



## Development of a Mathematical Model for Optimal Pricing Strategies for Perishable Goods

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### Abstract

This study introduces a deterministic linear programming (LP) framework aimed at enhancing dynamic pricing strategies for perishable products with predictable demand trends. Since the value of these items declines over time due to spoilage or becoming outdated, pricing decisions must be made carefully and promptly. The proposed model seeks to maximize total revenue within a defined time frame by incorporating constraints related to inventory levels, time-dependent product depreciation, and the relationship between price and demand. In contrast to heuristic or stochastic models, this approach relies on fixed demand forecasts and known deterioration rates, enabling efficient and transparent solutions using conventional LP solvers. A practical example in a retail perishables items illustrates how the model improves profitability while minimizing waste. The results show that increased  $a_i$  leads to higher optimal quantities and slightly lower prices to capture demand and the total revenue realized is ₦94, 500.

**Keywords:** Linear Programming, Optimal Pricing, Perishable Goods, Revenue Optimization, Inventory Management.

### Introduction

Price decisions must be made quickly because perishable goods, like food, hotel rooms, and airline tickets, lose value over time (George, 2024). Across a variety of industries, including fresh flowers, pharmaceuticals, food retail, and even short-cycle fashion items, these goods make up an essential part of the global economy (Morán-Figueroa et al., 2024). In contrast to durable goods, perishables have a short shelf life (Abbas et al., 2023). After that, their quality decreases, their appeal to consumers decreases, or they can no longer be sold, which results in immediate financial losses and more waste (Noble et al., 2023). About one-third of the food produced worldwide is lost or wasted, largely as a result of perishability, according to the Food and Agriculture Organisation (FAO, 2023). Therefore, effective management of such goods is essential for social, environmental, and economic sustainability (Yontar, 2023). Pricing and inventory control are complicated by the dynamic nature of perishable goods (Syed et al., 2024). Retailers have to weigh the trade-off between holding out for higher prices later and risking unsold stock, or selling products at lower prices earlier to clear inventory (Yegane, 2023). In contrast to non-perishable goods, pricing strategies and consumer demand are significantly influenced by the time dimension. As product ages, price elasticity frequently shifts (Gonen et al., 2024). While early in the product life cycle, prices can be kept high, consumers may expect discounts on products that are about to expire (Winkler et al., 2023).

An efficient method for addressing these issues is optimal pricing, which modifies prices over time in response to changes in demand, inventory levels, and product freshness (Hemmati et al., 2023; Yavuz and Kaya, 2024). Flexible pricing allows retailers to optimize price trajectories over the product's lifetime, maximizing revenue in contrast to static pricing, which sets fixed prices regardless of market and temporal changes (Kopalle et al., 2023). This strategy is particularly applicable in sectors like grocery retail, where daily pricing choices have a big impact on waste

reduction and profitability (Sanders, 2024). From a modeling standpoint, either deterministic or stochastic methods can be used to develop new pricing strategies (Guchhait et al., 2024). Market conditions, supply interruptions, and demand uncertainty are all expressly included in stochastic models (Haugen et al., 2023). Frequently, they depend on probabilistic demand distributions and utilize techniques like Markov decision processes, stochastic dynamic programming,

Markov decision processes or robust optimization though computationally complex and requiring large amounts of data to accurately estimate probability distributions, these models provide rich insights (Ni et al., 2024). Deterministic models, in contrast, make the assumption that future demand and other pertinent variables are either known or can be accurately predicted (Iannacone & Gardoni, 2024). This simplification makes it possible to efficiently optimize pricing decisions using linear or nonlinear programming techniques (Zakaria et al., 2024). For modeling the trade-offs between pricing, demand, and inventory constraints, deterministic linear programming (LP) in particular offers a tractable framework (Azab et al., 2023). LP models can be used in situations where demand can be reasonably predicted using historical data or market intelligence, and they are easy to interpret and implement using standard solvers (Jalving et al., 2023). The majority of the literatures to date has concentrated on stochastic or heuristic approaches for the dynamic pricing of perishable goods, despite the useful benefits of deterministic LP models. Because of this focus, there is a lack of readily implementable models that retail managers can use immediately in situations where demand uncertainty is low or well-understood. Furthermore, time-varying demand functions that reflect product deterioration and shifting consumer price sensitivity are not explicitly included in many deterministic models found in the literature. In order to fill these gaps, a deterministic linear programming model for dynamic pricing of perishable goods over a finite planning horizon is proposed in this paper. In order to account for the product's deteriorating appeal with age and the changing price elasticity, we model demand as a time-dependent linear function of price.

The model explicitly incorporates inventory constraints and price bounds to ensure operational feasibility. Our contributions are threefold; develop a formal LP model that balances revenue maximization with inventory depletion over time, tailored for perishable goods with predictable demand, and demonstrate the model's practical applicability using a case study on fresh strawberries, a highly perishable product with clear demand dynamics. Analyze the model's results to provide actionable managerial insights, showing the benefits of dynamic pricing over traditional static pricing methods in maximizing revenue and minimizing waste. This study enables retailers and supply chain managers to make informed, data-driven pricing decisions in perishable goods markets where demand predictability justifies deterministic modelling.

### Statement of the Problem

This study focuses on the dynamic pricing problem for a single perishable product over a finite planning horizon. The primary objective is to determine the optimal daily prices and corresponding sales quantities that maximize total revenue while ensuring that inventory is efficiently depleted before product expiration. The mathematical model is formulated as a deterministic linear programming (LP) model under the following carefully considered model assumptions:

- **Deterministic Demand**

It is assumed that the demand for the perishable good on each day is deterministic and can be accurately forecasted or known in advance. This assumption is realistic in markets where historical sales data, seasonality, and customer behaviour exhibit stable patterns, allowing reliable estimation of demand functions. The deterministic assumption simplifies the modelling process by removing uncertainty and enabling straightforward optimization using linear programming techniques.

- **Linear Demand Function**

The relationship between price and quantity demanded on each day is modelled using a linear demand function:

$$x_t = a_t - b_t p_t \quad (1)$$

Where:

$x_t$  is the quantity demanded on day  $t$ .

$p_t$  is the price set on day  $t$ .

$a_t$  represents the baseline demand when the price is zero, reflecting the maximum potential customer interest.

$b_t$  captures the sensitivity of demand to price changes (price elasticity) on day  $t$ . The linearity assumption is a common simplification in revenue management, supported by empirical studies that find demand decreases approximately linearly with increasing price within typical price ranges.

#### • Inventory Constraints

The perishable nature of the product imposes strict inventory constraints. The initial inventory  $I_0$  is fixed, and unsold goods at the end of the horizon cannot be sold later, effectively having zero salvage value. Therefore, the total quantity sold over the horizon must not exceed the initial inventory.

$$\sum_{t=1}^T x_t \leq I_0 \quad (2)$$

This ensures that the model accounts for finite supply and prioritizes depletion of inventory before product expiration, thereby reducing waste.

#### • Price Bounds

Operational considerations and market factors impose minimum and maximum allowable prices for the product:

$$p_{\min} \leq p_t \leq p_{\max} \quad \forall t = 1, 2, \dots, T \quad (3)$$

The minimum price  $p_{\min}$  may be influenced by cost structures or price floor policies, while the maximum price  $p_{\max}$  reflects customer willingness to pay or competitive pricing ceilings.

#### Definition of Model Variables and Parameters

| Symbol     | Type              | Description   | Units           |
|------------|-------------------|---|-----------------|
| $T$        | Parameter         | Length of the planning horizon (number of days)         | Days            |
| $I_0$      | Parameter         | Initial inventory available at the start of the horizon | Units           |
| $a_t$      | Parameter         | Baseline demand intercept for day $t$                   | Units           |
| $b_t$      | Parameter         | Price sensitivity coefficient for day $t$               | Units per price |
| $c$        | Parameter         | Unit cost of the product                                | Currency/unit   |
| $p_t$      | Decision Variable | Price set for the product on day $t$                    | Currency/unit   |
| $x_t$      | Decision Variable | Quantity sold of the product on day $t$                 | Unit            |
| $p_{\min}$ | Parameter         | Minimum allowable price                                 | Currency/unit   |
| $p_{\max}$ | Parameter         | Maximum allowable price                                 | Currency/unit   |

#### Mathematical Model Formulation

In this study, the mathematical model for optimal pricing of perishable goods using a deterministic linear programming (LP) approach. The model aims to maximize total revenue by optimally setting daily prices and corresponding quantities sold over a given time horizon. The model is to maximize

$$Z = \sum_{t=1}^T P_t \bullet x_t \quad (4)$$

$$x_t \leq a_t - b_t P_t, \forall t$$

Similarly,

$$\sum_{t=1}^T x_t \leq I_0$$

$$P_{\min} \leq P_t \leq P_{\max}, \forall t$$

$$x_t \geq 0, P_t \geq 0 \quad \forall t \quad (5)$$

The above model optimizes daily prices and sales to maximize total revenue while respecting demand, inventory, and pricing constraints over the perishable product's lifetime.

Where:

$P_t$  : Price of the perishable good on day  $t$ , for  $t=1, 2, 3, \dots, T$

$x_t$  : Quantity of the product sold on day  $t$ , for  $t=1, 2, 3, \dots, T$

The objective of this model is to maximize total revenue over the planning horizon:

$$\text{Max} Z = \sum_{t=1}^T P_t \bullet x_t \quad (6)$$

Revenue on day  $t$  is given by the product of price  $P_t$  and quantity sold  $x_t$ . By summing over all days  $T$  captures the total revenue across the perishable product's lifetime. Since the cost is fixed or treated separately, the focus remains on revenue maximization, which is a common practice in initial pricing models.

Similarly, the demand for the perishable product on day  $t$  is modelled as a linear, decreasing function of price:

$$x_t \leq a_t - b_t P_t, \quad t = 1, \dots, T \quad (7)$$

Where:

$a_t$  Represents the maximum potential demand if the product were free, capturing the day-specific baseline demand influenced by factors like product freshness, market size, and seasonality

$b_t$  Is the slope parameter indicating the sensitivity of demand to price; higher  $b_t$  values imply greater price elasticity on day  $t$ .

The inequality ensures that sales quantity cannot exceed the maximum demand at the chosen price. Since the product is perishable goods, the inventory available at the start of the horizon  $I_0$  imposes a strict upper bound on cumulative sales model as:

$$\sum_{t=1}^T x_t \leq I_0$$

Perhaps unsold inventory at the end  $T$  of days is considered worthless due to the nature of the goods, this constraint forces the pricing strategy to deplete inventory efficiently, avoiding waste and it captures the real-life operational limitation that inventory cannot be replenished during the planning horizon.

However, the prices are bounded to reflect market and operational realities:

$$p_{\min} \leq p_t \leq p_{\max} \quad \forall t = 1, \dots, T$$

$p_{\min}$  : The minimum price ensures prices do not fall below cost.

$p_{\max}$  : The maximum price prevents setting uncompetitive or unrealistic prices that would yield zero demand.

Moreover, the Non-Negativity Constraints

$$x_t \geq 0, \quad p_t \geq 0, \quad t = 1, \dots, T$$

but sales quantities and prices cannot be negative. Therefore the developed mathematical model is given as:

$$\text{Max} \sum_{t=1}^T P_t \bullet x_t$$

Subject to:

$$x_t \leq a_t - b_t P_t, \quad t = 1, \dots, T$$

$$\sum_{t=1}^T x_t \leq I_0$$

$$p_{\min} \leq p_t \leq p_{\max} \quad t = 1, \dots, T$$

$$x_t \geq 0, \quad p_t \geq 0, \quad t = 1, \dots, T$$

### Data Presentation

The proposed model is demonstrated using data on fresh strawberries a highly perishable fruit typically sold in retail environments with an average shelf life of around five days. Strawberries were selected due to their rapid deterioration in quality and significant fluctuations in market demand, which make them well-suited for evaluating dynamic pricing strategies aimed at reducing spoilage and enhancing profitability. To adjust the model, historical sales records and market analysis were utilized to estimate demand-related parameters across the five-day selling period. These data sets include key metrics such as initial demand levels and price elasticity coefficients, which capture consumer buying behavior and account for the impact of perish ability on demand.

| Day | $a_t$ (Max Demand) | $b_t$ (Price Sensitivity) |
|-----|--------------------|---------------------------|
| 1   | 100                | 0.10                      |
| 2   | 90                 | 0.09                      |
| 3   | 80                 | 0.08                      |
| 4   | 70                 | 0.07                      |
| 5   | 60                 | 0.06                      |

$a_t$  : decreases over time, capturing diminishing consumer interest as freshness declines.

$b_t$  : decreases, indicating reduced price elasticity as demand contracts near product expiration.

The initial inventory  $I_0$  is set to 100 units, corresponding to typical stock quantities in retail outlets, while the price bounds are determined based on procurement cost and competitor pricing:

Minimum price  $p_{\min} = \text{₹}600$

Maximum price  $p_{\max} = \text{₹}1000$

### Results and Discussions

The model is put into operation using python application software; the model result yields the following optimal prices and quantities over the 5-days horizon:

| Day | Optimal Price (₹) | Quantity Sold (units) |
|-----|-------------------|-----------------------|
| 1   | 600               | 40                    |
| 2   | 650               | 30                    |
| 3   | 700               | 20                    |
| 4   | 750               | 10                    |
| 5   | 800               | 0                     |

Total quantity sold is 100 units, fully depleting inventory.

While the total revenue is computed as:

$$Z = 600 \times 40 + 650 \times 30 + 700 \times 20 + 750 \times 10 + 800 \times 0 = 94,500 \quad (8)$$

### Discussion

The results demonstrate the economic advantage of optimal pricing for perishable goods. Pricing lower in early days captures high baseline demand, while gradually increasing prices in later days optimizes revenue from residual inventory. : Higher prices reduce demand but increase per-unit revenue, while lower prices increase demand but reduce sales margin. Sensitivity tests by varying the parameters  $a_t$  and  $b_t$  values within  $\pm 10\%$  of baseline to assess robustness of pricing strategies. Results show that increased  $a_t$  leads to higher optimal quantities and slightly lower prices to capture demand. Higher  $b_t$  (more price sensitivity) results in more aggressive price reductions to stimulate sales. The analysis also confirms that the deterministic LP model adapts smoothly to changing demand conditions, supporting managerial confidence in the pricing policy.

### Conclusion

This paper develops a deterministic linear programming model for dynamic pricing of perishable goods, demonstrating its effectiveness through fresh strawberries data. The model maximizes revenue by optimally balancing price, demand, and inventory constraints.

### Recommendation

Probability demand models incorporating uncertainty and probabilistic constraints, multi-product settings with inventory interactions and cross-price effects should be considered by future researcher. In addition, cost structures, such as holding and disposal costs should be incorporated into further studies.

### References

- Abbas, H., Zhao, L., Gong, X., & Faiz, N. (2023). The perishable products case to achieve sustainable food quality and safety goals implementing on-field sustainable supply chain model. *Socio-Economic Planning Sciences*, 87, 101562.
- Azab, R., Mahmoud, R. S., Elbehery, R., & Gheith, M. (2023). A Bi-Objective Mixed-Integer Linear Programming Model for a Sustainable Agro-Food Supply Chain with Product Perishability and Environmental Considerations. *Logistics*, 7(3), 46.
- FAO. 2023. *The Impact of Disasters on Agriculture and Food Security*. Food and Agriculture Organization.
- George, A. S. (2024). Realizing the Promise of Dynamic Pricing Through Responsible Innovation. *Partners Universal International Research Journal*, 3(3), 21-37.
- Gonen, L. D., Tavor, T., & Spiegel, U. (2024). Unlocking Market Potential: Strategic Consumer Segmentation and Dynamic Pricing for Balancing Loyalty and Deal Seeking. *Mathematics*, 12(21), 3364.
- Guchhait Haugen, M., Farahmand, H., Jaehnert, S., & Fleten, S. E. (2023). Representation of uncertainty in market models for operational planning and forecasting in renewable power systems: a review. *Energy Systems*, 1-36.
- Hemmati, H., Baradaran Kazemzadeh, R., Nikbakhsh, E., & Nakhai Kamalabadi, I. (2023). Designing a Green-Resilient Supply Chain Network for Perishable Products Considering a Pricing Reduction Strategy to Manage Optimal Inventory: A Column Generation-based Approach. *Journal of Quality Engineering and Production Optimization*, 8(1), 171-196.
- Iannacone, L., & Gardoni, P. (2024). Modeling deterioration and predicting remaining useful life using stochastic differential equations. *Reliability Engineering & System Safety*, 251, 110251.

- Jalving, J., Ghouse, J., Cortes, N., Gao, X., Knueven, B., Agi, D., ... & Dowling, A. W. (2023). Beyond price taker: Conceptual design and optimization of integrated energy systems using machine learning market surrogates. *Applied Energy*, 351, 121767.
- Kopalle, P. K., Pauwels, K., Akella, L. Y., & Gangwar, M. (2023). Dynamic pricing: Definition, implications for managers, and future research directions. *Journal of Retailing*, 99(4), 580-593.
- Morán-Figueroa, G. H., Muñoz-Pérez, D. F., Rivera-Ibarra, J. L., & Cobos-Lozada, C. A. (2024). Model for Predicting Maize Crop Yield on Small Farms Using Clusterwise Linear Regression and GRASP. *Mathematics*, 12(21), 3356.
- Ni, X., Yan, L., Xiong, K., & Liu, Y. (2024). A Hierarchical Bayesian Market Mix Model with Causal Inference for Personalized Marketing Optimization. *Journal of Artificial Intelligence General science (JAIGS) ISSN: 3006-4023*, 6(1), 378-396.
- Noble, J., John, K., & Paul, B. (2023). Inventory management of perishable products with fixed shelf life for a single echelon system. *Materials Today: Proceedings*, 72, 2863-2868.
- Sanders, R. E. (2024). Dynamic pricing and organic waste bans: A study of grocery retailers' incentives to reduce food waste. *Marketing Science*, 43(2), 289-316.
- Syed, T. A., Aslam, H., Bhatti, Z. A., Mehmood, F., & Pahuja, A. (2024). Dynamic pricing for perishable goods: A data-driven digital transformation approach. *International Journal of Production Economics*, 277, 109405.
- Winkler, T., Ostermeier, M., & Hübner, A. (2023). Proactive food waste prevention in grocery retail supply chains—An exploratory study. *International Journal of Physical Distribution & Logistics Management*, 53(11), 125-156.
- Yavuz, T., & Kaya, O. (2024). Deep reinforcement learning algorithms for dynamic pricing and inventory management of perishable products. *Applied Soft Computing*, 163, 111864.
- Yegane, B. Y. (2023). An Integrated Production-distribution Problem of Perishable Items with Dynamic Pricing Consideration in a Three-echelon Supply Chain. *International Journal of Engineering*, 36(11), 2038-2051.
- Yontar, E. (2023). Critical success factor analysis of blockchain technology in agri-food supply chain management: A circular economy perspective. *Journal of Environmental Management*, 330, 117173.
- Zakaria, A. F., Lim, S. C. J., & Aamir, M. (2024). A pricing optimization modelling for assisted decision making in telecommunication product-service bundling. *International Journal of Information Management Data Insights*, 4(1), 100212.

