



Mathematical Modelling of the Fifth Law of Library Science

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Abstract

Over the years, most students see Mathematics as a chain of abstract ideology that has no application in real life. These beliefs tend to make the course more difficult to learn, as memorization and deduction seemed impossible. Mathematical modelling has increasingly become a valuable tool in different disciplines, library and information science inclusive. It helps to quantify and predict library operations, user behaviors, and resource utilization. The principles of library science have been foundational in shaping the organization, management, and accessibility of information. In 1931, Shiyali Ramamrita Ranganathan, an Indian mathematician and librarian, formulated five laws of library science that have since become cornerstones of modern library practice. Among these, is the Fifth Law of Library Science which states that; “The library is a growing organism” captures the dynamic, evolving nature of libraries as institutions that must adapt to changing societal, technological, and informational needs. This study focuses on the application of Mathematical modelling to the fifth law. The paper also discusses various mathematical frameworks, including differential equations and growth models that can be employed to simulate the continual development of libraries

Keywords: Mathematical Modelling, Fifth Law, Library Science, Library Growth, Differential Equations.

Introduction

Library science is the study of the principles, practices, and systems used to manage and organize information in libraries. It encompasses a wide range of topics, including cataloging, classification, information retrieval, information preservation etc. Library science is a multidisciplinary field that draws on elements of information science, archival studies, and computer science, with an emphasis on improving access to knowledge and resources for users. Over the years, library science has developed several guiding principles and laws that help librarians and information professionals navigate the complex process of managing and sharing information.

The Principles and Laws of Library Science

One of the foundational elements of library science is the development of a set of laws or guiding principles that reflect how libraries should operate to best serve their users. These laws were first articulated by S. R. Ranganathan, an Indian librarian and scholar, in his landmark work *The Five Laws of Library Science* (Ranganathan, 1931). These laws have shaped modern library practices and continue to be influential in how libraries are run today.

The Five Laws of Library Science

The five laws of library science simply states that:

- (i) **Books are for use:** This law emphasizes that the primary purpose of a library is to provide access to information, and that materials should be easily accessible to users.
- (ii) **Every reader his or her book:** This law highlights the importance of matching users with the materials that meet their needs, ensuring that everyone has access to the right information.
- (iii) **Every book its reader:** This principle complements the second law by asserting that every item in the library’s collection has value for some user, whether it’s an academic researcher or a casual reader.
- (iv) **Save the time of the reader:** Libraries should be organized in a way that allows users to find information quickly and easily, reducing unnecessary effort or confusion.

- (v) **The library is a growing organism:** This law emphasizes the dynamic nature of libraries. It asserts that libraries should continuously evolve and expand, adapting to changes in technology, user needs, and the broader information environment.

Focusing on the Fifth Law: "The Library is a Growing Organism"

The fifth of Shiyali Ramamrita Ranganathan's Five Laws of Library Science states, "The library is a growing organism." This law is perhaps the most dynamic and forward-thinking of all the principles in library science. It emphasizes that libraries should be viewed not as static institutions, but as living, evolving entities that must continuously adapt to meet the changing needs of users, advances in technology, and shifts in cultural and societal contexts (Ranganathan, 1931). This law has profound implications for library management, service delivery, and the future of libraries.

Understanding the Fifth Law

The metaphor of a "growing organism" suggests that a library is like a living being, with needs for nurturing, development, and adaptation. Just as organisms grow in response to environmental changes and internal development, libraries must also evolve to remain relevant, effective, and valuable to their communities. The law implies that libraries should be dynamic, continuously expanding their collections, services, and roles in response to new information, changing user demands, and societal trends (Rath, 2014). In practical terms, this law advocates for a library system that:

- Adapts to technological advancements: Libraries must integrate new technologies, from digital cataloging systems to online resources, e-books, and databases (Emmanuel, 2024).
- Expands collections and services: Libraries should actively grow their collections, acquiring new resources that reflect the interests and needs of their community. This could include both physical and digital materials (Opele, 2023).
- Responds to changing user needs: A library should always consider how it can better serve its users, including diversifying its programs, providing access to emerging content types, and offering personalized services (Rath, 2014).
- Engages with evolving cultural and social contexts: Libraries must reflect the cultural diversity of the communities they serve and evolve in tandem with changing social, political, and economic landscapes (Emmanuel, 2024).

Implications for Library Management

The "growing organism" law has direct implications for library management. It calls for a library to be a flexible, forward-thinking institution that fosters a culture of continuous improvement. Below are key areas where this law impacts library management:

- **Collection Development and Acquisition**
A library's collection should be seen as a living body of knowledge that requires constant nurturing and renewal. The "growing organism" law stresses that libraries must constantly update their collections to meet evolving user needs. This means acquiring new materials, weeding out outdated or irrelevant items, and integrating new formats such as e-books, online databases, and multimedia content. The growth of a library's collection should reflect changing areas of knowledge, emerging academic fields, and community interests (Rath, 2014).
- **Staff Training and Professional Development**
A growing library requires a staff that is adaptable, knowledgeable, and open to learning. Librarians and library staff must continuously update their skills to keep up with technological changes (such as digital cataloging, virtual reference, or data management tools) and evolving best practices in user service. Staff development programs, professional conferences, and continuous learning are essential for fostering an environment where growth and innovation are prioritized (Amna, 2021).
- **Technology Integration**
As an organism grows and adapts to its environment, so must a library integrate new technologies to better serve its users. This might include implementing digital catalogs, providing access to online resources, integrating virtual reference services, and exploring artificial intelligence to enhance user experience (Opele, 2023). Libraries also need to manage and preserve digital collections, which requires new forms of expertise in data curation, metadata, and digital preservation (Emmanuel, 2024).
- **Responsive to User Needs**
A library that functions as a growing organism must also be responsive to the changing needs of its users. This means regularly gathering feedback from the community, assessing the evolving demographics, and being attuned to shifts in educational, cultural, and social needs. Libraries may need to adjust their programs and services based on user feedback, introducing new services like maker spaces, study pods, community events, and educational workshops (Mirna & Lofty, 2022).

- **Community Engagement**

Libraries must grow in connection with the communities they serve. Community engagement is crucial to ensure that libraries evolve in ways that are both relevant and inclusive. This includes offering programs and services for diverse populations, providing resources in different languages, supporting local culture, and addressing the information needs of all segments of society whether they are students, job seekers, seniors, or marginalized groups.

- **Space and Infrastructure Management**

Just as an organism requires space to grow, a library must also evolve in terms of physical space and infrastructure. Libraries need to plan for future growth, which may involve expanding physical facilities, repurposing underused spaces (e.g., converting traditional reading rooms into collaborative spaces), and redesigning spaces to accommodate new technologies (e.g., multimedia labs, VR spaces, etc.). Flexible, user-centered space planning becomes an essential part of library management to support the continual growth and adaptability of the institution (Negi, 2023).

- **Sustainability and Preservation**

- Growth doesn't only imply expansion but also sustainability. Libraries must find a balance between new acquisitions and the preservation of valuable and historical materials. This involves maintaining archives, digitizing older works, and ensuring that the library's collections can be accessed and preserved for future generations. The library must be aware of the challenges of resource preservation and management in both physical and digital formats (Mirna & Lofty, 2022).

Implications for Library Services

The "growing organism" law is also highly relevant for the types of services libraries provide to their communities. As libraries continue to grow, the following service-oriented practices become increasingly important:

- (i) **Access to Information:** The library must ensure its information systems are continually updated to allow seamless access to the growing body of knowledge. This means that library users should have easy access to both print and digital resources, wherever they are located. Cloud-based systems, remote access platforms, and interlibrary loan services should be part of the library's growing infrastructure (Opele, 2023).
- (ii) **Diversity of Formats:** The diversity of information formats is an essential aspect of a growing library. Libraries should provide not only books and journals but also digital media, databases, podcasts, online videos, and interactive content. This helps meet the varying preferences of users who may prefer e-books, audiobooks, or multimedia learning resources (Rath, 2014).
- (iii) **User-Centered Services:** The law encourages libraries to be more user-centered, continuously adapting to the diverse needs and desires of their community. Personalized services like tailored reading recommendations, learning resources, or even community-based programs allow libraries to stay relevant. Additionally, the development of special services for underrepresented groups (such as senior citizens, students, or non-native speakers) ensures that libraries remain inclusive spaces (Tom, 2010).
- (iv) **Collaboration and Partnerships:** Libraries should also embrace collaboration, partnering with schools, universities, community organizations, and even tech companies to enhance their services and reach. Through partnerships, libraries can offer specialized services like tutoring, career advice, or access to specialized content, fostering a more holistic approach to community support (Gubbin, 2010).

Basics of Mathematical Modelling: Introduction to Mathematical Modelling Concepts

Mathematical modelling is a fundamental tool used to represent real-world phenomena through mathematical structures and equations. It involves translating complex systems or processes into mathematical formulations, which can then be analyzed, solved, and used to make predictions or inform decision-making (Sefa, 2012). Mathematical models are widely applied in various disciplines, from physics and engineering to economics, biology, and social sciences. By simplifying and quantifying complex systems, mathematical modelling has become an essential technique in many scientific and practical applications.

Key Concepts in Mathematical Modelling

- (i) **Definition of Mathematical Modelling:** A mathematical model is a representation of a system or process in the form of mathematical concepts and language. The goal of a mathematical model is to describe the relationships between the various components of the system, often resulting in equations that can be solved or analyzed to predict behavior or outcomes (Srivastava, 2021). The

process of creating a mathematical model involves translating real-world phenomena into assumptions, variables, and equations.

- (ii) **Components of a Mathematical Model:** A typical mathematical model consists of several key components:
- **Variables:** These are the quantities that are measured or controlled in the system. They can be independent (inputs) or dependent (outputs).
 - **Parameters:** These are constants or coefficients that define the system's characteristics and remain fixed during the modelling process (Srivastava, 2021).
 - **Equations:** These describe the relationships between the variables and parameters, often taking the form of algebraic, differential, or integral equations (Srivastava, 2021).
 - **Boundary Conditions/Initial Conditions:** These are constraints or starting values that help solve the equations in a specific context, ensuring the model reflects the real-world scenario (Blum, 2009).
- (iii) **The Modelling Process:** The process of mathematical modelling typically follows these steps:
- **Problem Definition:** Understanding and clearly defining the problem or phenomenon to be modeled (Tzafiriri, 2003).
 - **Assumptions and Simplifications:** Making necessary assumptions to simplify the real-world system to a manageable mathematical framework (Sefa, 2012).
 - **Formulating the Model:** Translating the assumptions into mathematical equations (Tzafiriri, 2003).
 - **Solving the Model:** Using mathematical techniques to solve the equations, either analytically or numerically (Allen, 2018).
 - **Validation:** Comparing the model's predictions with real-world data to ensure its accuracy and relevance (Allen, 2018).
 - **Analysis and Interpretation:** Using the model to gain insights into the system and make predictions or guide decisions (Sefa, 2012).

Types of Mathematical Models

Mathematical models can be broadly classified based on their structure, purpose, and application:

- (i) **Deterministic vs. Stochastic Models:** Deterministic Models: These models assume that outcomes are precisely determined by initial conditions and parameters. There is no randomness or uncertainty in the system. For example, deterministic models are commonly used in physics to describe systems under known forces (Tzafiriri, 2003).
- (ii) **Stochastic Models:** These models incorporate randomness and uncertainty, acknowledging that outcomes can vary even with identical starting conditions. Stochastic models are widely used in fields like finance, biology, and queueing theory (Allen, 2018).
- (iii) **Continuous vs. Discrete Models:**
- **Continuous Models:** These models describe systems where changes occur continuously, such as the population growth of a species or the flow of fluids. The variables are treated as continuous functions of time or space, often leading to differential equations (Last, 2007).
 - **Discrete Models:** These models are used for systems where changes happen in distinct steps, such as the number of people in a queue or the outcomes of a series of events. Discrete models are often formulated using difference equations or algorithms (Srivastava, 2021).
- (iv) **Static vs. Dynamic Models**
- I. **Static Models:** These models represent systems at a specific point in time or in equilibrium. They do not account for changes over time, such as static economic models that describe equilibrium states at a moment (Burgess, 2023).
 - II. **Dynamic Models:** These models describe systems that evolve over time and typically involve differential or difference equations that describe how the system changes. Dynamic models are used to model phenomena like population dynamics, economic growth, or the spread of diseases (Burgess, 2023).
- (v) **Linear vs. Nonlinear Models**
- **Linear Models:** In these models, the relationships between variables are linear, meaning that changes in one variable produce proportional changes in another. Linear models are simpler to analyze but may not capture the complexity of many real-world systems (Michael, 2013).

- **Nonlinear Models:** These models involve relationships where changes in variables do not produce proportional responses. Nonlinear models are used to describe systems with feedback loops, chaos, and complex dynamics, such as in climate models or epidemic modelling (Michael, 2013).

Applications of Mathematical Models

Mathematical models are used in a variety of fields to help scientists, engineers, economists, and decision-makers make predictions, optimize systems, and solve complex problems.

- (i) **Physics and Engineering:** In physics, mathematical models describe the behavior of physical systems under various conditions. For instance, models of motion, thermodynamics, and electromagnetism are based on mathematical equations such as Newton's laws and Maxwell's equations (Rasa, 2022). In engineering, these models are used to design systems and predict performance, such as in fluid dynamics or structural engineering.
- (ii) **Biology and Medicine:** Mathematical models are extensively used in biology to describe processes like population dynamics, ecological interactions, and the spread of diseases. Models such as the SIR (Susceptible-Infected-Recovered) model are widely used to understand and predict the spread of infectious diseases, including the COVID-19 pandemic (Padamwar, 2019). In medicine, mathematical models optimize resource allocation, such as vaccine distribution or the scheduling of surgeries (Padamwar, 2019).
- (iii) **Economics and Finance:** In economics, mathematical models are used to study market dynamics, economic growth, and policy impacts. The IS-LM (Investment Saving-Liquidity Preference Money Supply) model helps explain macroeconomic equilibrium (Wainwright, 2005). In finance, models like the Black-Scholes model are used to price options, while stochastic models predict stock prices and manage risk in investment portfolios (Mikosch, 2014).
- (iv) **Environmental Science:** Environmental models are essential for understanding and predicting human impacts on the environment. Climate models, for instance, use differential equations to simulate temperature changes and greenhouse gas concentrations (Suguntha, 2024). Other models study the dynamics of ecosystems and the effects of pollution or conservation strategies (Justin, 2019).
- (v) **Social Sciences:** Mathematical models are used in the social sciences, particularly in sociology, psychology, and political science. Models of social networks, for example, can predict the spread of behaviors or information (Burgess, 2023). Game theory, which is a mathematical framework, is applied in political science and economics to study strategic interactions among rational agents (Sefa, 2012).
- (vi) **Operations Research and Management:** In operations research, mathematical models are used to optimize processes like cost minimization, efficiency maximization, or logistical improvements. Linear programming models are commonly used to determine the best allocation of resources, while queuing theory models help manage waiting times in systems like hospitals or call centers (Allen, 2018).

Growth Model

A growth model is a mathematical representation that describes how a quantity grows over time. In various fields, such as economics, biology, and business, growth models help analyze and predict the behavior of populations, economies, or companies.

Types of Growth Models

Mathematically, growth models are basically classified into three, namely; linear growth model, exponential growth model and logistic growth model.

- (i) **Linear Growth Models:**
Linear growth models describes a constant amount of growth over time. The mathematical equation of such model is given by:

$$y(t) = y_0 + kt$$
 Where: $y(t)$ is the quantity at a time t .
 y_0 is the initial quantity, and k is the constant growth rate.
- (ii) **Exponential Growth Model:**
The exponential growth model describes growth that occurs at a constant percentage rate. The equation of such growth rate is:

$$y(t) = y_0 e^{rt}$$
 In this case r is the growth rate while e is the base of the natural logarithm.
- (iii) **Logistic Growth Model:**

The logistic growth model describes a growth that starts exponentially but slows as it approaches a maximum limit (carrying capacity). The equation is given as:

$$y(t) = \frac{K}{1 + \frac{K - y_0}{y_0} e^{-rt}}$$

Where: K is the carrying capacity and r is the growth rate.

Exponential growth is a fundamental concept across various fields, including biology, economics, and population dynamics. The factors necessary for exponential growth include:

- (i) **Constant Growth Rate:** Exponential growth requires a growth rate that is consistent over time. This ensures that the rate of increase is proportional to the current value, leading to rapid growth as time progresses (Rowthorn, 2019).
- (ii) **Unlimited Resources:** For exponential growth to persist, the system must have access to abundant resources. In biological populations, this typically means an environment free from resource shortages, where food, space, and other necessities are readily available (Rowthorn, 2019). In practice, this condition is rarely met, which eventually leads to limiting factors.
- (iii) **Positive Feedback Loop:** Exponential growth often relies on positive feedback, where the growth of the population or quantity accelerates as it increases. In population biology, this is seen when each new individual in a population contributes to further reproduction, leading to increasingly rapid growth (Padamwar, 2019; Hans-Stefan, 2022).
- (iv) **Absence of Constraints:** Exponential growth assumes the absence of significant constraints such as predation, disease, or competition. Once these factors come into play, growth shifts to a more controlled pattern, often transitioning to logistic growth. These constraints become more significant as the population or quantity increases.
- (v) **Time:** Exponential growth occurs over time and the longer the time frame, the greater the increase in the quantity. This continuous compounding of growth makes exponential growth particularly impactful over long periods (Barro, 2015).
- (vi) **Initial Population or Quantity:** A starting value is crucial in exponential growth. Larger initial populations or quantities lead to more rapid increases, as the rate of growth is proportional to the size of the current quantity (Kormondy, 2013).

Issue Statement

- **Lack of Metrics:** Existing qualitative interpretations of the Fifth Law lacks measurable parameters to evaluate the dynamics of libraries systematically.
- **Sustainable Challenges:** Financial and physical constraints limits the unrestricted (exponential) growth,
- **Unpredictable growth pattern:** Libraries often expands their collections and services without a clear understanding of the long-term impact on space, infrastructure, and financial resources.
- **User Resource Mismatch:** Growing Library resources do not always correlate with proportional growth in user engagement, leading to inefficiencies.

Modelling of Library Growth using Differential Equations

- (i) **Definition of Variables:**
 - Let $G(t)$ represent the expected population growth of patrons at a time “ t ”
 - Let r represent the growth rate of the population of patrons.
 - Let “ G ” denotes the current population size of patrons.
- (ii) Assume the population of patron grows at a rate proportional to its current population size.
 - The above assumption can be mathematically expressed as:

$$\frac{dG}{dt} = rG$$
- (iii) Separate the variables to solve the differential equation:
 - The above equation can be rewritten as:

$$\frac{1}{G} \frac{dG}{dt} = r$$
 - Separate the variables G and t

$$\frac{1}{G} dG = r dt$$
- (iv) Integrate both sides:
 - Integrate the left side with respect to G and right side with respect to t :

$$\int \frac{1}{G} dG = \int r dt$$
 - Then integrals are:

$$\ln|G| = rt + C$$
 Where C is the constant of integration.

(v) Solve for G :

- Exponentiate both sides to solve for P ;
 $|G| = e^{rt+C}$
- Simplify the expression:
 $|G| = e^{rt} \cdot e^C$
- Let $e^C = G_0$, where G_0 is the initial population at the initial time $t = 0$
 $G = G_0 e^{rt}$

Thus, growth model $G(t) = G_0 e^{rt}$ is derived using differential equation. The model shows that the population $G(t)$ at any time t is equal to the product of the initial population by the exponential function of the growth rate and time.

The above population growth model is also called exponential growth phase. Initially, when resources are abundant, population grows rapidly, leading to J-shaped curve (Gause, 1934)

Modification of the population growth model to include carrying capacity

The exponential growth model above assumes unlimited resources, which is not realistic for most population. The logistic growth model describes how population grow in environments with limited resources, emphasizing the concept of carrying capacity, which is the maximum population size that an environment can sustain. To account for the carrying capacity, the research introduce a factor that slows down growth as the population approaches the carrying capacity “ K ” (Reed, 1920) . This factor is $(1 - \frac{G}{K})$.

Logistic Growth Differential Equation

The logistic growth model (Murray, :2002) is given by:

$$\frac{dG}{dt} = rG \left(1 - \frac{G}{K}\right)$$

Simplify the Equation

$$\frac{dG}{dt} = rG - \frac{rG^2}{K}$$

Solve the Logistic Growth Differential Equation

To solve the differential equation, there is need to use the method of separation of variables.

(i) Separate the variables:

$$\frac{dG}{G(1-\frac{G}{K})} = r dt$$

The left-hand side is the same as:

The left hand-side requires partial fraction decomposition

(ii) Simplify the left-hand side;

$$\frac{dG}{G(1-\frac{G}{K})} = \frac{dG}{G} + \frac{dG}{K-G}$$

(iii) Integrate both sides

$$\int \left(\frac{1}{G} - \frac{1}{K-G}\right) dG = \int r dt$$

(iv) Solve the integrals:

$$\ln|G| - \ln|K - G| = r t + C$$

Where C is the constant of integration

(v) Combine the logarithms:

$$\ln \left| \frac{G}{K-G} \right| = r t + C$$

(vi) Exponentiate both sides to solve for G :

$$\frac{G}{K-G} = e^{rt+C}$$

Let $e^C = A$, where A is a new constant.

$$\therefore \frac{G}{K-G} = A e^{rt}$$

(vii) Solve for G :

$$G = \frac{AK e^{rt}}{1 + A e^{rt}}$$

(viii) Determine the constant A using the initial condition $G(0) = G_0$:

$$G_0 = \frac{AK}{1+A}$$

Solving for A :

$$A = \frac{G_0}{K-G_0}$$

(ix) Substitute A back into the equation for G :

$$G(t) = \frac{K \frac{G_0}{K-G_0} e^{rt}}{1 + \frac{G_0}{K-G_0} e^{rt}}$$

(x) Simplify the expression:

$$G(t) = \frac{K G_0 e^{rt}}{K - G_0 + G_0 e^{rt}}$$

$$G(t) = \frac{K G_0 e^{rt}}{K + G_0 (e^{rt} - 1)}$$

Finally the logistic growth model is:

$$G_2(t) = \frac{K G_0 e^{rt}}{K + G_0 (e^{rt} - 1)}$$

This equation describes how the population $G_2(t)$ grows over time, taking into account the carrying capacity K and the initial population size G_0 .

Note:

$$r = \frac{G_0}{K} \times 100\%$$

Simulation:

S/N	Capacity (K)	Present Population (G ₀)	Rate (r) $r = \frac{G_0}{K} \times 100\%$	Time (t) in years	$G_1(t) = G_0 e^{rt}$	$G_2(t) = \frac{K G_0 e^{rt}}{K + G_0 (e^{rt} - 1)}$
1	300	100	33.33%	1	140	123
2	300	100	33.33%	2	195	148
3	300	100	33.33%	3	272	173
4	300	100	33.33%	4	379	196
5	300	100	33.33%	5	529	218
6	300	100	33.33%	6	739	236
7	300	100	33.33%	7	1031	251
8	300	100	33.33%	8	1439	263
9	300	100	33.33%	9	2008	273
10	300	100	33.33%	10	2802	280

$$r = \frac{200}{300} \times 100 = 66.67\% = 0.6667$$

Discussion

The Library of three hundred (300) seating capacity (K) was used, the initial population of patrons (G_0) is one-hundred(100), at the growth rate of 33.33% was also used, within the time interval of ten years. The result shows that; the growth of the Library $G_1(t)$ without constrains exceed the carrying capacity within the first four years while the growth of the Library $G_2(t)$ with constrains, did not exceed the carrying capacity up to the tenth year. This implies that, every Library’s growth can be calculated, with or without a carrying capacity.

Conclusion

The Fifth Law of Library Science remain as relevant today as it was when it was first proposed. Libraries are, indeed, growing organisms that must continuously grow, evolve, and adapt to meet the needs of their users. The digital age has amplified this need for growth, as libraries embrace new technologies, expand their services, and redefine their roles within society. The growth of the library can be evaluated and calculated. Mathematical Model derived can be used to calculate the growth of the library.

Suggestions

- (i) The growth of the library $G_1(t)$ can be calculated using $G_1(t) = G_0 e^{rt}$, in situations where there are no constraints.
- (ii) The growth of the library $G_2(t)$ can be calculated using $G_2(t) = \frac{K G_0 e^{rt}}{K + G_0 (e^{rt} - 1)}$, in situations where there are constraints.
- (iii) The growth rate of the patrons is $r = \frac{G_0}{K} \times 100\%$. Where K is the carrying capacity the Library.

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