2248	1.7556	80.9691	9.2248	1.8100	80.3791
9.2247	1.7751	80.7572	9.2247	1.7907	80.5877
9.2249	1.7807	80.6968	9.2249	1.7852	80.6482
9.2248	1.7823	80.6793	9.2248	1.7836	80.6654
9.2248	1.7828	80.6742	9.2248	1.7831	80.6702
9.2248	1.7829	80.6729	9.2248	1.7830	80.6717
9.2248	1.7829	80.6724	9.2248	1.7830	80.6721
9.2248	1.7829	80.6722	9.2248	1.7830	80.6721
9.2248	1.7829	80.6723	9.2248	1.7829	80.6723
9.2248	1.7829	80.6724	9.2248	1.7829	80.6724
9.2248	1.7829	80.6723	9.2248	1.7829	80.6723
9.2248	1.7829	80.6723	9.2248	1.7829	80.6723
9.2248	1.7830	80.6723	9.2248	1.7830	80.6723
9.2248	1.7830	80.6721	9.2248	1.7830	80.6721
9.2248	1.7830	80.6721	9.2248	1.7830	80.6721

Table 3: Quantifying the effect of decreasing α_1 and α_2 by 30% on the yield of cassava and yam using the ODE45 numerical method.888888

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	3.5765	61.2111	9.2293	3.7077	59.8270
9.2248	3.6383	60.5598	9.2248	3.6485	60.4488
9.2247	3.6430	60.5081	9.2247	3.6438	60.4995
9.2249	3.6433	60.5051	9.2249	3.6434	60.5045
9.2248	3.6434	60.5044	9.2248	3.6434	60.5043
9.2248	3.6434	60.5042	9.2248	3.6434	60.5042
9.2248	3.6434	60.5043	9.2248	3.6434	60.5043
9.2248	3.6434	60.5041	9.2248	3.6434	60.5041
9.2248	3.6434	60.5041	9.2248	3.6434	60.5041
9.2248	3.6434	60.5043	9.2248	3.6434	60.5043
9.2248	3.6434	60.5046	9.2248	3.6434	60.5046
9.2248	3.6434	60.5039	9.2248	3.6434	60.5039
9.2248	3.6434	60.5039	9.2248	3.6434	60.5039

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

9.2248	3.6434	60.5042	9.2248	3.6434	60.5042
9.2248	3.6434	60.5041	9.2248	3.6434	60.5041
9.2248	3.6434	60.5047	9.2248	3.6434	60.5047
9.2248	3.6434	60.5041	9.2248	3.6434	60.5041

Table 4: Quantifying the effect of decreasing α_1 and α_2 by 50% on the yield of cassava and yam using the ODE45 numerical method

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	4.5271	50.9012	9.2293	4.6192	49.9503
9.2248	4.5717	50.4409	9.2248	4.5755	50.4003
9.2247	4.5736	50.4206	9.2247	4.5737	50.4189
9.2249	4.5736	50.4206	9.2249	4.5736	50.4205
9.2248	4.5736	50.4204	9.2248	4.5736	50.4204
9.2248	4.5736	50.4201	9.2248	4.5736	50.4201
9.2248	4.5737	50.4201	9.2248	4.5737	50.4201
9.2248	4.5737	50.4200	9.2248	4.5737	50.4200
9.2248	4.5736	50.4206	9.2248	4.5736	50.4206
9.2248	4.5737	50.4201	9.2248	4.5737	50.4201
9.2248	4.5736	50.4203	9.2248	4.5736	50.4203
9.2248	4.5736	50.4202	9.2248	4.5736	50.4202
9.2248	4.5736	50.4202	9.2248	4.5736	50.4202
9.2248	4.5737	50.4199	9.2248	4.5737	50.4199
9.2248	4.5737	50.4198	9.2248	4.5737	50.4198
9.2248	4.5736	50.4208	9.2248	4.5736	50.4208
9.2248	4.5737	50.4200	9.2248	4.5737	50.4200
9.2248	1.5736	50.4201	9.2248	4.5736	50.4201

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

Table 5: Quantifying the effect of decreasing α_1 and α_2 by 90% on the yield of cassava and yam using the ODE45 numerical method

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	8.2872	10.1216	9.2293	8.3020	10.0475
9.2248	8.2946	10.0838	9.2248	8.2946	10.0835
9.2247	8.2946	10.0835	9.2247	8.2946	10.0835
9.2249	8.2946	10.0849	9.2249	8.2946	10.0849
9.2248	8.2946	10.0844	9.2248	8.2946	10.0844
9.2248	8.2946	10.0838	9.2248	8.2946	10.0838
9.2248	8.2946	10.0846	9.2248	8.2946	10.0846
9.2248	8.2946	10.0842	9.2248	8.2946	10.0842
9.2248	8.2946	10.0836	9.2248	8.2946	10.0836
9.2248	8.2946	10.0843	9.2248	8.2946	10.0843
9.2248	8.2945	10.0845	9.2248	8.2945	10.0845
9.2248	8.2945	10.0845	9.2248	8.2945	10.0845
9.2248	8.2946	10.0839	9.2248	8.2946	10.0839
9.2248	8.2946	10.0836	9.2248	8.2946	10.0836
9.2248	8.2946	10.0836	9.2248	8.2946	10.0836
9.2248	8.2945	10.0844	9.2248	8.2945	10.0844
9.2248	8.2945	10.0851	9.2248	8.2945	10.0851
9.2248	8.2946	10.0836	9.2248	8.2946	10.0836

Table 6: Quantifying the effect of decreasing α_1 and α_2 by 99% on the yield of cassava and yam using the ODE45 numerical method

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	9.1271	1.0124	9.2293	9.1364	1.0063
9.2248	9.1318	1.0079	9.2248	9.1318	1.0079
9.2247	9.1318	1.0071	9.2247	9.1318	1.0071
9.2249	9.1317	1.0096	9.2249	9.1317	1.0096
9.2248	9.1318	1.0088	9.2248	9.1318	1.0088
9.2248	9.1319	1.0068	9.2248	9.1319	1.0068
9.2248	9.1317	1.0095	9.2248	9.1317	1.0095
9.2248	9.1317	1.0093	9.2248	9.1317	1.0093
9.2248	9.1319	1.0072	9.2248	9.1319	1.0072
9.2248	9.1317	1.0093	9.2248	9.1317	1.0093
9.2248	9.1317	1.0093	9.2248	9.1317	1.0093
9.2248	9.1318	1.0075	9.2248	9.1318	1.0095
9.2248	9.1317	1.0091	9.2248	9.1317	1.0091
9.2248	9.1317	1.0091	9.2248	9.1317	1.0091
9.2248	9.1318	1.0078	9.2248	9.1318	1.0078
9.2248	9.1317	1.0088	9.2248	9.1317	1.0088
9.2248	9.1317	1.0089	9.2248	9.1317	1.0089
9.2248	9.1318	1.0079	9.2248	9.1318	1.0079

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

Table 7: Quantifying the effect of decreasing α_1 and α_2 by 101% on the yield of cassava and yam using the ODE45 numerical method

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	9.3136	1.0101	9.2293	9.3220	1.0042
9.2248	9.3178	1.0080	9.2248	9.3178	1.0079
9.2247	9.3178	1.0088	9.2247	9.3178	1.0088
9.2249	9.3178	1.0075	9.2249	9.3178	1.0075
9.2248	9.3178	1.0080	9.2248	9.3178	1.0080
9.2248	9.3178	1.0086	9.2248	9.3178	1.0086
9.2248	9.3178	1.0077	9.2248	9.3178	1.0077
9.2248	9.3178	1.0076	9.2248	9.3178	1.0076
9.2248	9.3177	1.0077	9.2248	9.3177	1.0077
9.2248	9.3177	1.0070	9.2248	9.3177	1.0070
9.2248	9.3177	1.0071	9.2248	9.3177	1.0071
9.2248	9.3178	1.0079	9.2248	9.3178	1.0079
9.2248	9.3178	1.0080	9.2248	9.3178	1.0080
9.2248	9.3178	1.0080	9.2248	9.3178	1.0080
9.2248	9.3178	1.0085	9.2248	9.3178	1.0085
9.2248	9.3178	1.0083	9.2248	9.3178	1.0083
9.2248	9.3178	1.0082	9.2248	9.3178	1.0082
9.2248	9.3178	1.0085	9.2248	9.3178	1.0085

Table 8: Quantifying the effect of decreasing α_1 and α_2 by 110% on the yield of cassava and yam using the ODE45 numerical method

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	10.1524	10.1078	9.2293	10.1577	10.0587
9.2248	10.1551	10.0846	9.2248	10.1551	10.0845
9.2247	10.1549	10.0837	9.2247	10.1549	10.0837
9.2249	10.1550	10.0831	9.2249	10.1550	10.0831
9.2248	10.1550	10.0829	9.2248	10.1550	10.0829
9.2248	10.1551	10.0847	9.2248	10.1551	10.0847
9.2248	10.1551	10.0838	9.2248	10.1551	10.0838
9.2248	10.1552	10.0849	9.2248	10.1552	10.0849
9.2248	10.1550	10.0840	9.2248	10.1550	10.0840
9.2248	10.1550	10.0837	9.2248	10.1550	10.0837
9.2248	10.1550	10.0834	9.2248	10.1550	10.0834
9.2248	10.1550	10.0837	9.2248	10.1550	10.0837
9.2248	10.1550	10.0839	9.2248	10.1550	10.0839
9.2248	10.1550	10.0833	9.2248	10.1550	10.0833
9.2248	10.1551	10.0847	9.2248	10.1551	10.0847
9.2248	10.1551	10.0844	9.2248	10.1551	10.0844

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

9.2248	10.1551	10.0850	9.2248	10.1551	10.0850
9.2248	10.1550	10.0838	9.2248	10.1550	10.0838

Table 9: Quantifying the effect of decreasing α_1 and α_2 by 120% on the yield of cassava and yam using the ODE45 numerical method

x (old)	x (new)	EBL	y(old)	y(new)	EBL
2.0000	2.0000	0	3.0000	3.0000	0
9.2204	11.0837	20.2083	9.2293	11.0868	20.1259
9.2248	11.0853	20.1682	9.2248	11.0853	20.1681
9.2247	11.0854	20.1698	9.2247	11.0854	20.1698
9.2249	11.0853	20.1672	9.2249	11.0853	20.1672
9.2248	11.0852	20.1674	9.2248	11.0852	20.1674
9.2248	11.0853	20.1683	9.2248	11.0853	20.1683
9.2248	11.0853	20.1676	9.2248	11.0853	20.1676
9.2248	11.0854	20.1686	9.2248	11.0854	20.1686
9.2248	11.0853	20.1682	9.2248	11.0853	20.1682
9.2248	11.0852	20.1675	9.2248	11.0852	20.1675
9.2248	11.0853	20.1677	9.2248	11.0853	20.1677
9.2248	11.0853	20.1684	9.2248	11.0853	20.1684
9.2248	11.0854	20.1688	9.2248	11.0854	20.1688
9.2248	11.0853	20.1678	9.2248	11.0853	20.1678
9.2248	11.0852	20.1680	9.2248	11.0852	20.1680
9.2248	11.0853	20.1680	9.2248	11.0853	20.1680
9.2248	11.0853	20.1680	9.2248	11.0853	20.1680
9.2248	11.0853	20.1692	9.2248	11.0853	20.1692
9.2248	11.0853	20.1680	9.2248	11.0853	20.1680

3.3 Graphical Simulation on Biodiversity Loss and Gain

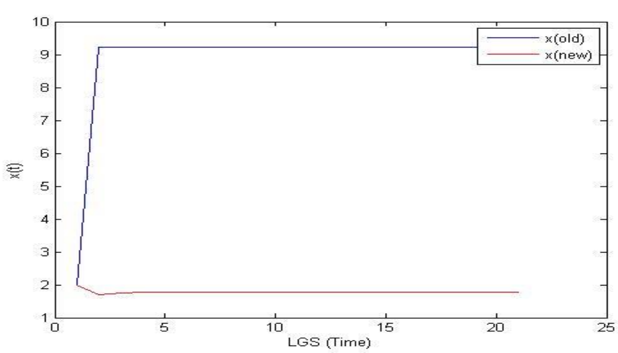


Figure 1. Graphical Simulation on biodiversity scenario when α_1 and α_2 are varied together by 20%

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

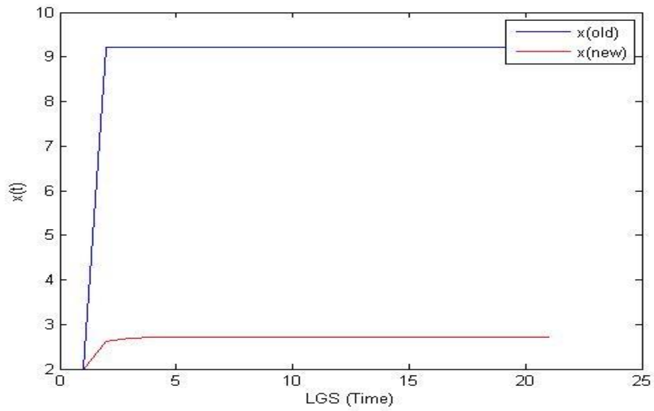


Figure 2: Graphical simulation on biodiversity scenario when α_1 and α_2 are varied together by 30%

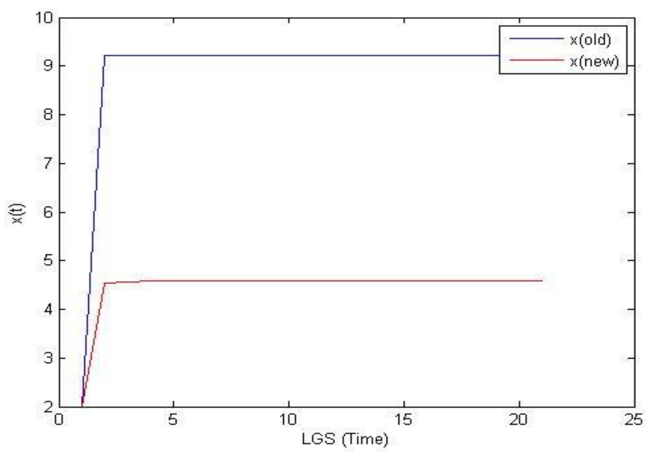


Figure 3. Graphical simulation on biodiversity scenario when α_1 and α_2 are varied together by 50%

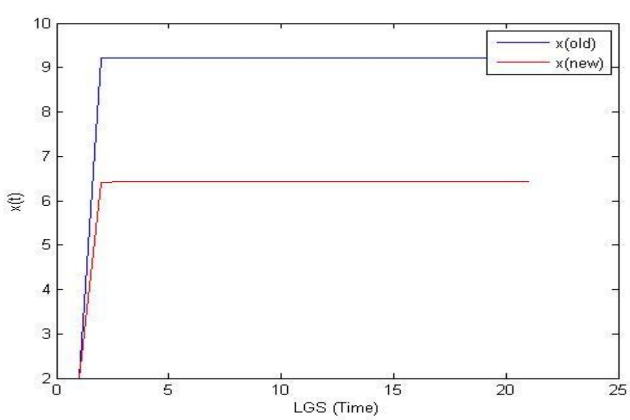


Figure 4. Graphical simulation on biodiversity scenario when α_1 and α_2 are varied together by 70%

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

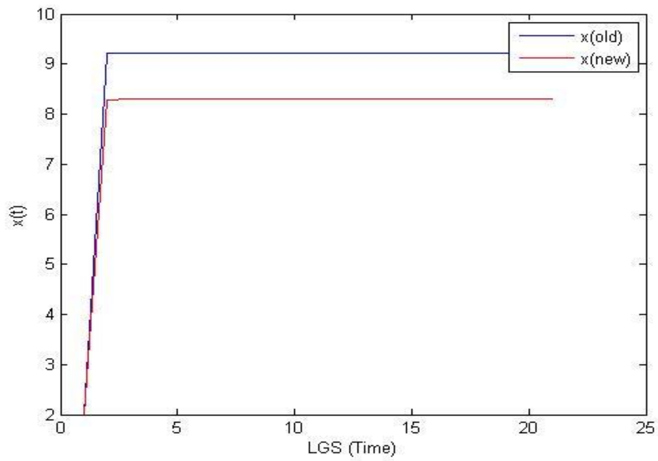


Figure 5. Graphical simulation on biodiversity scenario when α_1 and α_2 are varied together by 90%

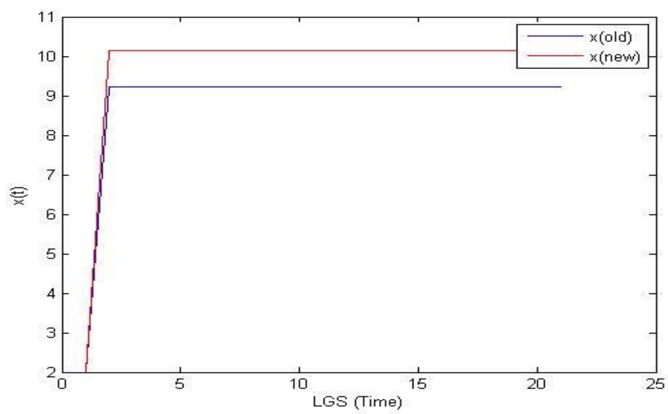


Figure 6. Graphical simulation on biodiversity scenario when α_1 and α_2 are varied together by 110%

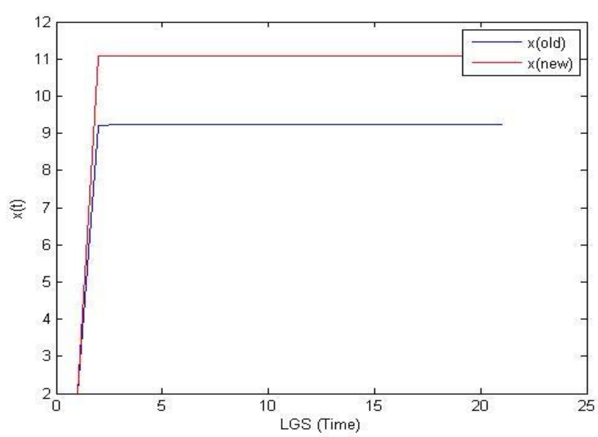


Figure 7. Graphical simulation on biodiversity scenario when α_1 and α_2 are varied together by 120%

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

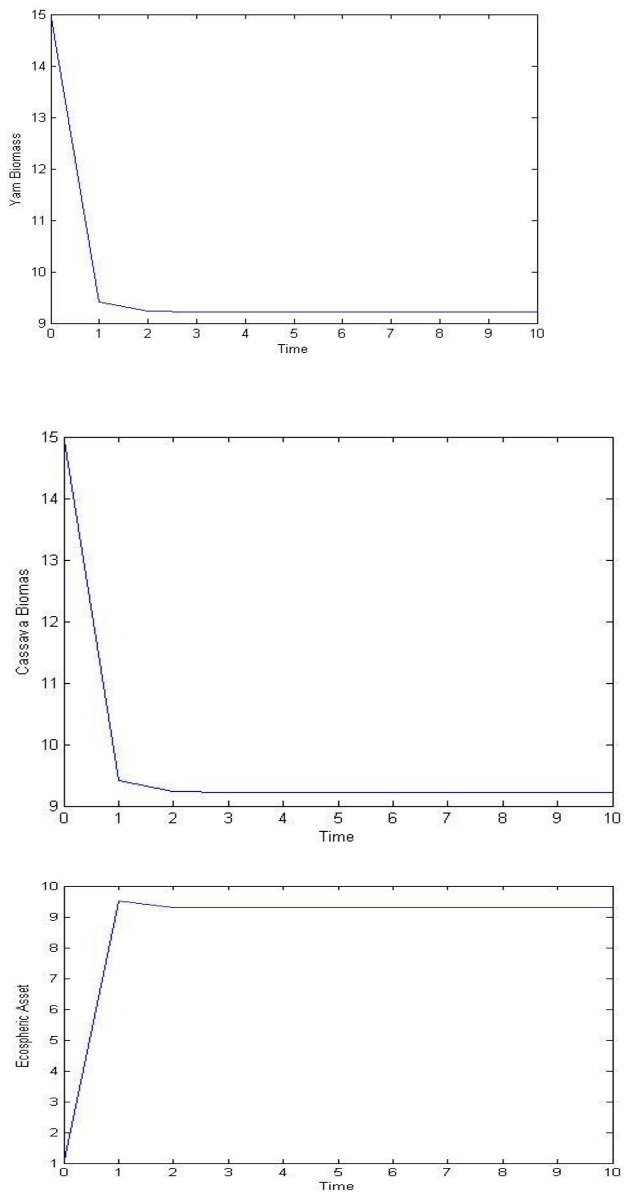
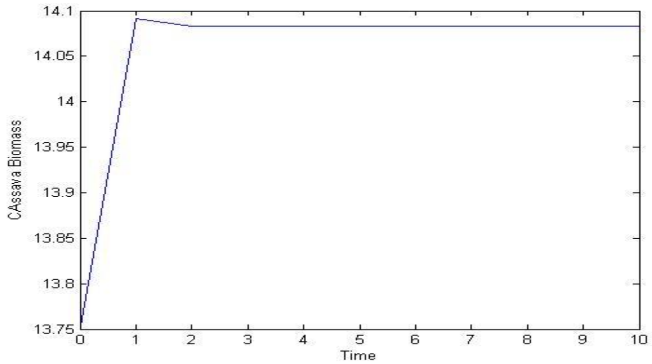
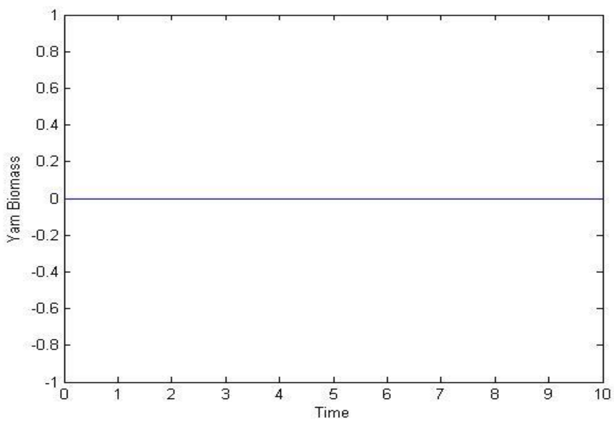


Figure 8. G$xyz\left(\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, 1\right)$ is globally asymptotically stable

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868





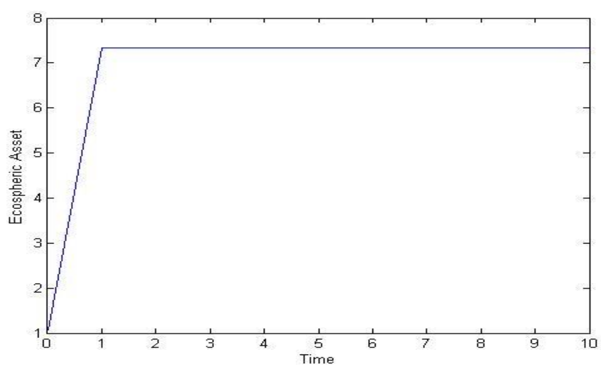


Figure 9. $Gxyz\left(\frac{\alpha_1-\rho}{\beta_1},0,1\right)$ is globally asymptotically stable. Yam is driven extinct.

Discussion

Table 1 to Table 5 shows the results of quantifying the effect of decreasing the growth rates of cassava and yam. We observed from the prediction that x (old) and y (old) remain the same because all the model parameters are fixed at 100% while x (new) and y (new) varies due to the variation of the growth rates of cassava and yam.

The biomass of cassava x (new) and y (new) when the growth rates are decreased from 5% to 99% has lost biomass over that x (old) and yam y (new) when the growth rates are decreased from 5% to 99% has lost biomass over that of x (old) and y (old), that is, when all the model parameters are fixed at 100%. However, we have discussed from the results that x (new) and yam (new), which remain, due to the increase of the growth rates coefficients from 101% to

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

120%, are greater than each value of x (old) and yam (old), that is, the biomass of cassava and yam when growth rates are varied together is greater than the model parameter values are fixed at 100%. Also, we have established the conditions for stability, asymptotic stability, and its uniqueness.

Conclusion

In this study, a mathematical model was proposed to study the interaction between Cassava and Yam biomass, and Ecospheric assets using a system of non-linear ordinary differential equations. The study aim to quantify the effect of decreasing and increasing the growth rates of cassava and yam together and to check the pattern of biodiversity loss and biodiversity gain. The ordinary differential equation of order 45 (ODE45) was applied in this research. Though continuous biodiversity loss is as a result of decreasing the growth rate coefficients and other natural factors like the infertility of the soil, continuous cultivation is a major threat to our Cassava and Yam assets growth and socio-economic development in Africa. These problems cannot be totally removed or eliminated, however, the administrator, stakeholder, and government need to take appropriate measures to minimize the effect of reducing the yield of these assets for the overall benefit of mankind. Some of these measures are suggested based on the results of this work:

1. Socio-Economic policies that encourage biodiversity loss should be corrected to suit the growth of cassava and Yam asset.
2. There should be an appropriate measure by the local communities and government at all levels to protect and also preserve the parameters that enhance the growth of Cassava and Yam production.

References

Abanum, G., Omoregbe, C., & Ekakaa, N. (2020). Numerical simulation of biodiversity loss: comparison of numerical methods. *International Journal of Mathematics Trends and Technology*, 66(3), 53-64.

Abanum, G., & Eli, I. (2022). Computational and mathematical modeling of industrial assets: Alternative numerical approach. *Asian journal of pure and applied mathematics*, 4(2), 45-59.

Abanum, G., Eli, I., & Iweobodo, D. (2024). Computational Modelling of Agricultural Assets. *Universal Journal of Applied Mathematics*, 12(1), 1-17. DOI: 10.13189/ujam.2024.120101.

Abanum, G., & Oke, K. (2019). Modified variation iteration method for the solution of seventh order boundary value problem using canonical polynomial. *International journal of modern mathematical science,* 17(1), 57-67

Agyemang, I., & Freedman, I. (2009). An Environmental Model for the Interactions of Industry with two Competing Agricultural Resources. *Mathematical and Computer Modelling, ELSEVIER*, 49, 1618-1643.

Agyemang, I., Freedman, I., & Macki, J. (2007). An ecospheric recovery model for agriculture industry interactions. *Diff. Eqns. Dyn. Syst*, 15, 185-208.

Antle, J., & Valdivia, R. (2006). Modelling the Supply of Ecosystem Services from Agriculture: A Minimum Data Approach. *The Australian journal of agricultural and resource economics,* 50, 1-15.

Donatelli, M., Mgarey, D., Bregagtio, S., Willecquet, L., Wish, J., & Savary, S. (2017). Modelling the Impacts of Pests and Diseases on Agricultural Systems. *Agricultural System, Elsevier* 155, 213-224.

Eli, I., & Abanum, G. (2020). Comparison between analytical and numerical result of stability analysis of a dynamical system. *Communication in physical sciences*, 5(4), 473-440

Eleki, G., & Ekaka, E. (2019). Computational Modelling of Ecospheric assets: alternative Numerical Approach. *African Scholar Journal of Sustainable Development*, 15(2), 105-115.

Eleki, G., & Ekakeaa, E. (2019). Differential Effects of a Scenario of the non-additive environmental perturbation on biodiversity of the Ecospheric Assets: Kalabari Kingdom Ecosystem. *African Scholar Publications and Research Internationally*, 1-15.

Imoni, S., Edogbayan, H., Badaki, J., Oyem, A., & Koffa, J. (2019). Existence and Uniqueness Analysis of Transmission Model of Schistosomiasis. *FUW Trends in Science and Technology Journal*, 4(2), 530-533.

Iweobodo, D., Abanum, G., Ochonogor, N., Apanapudor, J., & Njoseh, I. (2024). A Galerkin Finite Element Technique With Iweobodo-Mamadu-Njoseh Wavelet (IMNW) Basis Function For The Solution of Fractional Advection-Diffusion Problems. *Partial Differential Equations in Applied Mathematics*, 12. https://doi.org/10.1016/j.padiff.2024.100965.

Lanlege I., & Imoni, S. (2018). Using Mathematical Modelling of Understanding the Cause and theWay to Prevent Outbreak Diseases (Zika Virus), *Abacus, Mathematical Science Series*, 46(1), 239-257.

Liu, P. (2010). An Analysis of a Predator-Prey Model with Diffusion and Migration. *Mathematical and Computer Modelling, Elsevier*, 51, 11064-1070.

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868

Li, Z., Liang, Z., & Yan, Y. (2017) High-order Numerical Methods for Solving Time Fractional Partial Differential Equations. *Scientific Computing*, 71(2), 785-803.

Mittelbach, G. (2012). *Community Ecology*. Sinauer Associates, Sunderland, Massachusetts.

Morin, P. J. (2002). *Community Ecology*.Blackwell Science, Inc.

Quinn, J., Brandle, J., & Johnson, J. (2013). A farm-scale biodiversity and ecosystem services assessment tools: The healthy farm index. *International journal of sustainability*, 11(2), 176-192.

Vandermeer, J., & Goldberg, D. (2013). Population Ecology: First Principles. 2nd edition, Princeton University Press, Princeton.

Wang, Z. H. (2008). Numerical stability test of neutral delay differential equations. *Mathematical problems in Engineering,* 3(4), 1-10. Dio:10.1155/2008/698043.

Cite this article as:
Abanum ,G. C., Iweobodo, D. C., Ogoegbulem ,O.,& Imoni, S. O. (2025). A mathematical model of competition between cassava and yam. *FNAS Journal of Mathematical Modeling and Numerical Simulation, 2*(3), 96-112. https://doi.org/10.63561/jmns.v2i3.868