



Numerical Evaluation of the Adverse Effects of Economic Instability on Consumer Buying Behaviour in Nigeria

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Abstract

The buying behaviour in consumer research encompasses the decision-making processes customers undergo when purchasing goods and services. The buying behaviour of consumers under economic instability is influenced by economic instabilities such as hyperinflation, unstable dollar to naira exchange rate, removal of fuel subsidy without predictive feasibility study analysis, increment of value added tax (VAT) and high electricity tariff. These economic instabilities result to volatility adverse effects of devaluation of naira, income fluctuations, disruption of financial planning and reduction of household expenditure, increased cost of imported raw materials leading to an increase in production and marketing costs. Amid these economic instabilities, there is urgent need to evaluate its volatility adverse effects on consumers' buying behaviour and this can be modeled as Econometric Instability Time-Delay Differential Equation (EITDDE). Numerically, this study aims to evaluate and proffer best ways to overcome the volatility adverse effects of economic instability on consumers' buying behaviour in Nigeria. Some examples of the modeled equation were solved using Extended Block Adams Moulton Methods (EBAMM) with the use of modeled mathematical sequences for evaluation of the instability delay and noise terms. The instability delay and noise terms were evaluated by applying acceptable ideas of developed sequences. The discrete schemes of the applied numerical method were obtained through the use of linear multistep collocation procedure using matrix inversion approach. Following the volatility display of the numerical results of the method which represents the volatility adverse effects of five dimensions of economic instability on consumers' buying behaviour, the Absolute Instability Errors (AIEs) of partition number $k = 4$ of EBAMM produced better and faster numerical results than the partition numbers $k = 3$ and 2 by giving the Least Minimum Absolute Instability Errors (LMAIEs) at a Lower Computational Time (LCT) when compared with established methods. This study recommends that the Nigerian government should adopt more robust and consistent monetary policies and diversify its economy by investing in non-oil sectors such as agriculture, technology, and manufacturing in order to create a more stable and resilient economic environment and to improve consumers' buying behaviour.

Keywords: Economic Instability, Consumers' Buying Behaviour, Nigeria, EITDDE, EBAMM

Introduction

In periods of economic instability, consumers face increased financial constraints, prompting unstable changes in their purchasing decisions in Nigeria. Rising economic instability reduces household incomes, and heightened uncertainty about future financial stability which often pose severe challenges on consumers buying of goods and services. (Menekes & Schramm-Klein, 2025). The behaviors, intentions, desires, and decisions of customers when purchasing a product or service are all part of consumer buying behavior. Understanding market procurement helps detect and forecast the buying actions of consumers (Loxton et al., 2020). Consumers of usually make daily

purchase decisions for buying a directly from the manufacturing companies and many of the consumers buying behaviours may be influenced by the adverse effects of economic volatility. The buying behaviour of consumers under economic instability is influenced by economic instabilities such as hyperinflation, unstable dollar to naira exchange rate, removal of fuel subsidy without predictive feasibility study analysis, increment of value added tax (VAT) and high electricity tariff. According to (Hofmann & Baier, 2019) hyperinflation, a severe persistent increase in the general price level of goods and services, remains a fundamental economic challenge affecting consumer decision-making across various sectors of industries across the globe. In both developed and emerging economies, economic instability disrupts financial planning by eroding the purchasing power of individuals and households, thereby influencing their spending behavior (Katnic et al., 2024). When economic instability pressure intensifies, customers often reassess their financial commitments and expenditures leading to shifts in their buying behavior (Lakshmanamoorthy et al., 2024; Olamide et al., 2022) argues that high and volatile inflation rates can disrupt economic stability, leading to uncertainty, reduced investment, and distortions in resource allocation, contributing to economic fluctuations. During periods of economic instability, money loses its purchasing power, making it imperative for consumers to be cautious when purchasing any product to comply with solvency regulations (Kushwaha et al., 2015). Economic instability makes it tougher for people to buy what they need and can change what they choose to spend on (Olusola et al., 2022). Unstable dollar to naira exchange rate, increment of value added tax (VAT) and high electricity tariff can lead to unstable prices, especially for imported items and materials needed for production (Sugiharti et al., 2020). Together, these economic challenges create a level of uncertainty that pushes people and production companies in Nigeria to rethink how they spend their money and find ways to be financially stable.

The sensitivity of removal of fuel subsidy without predictive feasibility study analysis to changes in dollar to naira exchange rates influences the economic stability and consumers buying behavior in Nigeria. The effects of this sensitivity results to economic instability which can be seen in the high cost of importation of basic goods and raw materials because a weaker naira makes living costs go up. Also, the ongoing economic instability in Nigeria has cuts into people's disposable income, leaving them with fewer choices. This pressure leads to noticeable changes in how people behave as consumers, like cutting back on spending, opting for locally made products, and choosing cheaper brands over more expensive ones Oloko (2021). Understanding these changes help to grasp the bigger picture of how economic instability impact consumers' buying behavior in Nigeria.

These economic instability and its volatility adverse effects on consumers' buying behaviour are modeled as Econometric Instability Time-Delay Differential Equation (EITDDE) which requires thorough numerical examination to assess and evaluate the volatility adverse effects of economic instability on consumers' buying behaviour in Nigeria. Several numerical methods have been applied to solve the modeled equation by most scholars such as (Osu et al., 2021; Evelyn, 2000; Bahar, 2019; Wang et al., 2011; Zhang et al., 2009) who used interpolation system in analyzing the lag term and volatility term of the modeled equation and experienced setbacks in calculating the Least Absolute Instability Errors (LAIEs) at the Lowest Computational Time (LCT) which hindered the accuracy of their applied methods. The setbacks encountered by the applied methods and the volatility side effects of economic instability on consumers' buying behavior in Nigeria can be tackled through the numerical application of Extended Block Adams Moulton Method (EBAMM) in evaluating the modeled equation together with incorporation of new sequences developed by Osu et al. (2023) for delay term and volatility term evaluations. EVTDDE is an econometric instability operation that focuses on the present state as the drift part and the instability state as diffusion part of the modeled equation. An instability term is a stochastic process of any random variable $\{S_t, t \in T\}$ where S_t is the instability-time and t is the time difference of various consumers' buying behaviour of this study. Ugbebor, (1991) defined volatility as a contingent operation for collation of stochastic variables of countable time points govern by axioms of economic instabilities. Akhtari et al. (2015) developed a model equation of Instability Time-Delay Differential Equation (ITDDE) which contains the initial state and the volatility state as displayed below;

$$\begin{aligned} ds(t) &= D(s(t), s(t - \tau), t)dt + E(s(t), s(t - \tau), t)d\Phi(t) \text{ for } t > 0, \tau > 0. \\ y(t) &= \Phi(t)d\Phi(t), \quad \text{for } t > 0 \end{aligned} \quad (1)$$

Following the econometric model developed by Aigbedion et al. (2016), a multiple regression equation containing one econometric dependent variable as the Consumers' Buying Behaviour (CBB) in Nigeria and three dimensional econometric independent variables such as Hyperinflation (HI), Unstable Dollar to Naira Exchange Rate (UDNER), Removal of Fuel Subsidy without predictive feasibility study analysis (RFS), High Value Added Tax (HVAT) and

High Electricity Tariff (HET) of economic instability, a constant parameter (α), error term (ε_i) and $\beta_i = \beta_1, \beta_2, \beta_3, \dots$ are coefficients of three dimensional econometric independent variables of economic instability are developed in this study and can be presented as:

$$CBB = \alpha + \beta_1 HI + \beta_2 UDNER + \beta_3 RFS + \beta_4 HVAT + \beta_5 HET + \varepsilon_i \quad (2)$$

where the econometric variables constitute the constructs of this study. Incorporating the econometric multiple regression equation (2) into the Instability Time-Delay Differential Equation (ITDDE) (1), the modeled equation for this analysis results to Econometric Instability Time-Delay Differential Equation (EITDDE) showing the numerical functions of the econometric variables of the form;

$$dCBB(t) = D(CBB(t), CBB(t - \tau), t)dt + E(CBB(t), CBB(t - \tau), t)d|\varepsilon_i|(t) \text{ for } t > 0, \tau > 0. \quad (3)$$

where $\alpha(t)$ is the fundamental function, D, E are drift and volatility coefficients, $dCBB(t)$ indicates the differential equivalence of consumers' buying behavior in Nigeria, and t is the time variations of consumers' buying behavior fluctuations measured in months, τ is known as the delay, $(t - \tau)$ is denoted by the delay term to measure the rate of consumers' buying behavior fluctuations and $CBB(t - \tau)$ describe the delay term of the consumers' buying behavior in Nigeria. The variable $|\varepsilon_i|(t)$ represents the Absolute Instability Errors (AIEs) of consumers' buying behaviour instabilities with its differential correlation $d|\varepsilon_i|(t)$ as the instability term with the result of the delay term for consumers' buying behavior $CBB(t) = E(CBB(t), CBB(t - \tau), t)d|\varepsilon_i|(t)$ on the instability part of (3). The drift part of (3) $dCBB(t) = D(CBB(t), CBB(t - \tau), t)dt$ is deterministic which handles the consumers' buying behavior without any economic instability.

Aim and Objectives of the study

The aim of this study is to evaluate and proffer best ways to overcome the volatility adverse effects of economic instability on consumers' buying behaviour in Nigeria through numerical analysis approach with the following objectives;

1. To derive the discrete schemes of Extended Block Adams Moulton Methods (EBAMM) through the use of linear multistep collocation procedure following matrix inversion techniques.
2. To model Econometric Instability Time-Delay Differential Equation (EITDDE) containing the adverse effects-factors of economic instability.
3. To solve some examples of the modeled equations of this study using the derived discrete schemes of Extended Block Adams Moulton Methods (EBAMM).
4. To obtain the numerical solutions of the modeled equation of the study.
5. To analyze the obtained numerical results of the modeled equation to ascertain the advantage and superiority of the proposed method of this study over other existing methods in literature.

Methods and Materials

In this section, the discrete schemes of $k = 2, 3$ and 4 for Extended Block Adams Moulton Method (EBAMM) were constructed by the application of matrix inversion method on the k -step multistep collocation approach. The analysis of the axioms of the method base on the discrete schemes of $k = 2, 3$ and 4 shall be examined to determine the necessary and sufficient conditions of numerical analysis required in solving and obtaining numerical solutions of the modeled equation.

Development of the method

Extended Block Adams Moulton Methods (EBAMM) is a linear multistep method whose general mathematical expression was developed as a k -step linear multistep method. The k -step linear multistep collocation operation with m collocation points was derived in Onumanyi et al. (1994) as;

$$a(x) = \sum_{v=0}^{c-1} \alpha_v(x) g_{u+v} + d \sum_{v=0}^{z-1} \beta_v(x) h_{u+v}(x, y(x)) \quad (4)$$

From (4) the continuous expression of extended block adams moulton methods can be expressed as

$$a(x) = \sum_{v=0}^{c-1} \alpha_v(x) a_{u+v} + d \sum_{v=0}^{z-1} \beta_v(x) h_{u+v}(x, y(x)) + d \sum_{v=0}^{z-1} \gamma_v(x) g_{u+v}(x, y(x)) \quad (5)$$

where $\alpha_v(x)$, $\beta_v(x)$ and $\gamma_v(x)$ are continuous coefficients of the method defined as

$$\alpha_v(x) = \sum_{m=0}^{c+z-1} \alpha_{v,m+1} x^m \text{ for } v = \{0, 1, \dots, c-1\} \quad (6)$$

$$d\beta_v(x) = \sum_{m=0}^{c+z-1} d\beta_{v,m+1} x^m \text{ for } v = \{0, 1, \dots, z-1\}$$

(7)

$$d\gamma_v(x) = \sum_{m=0}^{c+z-1} d\gamma_{v,m+1} x^m \text{ for } v = \{0, 1, \dots, z-1\}$$

(8)

where $v = 0, 1, 2, \dots, z-1$ are the z collocation points, x_{u+v} , $v = 0, 1, 2, \dots, c-1$ are the c randomly chosen interpolation points and d is the fixed partition width.

To get $\alpha_v(x)$, $\beta_v(x)$ and $\gamma_v(x)$, Sirisena, (1997) formulated a matrix equivalence of the form

$$BA = I \quad (9)$$

where I is the square matrix of proportion $(c+z) \times (c+z)$ while B and A are matrices defined as

$$B = \begin{bmatrix} \alpha_{0,1} & \alpha_{1,1} & \cdots & \alpha_{c-1,1} & d\beta_{0,1} & \cdots & d\beta_{z-1,1} \\ \alpha_{0,2} & \alpha_{1,2} & \cdots & \alpha_{c-1,2} & d\beta_{0,2} & \cdots & d\beta_{z-1,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{0,c+z} & \alpha_{1,c+z} & \cdots & \alpha_{c-1,c+z} & d\beta_{0,c+z} & \cdots & d\beta_{z-1,c+z} \end{bmatrix}$$

(10)

$$A = \begin{bmatrix} 1 & x_u & x_u^2 & \cdots & x_u^{c+z-1} \\ 1 & x_{u+1} & x_{u+1}^2 & \cdots & x_{u+1}^{c+z-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{u+c-1} & x_{u+c-1}^2 & \cdots & x_{u+c-1}^{c+z-1} \\ 0 & 1 & 2x_0 & \cdots & (c+z-1)x_0^{c+z-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 2x_{z-1} & \cdots & (c+z-1)x_{z-1}^{c+z-2} \end{bmatrix}$$

(11)

In matrix equivalence (9), the columns of $B = A^{-1}$ give the continuous coefficients of the continuous scheme of (5).

Construction of the method

Following the matrix inversion approach of k -step linear multistep method developed by Sirisena, (1997), the discrete schemes of $k = 2, 3$ and 4 for Extended Block Adams Moulton Method (EBAMM) were derived as;

$k = 2$ EBAMM

$$a_u = a_{u+1} - \frac{3}{8} dh_u - \frac{19}{24} dh_{u+1} + \frac{5}{24} dh_{u+2} - \frac{1}{24} dh_{u+3}$$

$$a_{u+2} = a_{u+1} - \frac{1}{24} dh_u + \frac{13}{24} dh_{u+1} + \frac{13}{24} dh_{u+2} - \frac{1}{24} dh_{u+3}$$

$$a_{u+3} = a_{u+1} + \frac{1}{3}dh_{u+1} + \frac{4}{3}dh_{u+2} + \frac{1}{3}dh_{u+3} \quad (12)$$

k = 3 EBAMM

$$\begin{aligned} a_u &= a_{u+2} - \frac{29}{90}dh_u - \frac{62}{45}dh_{u+1} - \frac{4}{15}dh_{u+2} - \frac{2}{45}dh_{u+3} + \frac{1}{90}dh_{u+4} \\ a_{u+1} &= a_{u+2} + \frac{19}{720}dh_u - \frac{173}{360}dh_{u+1} - \frac{19}{30}dh_{u+2} + \frac{37}{360}dh_{u+3} - \frac{11}{720}dh_{u+4} \\ a_{u+3} &= a_{u+2} + \frac{11}{720}dh_u - \frac{37}{360}dh_{u+1} + \frac{19}{30}dh_{u+2} + \frac{173}{360}dh_{u+3} - \frac{19}{720}dh_{u+4} \\ a_{u+4} &= a_{u+2} - \frac{1}{90}dh_u + \frac{2}{45}dh_{u+1} + \frac{4}{15}dh_{u+2} + \frac{62}{45}dh_{u+3} + \frac{29}{90}dh_{u+4} \end{aligned} \quad (13)$$

k = 4 EBAMM

$$\begin{aligned} a_u &= a_{u+3} - \frac{51}{160}dh_u - \frac{219}{160}dh_{u+1} - \frac{57}{80}dh_{u+2} - \frac{57}{80}dh_{u+3} + \frac{21}{160}dh_{u+4} - \frac{3}{160}dh_{u+5} \\ a_{u+1} &= a_{u+3} + \frac{1}{90}dh_u - \frac{17}{45}dh_{u+1} - \frac{19}{15}dh_{u+2} - \frac{17}{45}dh_{u+3} + \frac{1}{90}dh_{u+4} \\ a_{u+2} &= a_{u+3} - \frac{11}{1440}dh_u + \frac{31}{480}dh_{u+1} - \frac{401}{720}dh_{u+2} - \frac{401}{720}dh_{u+3} + \frac{31}{480}dh_{u+4} - \frac{11}{1440}dh_{u+5} \\ a_{u+4} &= a_{u+3} - \frac{11}{1440}dh_u - \frac{77}{1440}dh_{u+1} - \frac{43}{240}dh_{u+2} + \frac{511}{720}dh_{u+3} + \frac{637}{1440}dh_{u+4} - \frac{3}{160}dh_{u+5} \\ a_{u+5} &= a_{u+3} + \frac{1}{90}dh_u - \frac{1}{15}dh_{u+1} + \frac{7}{45}dh_{u+2} + \frac{7}{45}dh_{u+3} + \frac{43}{30}dh_{u+4} + \frac{14}{45}dh_{u+5} \end{aligned} \quad (14)$$

Analysis of the Necessary and Sufficient Axioms of the Method

The necessary and sufficient axioms for the convergence and stability of the method are examined following the conditions developed by (Lambert, 1973; Dahlquist, 1956).

Order and error constant

As EBAMM is one of the classes of Linear Multistep Method (LMM), Lambert, (1973) studied that LMM is said to be of order n if $C_0 = C_1 = 0, \dots, C_u = 0$ but $C_{u+1} \neq 0$ and C_{u+1} is the error constant.

The order and error constants for (12) are obtained as;

$$C_0 = C_1 = C_2 = C_3 = C_4 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T \text{ but } C_5 = \begin{pmatrix} \frac{19}{720} & \frac{11}{720} & -\frac{1}{90} \end{pmatrix}^T$$

From the above, (12) has order $p = 4$ and error constants, $\frac{19}{720} \quad \frac{11}{720} \quad -\frac{1}{90}$

For (13),

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T \quad \text{but}$$

$$c_6 = \begin{pmatrix} -\frac{1}{90} & \frac{11}{1440} & \frac{11}{1440} & -\frac{1}{90} \end{pmatrix}^T$$

Therefore, (13) has order $p = 5$ and error constants, $-\frac{1}{90} \quad \frac{11}{1440} \quad \frac{11}{1440} \quad -\frac{1}{90}$

In the same step to (14),

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T \text{ and}$$

$$c_7 = \begin{pmatrix} -\frac{3}{32} & 0 & -\frac{11}{288} & -\frac{3}{32} & -\frac{197}{18} \end{pmatrix}^T$$

Therefore, (14) has order $p = 6$ and error constants, $-\frac{3}{32} \quad 0 \quad -\frac{11}{288} \quad -\frac{3}{32} \quad -\frac{197}{18}$

Consistency

According to Lambert, (1973), a computational application is said to be consistent if the order $p \geq 1$. Since the order of EBAMM as evaluated is $p \geq 1$, the consistency is satisfied.

Zero-stability computation

In Dahlquist, (1956), a computational method is said to be zero stable if the roots $a_i, i = 1, 2, 3, \dots, \infty$ of the initial characteristic polynomial $U(q)$ developed as $R(q) = \det(qJ_c^{(z)} - J_c^{(z)})$ is $|q_i| \leq 1$ and the roots $|q_i|$ is simple or distinct where $J_c^{(z)}$ and $J_c^{(z)}$, are the matrices of the initial characteristic polynomial evaluated from (12), (13) and (14).

The zero stability for (12) is computed as follows

$$\begin{pmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{u+1} \\ a_{u+2} \\ a_{u+3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_{u-2} \\ a_{u-1} \\ a_u \end{pmatrix} + d \begin{pmatrix} -\frac{19}{24} & \frac{5}{24} & -\frac{1}{24} \\ \frac{13}{24} & \frac{13}{24} & -\frac{1}{24} \\ \frac{1}{3} & \frac{4}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} h_{u+1} \\ h_{u+2} \\ h_{u+3} \end{pmatrix} + d \begin{pmatrix} 0 & 0 & -\frac{3}{8} \\ 0 & 0 & -\frac{1}{24} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} h_{u-2} \\ h_{u-1} \\ h_u \end{pmatrix}$$

$$\text{where } J_2^{(1)} = \begin{pmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, J_1^{(1)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, L_2^{(1)} = \begin{pmatrix} -\frac{19}{24} & \frac{5}{24} & -\frac{1}{24} \\ \frac{13}{24} & \frac{13}{24} & -\frac{1}{24} \\ \frac{1}{3} & \frac{4}{3} & \frac{1}{3} \end{pmatrix} \text{ and } M_2^{(1)} = \begin{pmatrix} 0 & 0 & -\frac{3}{8} \\ 0 & 0 & -\frac{1}{24} \\ 0 & 0 & 0 \end{pmatrix}$$

Implementing the constructed initial characteristic polynomial, (12) is evaluated and presented as:

$$R(q) = \det(qJ_c^{(z)} - J_c^{(1)}) = |qJ_2^{(1)} - J_1^{(1)}| = 0. \quad (15)$$

Now we have,

$$R(q) = \left| q \begin{pmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right| = \left| \begin{pmatrix} -q & 0 & 0 \\ -q & q & 0 \\ -q & 0 & q \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right| \Rightarrow R(q) = \begin{vmatrix} -q & 0 & -1 \\ -q & q & 0 \\ -q & 0 & q \end{vmatrix}$$

using Maple 18 software,

$$R(q) = -q^3 - q^2 \Rightarrow -q^3 - q^2 = 0$$

$$\Rightarrow q_1 = -1, q_2 = 0, q_3 = 0. \text{ As } |q_i| < 1, i = 1, 2, 3, (12) \text{ is zero stable.}$$

By the same procedure for (13)

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{u+1} \\ a_{u+2} \\ a_{u+3} \\ a_{u+4} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_{u-3} \\ a_{u-2} \\ a_{u-1} \\ a_u \end{pmatrix} + d \begin{pmatrix} -\frac{62}{45} & -\frac{4}{15} & -\frac{2}{45} & \frac{1}{90} \\ -\frac{173}{360} & -\frac{19}{30} & \frac{37}{360} & -\frac{11}{720} \\ -\frac{37}{360} & \frac{19}{30} & \frac{173}{360} & -\frac{19}{720} \\ \frac{2}{45} & \frac{4}{15} & \frac{62}{45} & \frac{29}{90} \end{pmatrix} \begin{pmatrix} h_{u+1} \\ h_{u+2} \\ h_{u+3} \\ h_{u+4} \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 0 & -\frac{29}{90} \\ 0 & 0 & 0 & \frac{19}{720} \\ 0 & 0 & 0 & \frac{11}{720} \\ 0 & 0 & 0 & -\frac{1}{90} \end{pmatrix} \begin{pmatrix} h_{u-3} \\ h_{u-2} \\ h_{u-1} \\ h_u \end{pmatrix}$$

$$\text{where } J_2^{(2)} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}, J_1^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, L_2^{(2)} = \begin{pmatrix} -\frac{62}{45} & -\frac{4}{15} & -\frac{2}{45} & \frac{1}{90} \\ -\frac{173}{360} & -\frac{19}{30} & \frac{37}{360} & -\frac{11}{720} \\ -\frac{37}{360} & \frac{19}{30} & \frac{173}{360} & -\frac{19}{720} \\ \frac{2}{45} & \frac{4}{15} & \frac{62}{45} & \frac{29}{90} \end{pmatrix} \text{ and}$$

$$M_2^{(2)} = \begin{pmatrix} 0 & 0 & 0 & -\frac{29}{90} \\ 0 & 0 & 0 & \frac{19}{720} \\ 0 & 0 & 0 & \frac{11}{720} \\ 0 & 0 & 0 & -\frac{1}{90} \end{pmatrix}$$

The initial characteristic polynomial is presented as

$$R(q) = \det(qJ_2^{(2)} - J_1^{(2)}) = |qJ_2^{(2)} - J_1^{(2)}| = 0. \quad (16)$$

Now we have,

$$R(q) = q \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -q & 0 & 0 \\ q & -q & 0 & 0 \\ 0 & -q & q & 0 \\ 0 & -q & 0 & q \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow R(q) = \begin{pmatrix} 0 & -q & 0 & -1 \\ q & -q & 0 & 0 \\ 0 & -q & q & 0 \\ 0 & -q & 0 & q \end{pmatrix}$$

Using Maple 18 software, we obtain

$$R(q) = q^4 + q^3 \Rightarrow q^4 + q^3 = 0$$

$\Rightarrow q_1 = -1, q_2 = 0, q_3 = 0, q_4 = 0$. As $|q_i| < 1, i = 1, 2, 3, 4$, (13) is zero-stable.

For (14),

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{u+1} \\ a_{u+2} \\ a_{u+3} \\ a_{u+4} \\ a_{u+5} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_{u-4} \\ a_{u-3} \\ a_{u-2} \\ a_{u-1} \\ a_u \end{pmatrix}$$

$$+ d \begin{pmatrix} -\frac{219}{160} & -\frac{57}{80} & -\frac{57}{80} & \frac{21}{160} & -\frac{3}{160} \\ -\frac{17}{45} & -\frac{19}{15} & -\frac{17}{45} & \frac{1}{90} & 0 \\ -\frac{31}{480} & -\frac{401}{720} & -\frac{401}{720} & \frac{31}{480} & -\frac{11}{1440} \\ \frac{77}{1440} & -\frac{43}{240} & \frac{511}{720} & \frac{637}{1440} & -\frac{3}{160} \\ -\frac{1}{15} & \frac{7}{45} & \frac{7}{45} & \frac{43}{30} & \frac{14}{45} \end{pmatrix} \begin{pmatrix} h_{u+1} \\ h_{u+2} \\ h_{u+3} \\ h_{u+4} \\ h_{u+5} \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{51}{160} \\ 0 & 0 & 0 & 0 & \frac{1}{90} \\ 0 & 0 & 0 & 0 & -\frac{11}{1440} \\ 0 & 0 & 0 & 0 & -\frac{11}{1440} \\ 0 & 0 & 0 & 0 & \frac{1}{90} \end{pmatrix} \begin{pmatrix} h_{u-4} \\ h_{u-3} \\ h_{u-2} \\ h_{u-1} \\ h_u \end{pmatrix}$$

where

$$J_2^{(3)} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}, J_1^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, L_2^{(3)} = \begin{pmatrix} -\frac{219}{160} & -\frac{57}{80} & -\frac{57}{80} & \frac{21}{160} & -\frac{3}{160} \\ -\frac{17}{45} & -\frac{19}{15} & -\frac{17}{45} & \frac{1}{90} & 0 \\ -\frac{31}{480} & -\frac{401}{720} & -\frac{401}{720} & \frac{31}{480} & -\frac{11}{1440} \\ \frac{77}{1440} & -\frac{43}{240} & \frac{511}{720} & \frac{637}{1440} & -\frac{3}{160} \\ -\frac{1}{15} & \frac{7}{45} & \frac{7}{45} & \frac{43}{30} & \frac{14}{45} \end{pmatrix}$$

$$\text{and } M_2^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{51}{160} \\ 0 & 0 & 0 & 0 & \frac{1}{90} \\ 0 & 0 & 0 & 0 & -\frac{11}{1440} \\ 0 & 0 & 0 & 0 & -\frac{11}{1440} \\ 0 & 0 & 0 & 0 & \frac{1}{90} \end{pmatrix}$$

The initial characteristic polynomial is presented as

$$\begin{aligned} R(q) &= \det(qJ_2^{(3)} - J_1^{(3)}) \\ &= |qJ_2^{(3)} - J_1^{(3)}| = 0. \end{aligned} \quad (17)$$

Now we have,

$$\begin{aligned} R(q) &= q \begin{vmatrix} \begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ &= \begin{vmatrix} \begin{pmatrix} 0 & 0 & -q & 0 & 0 \\ q & 0 & -q & 0 & 0 \\ 0 & q & -q & 0 & 0 \\ 0 & 0 & -q & q & 0 \\ 0 & 0 & -q & 0 & q \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ &\Rightarrow R(q) = \begin{pmatrix} 0 & 0 & -q & 0 & -1 \\ q & 0 & -q & 0 & 0 \\ 0 & q & -q & 0 & 0 \\ 0 & 0 & -q & q & 0 \\ 0 & 0 & -q & 0 & q \end{pmatrix} \end{vmatrix} \end{aligned}$$

Using Maple 18 software, we obtain

$$\begin{aligned} R(q) &= -r^5 - r^4 \Rightarrow -r^5 - r^4 = 0 \\ &\Rightarrow q_1 = -1, q_2 = 0, q_3 = 0, q_4 = 0, q_5 = 0. \text{ As } |q_i| < 1, i = 1, 2, 3, 4, 5, (14) \text{ is zero-stable.} \end{aligned}$$

Convergence of the proposed method

Since the basic properties for consistency and zero-stability for a numerical method to be convergent as developed by (Lambert, 1973; Dahlquist, 1956) are satisfied following the consistency and zero-stability of the evaluated discrete schemes (12), (13) and (14) of EBAMM, therefore the method is convergent.

Region of absolute stability

The regions of absolute stability of the modeled equation (3) are considered. The G and H stability regions of absolute stability of (12), (13) and (14) are plotted and are represented in figures 1 to 6 below:

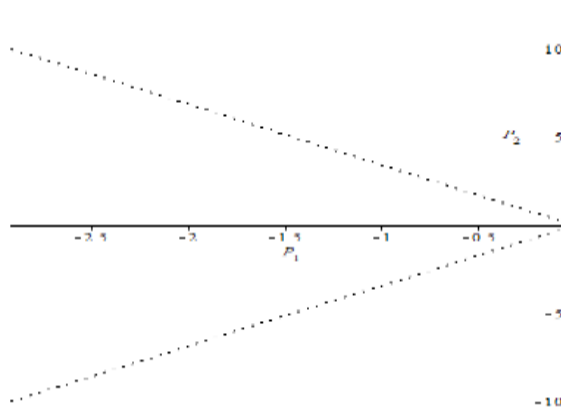


Figure 1: Region of G -stability (EBAMM) in (12)

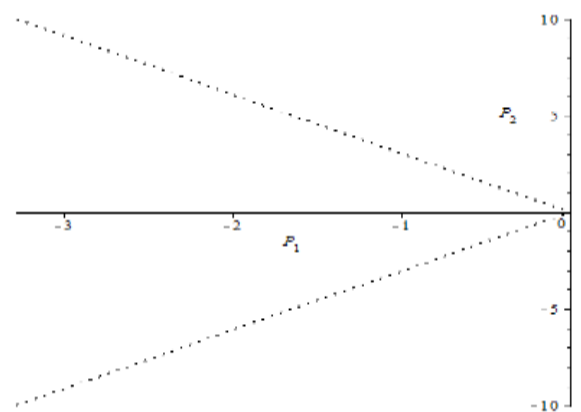


Figure 2: Region of G -stability (EBAMM) in (13)

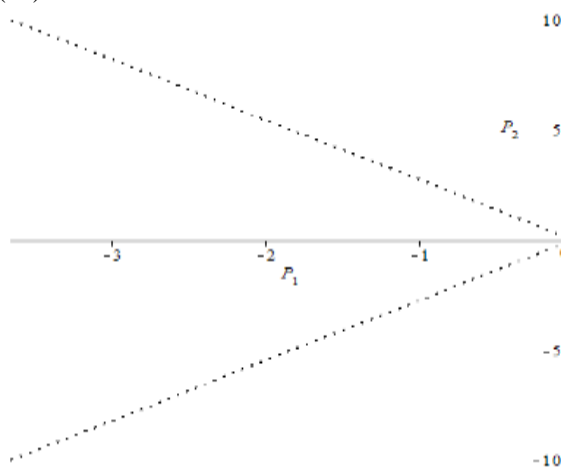


Figure 3: Region of G -stability (EBAMM) in (14)

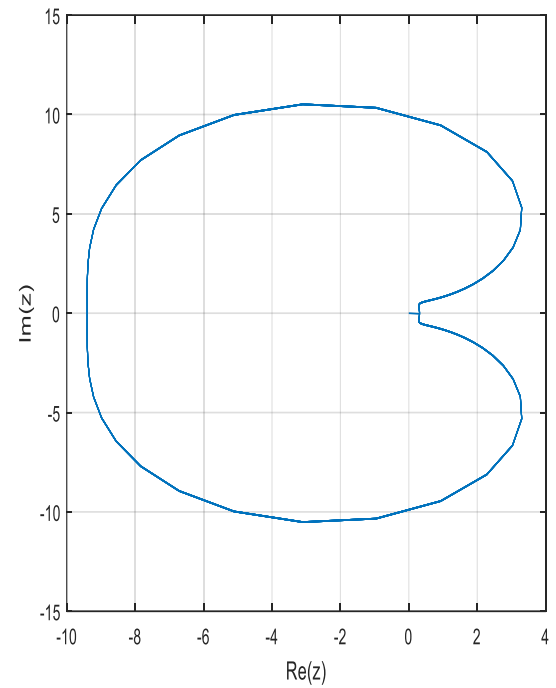


Figure 4: Region of H -stability (EBAMM) in (12)

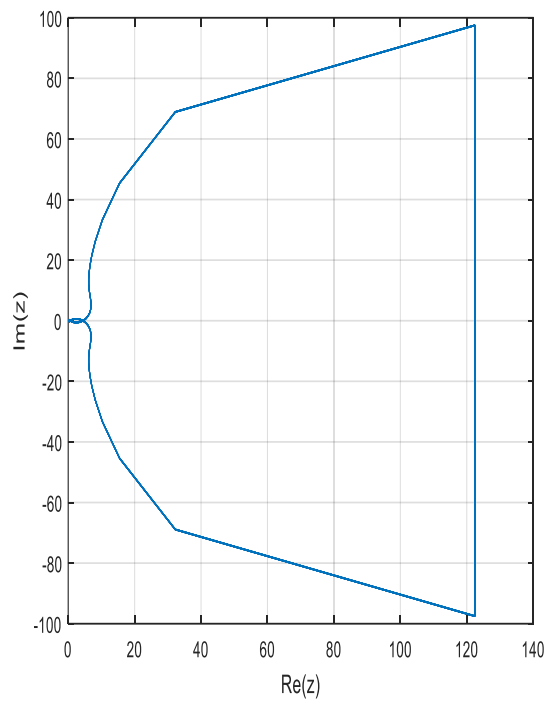


Figure 5: Region of H -stability (EBAMM) in (13)

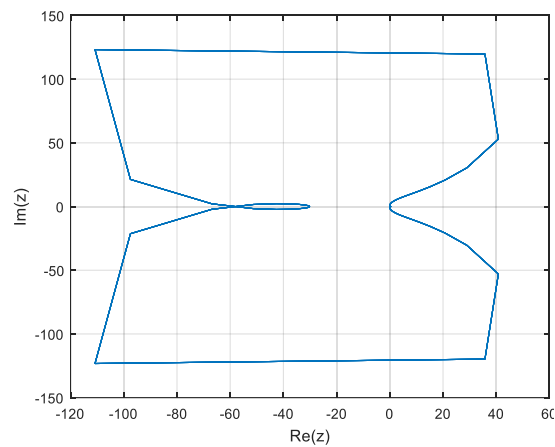


Figure 6: Region of H -stability (EBAMM) in (14)

The G -stability regions in figures 1 to 3 contained the entire left half of the complex plane regions while the H -stability regions in figures 4 to 6 lies inside the enclosed regions.

Numerical experiment

In this section, some formulated problems of the modeled equation of this study will be evaluated using the proposed method with the incorporation of the evaluated results of the delay and noise or instability terms applying the sequences developed by Osu et al. (2023) to obtain the numerical solutions and absolute instability errors of consumers' buying behavior in Nigeria modeled in (3).

Numerical problems

Problem1

$$dCBB(t) = \cos(t)((dCBB(t)(dCBB(t) - 2))dt + ((dCBB(t)(dCBB(t) - 2))d|\varepsilon_i|(t)), \\ 0 \leq t \leq 30.$$

$CBB(t) = (1 + \sin(t))d|\varepsilon_i|(t), t \geq 0$ is the instability nature of the exact solution

Problem 2

$$dCBB(t) = 1000 \left(dCBB(t) - dCBB(t) \left(t - (\ln(1000 - 1)) \right) \right) dt + \left(dCBB(t) - dCBB(t) \left(t - (\ln(1000 + 1)) \right) \right) d|\varepsilon_i|(t), 0 \leq t \leq 30.$$

$CBB(t) = (e^{-t})d|\varepsilon_i|(t), t \geq 0$ is the instability nature of the exact solution where $dCBB(t)$ representing the differential equivalence of consumers' buying behaviour in Nigeria.

Numerical Results

The above examples of model equation of this study were solved using the derived equations (12), (13) and (14) of EBAMM. The numerical results for step numbers 2, 3 and 4 are computed and presented in tables 1 to 2 below:

Table 1: Numerical Results of Problem 1 using the EBAMM for Partition Numbers $k = 2, 3$ & 4

t	Exact Results	K = 2 Approximate Results	K = 3 Approximate Results	K = 4 Approximate Results
1	0.367879441	0.589834933	0.689795572	0.789721979
2	0.135335283	0.575272659	0.67530279	0.875408765
3	0.049787068	0.960690853	0.86061934	0.674516706
4	0.018315639	0.246484521	0.519486611	0.536658324
5	0.006737947	0.279622773	0.604403141	0.48037731
6	0.002478752	0.83457778	0.821872587	0.703394864
7	0.000911882	0.452397377	0.28247501	0.082513608
8	0.000335463	0.638628718	0.715831259	0.904636565
9	0.00012341	0.668360541	0.617776391	0.608308945
10	4.54E-05	0.952582126	0.608806279	0.400583746
11	1.67E-05	0.34933964	0.512588258	0.708435037
12	6.14E-06	0.632108604	0.588560073	0.531806288
13	2.26E-06	0.73310202	0.523079062	0.412450784

14	8.32E-05	0.219651106	0.519008082	0.617642641
15	3.06E-05	0.215887702	0.317878601	0.432765197
16	1.13E-06	0.665822446	0.481676027	0.31510572
17	4.14E-07	0.339390617	0.57072879	0.838536316
18	1.52E-07	0.339902429	0.221224107	0.228405613
19	5.60E-06	0.428581313	0.167511394	0.166864098
20	2.06E-06	0.221901369	0.359838526	0.459057885
21	7.58E-07	0.112881121	0.260929513	0.312160267
22	2.79E-05	0.087295817	0.071089721	0.06414411
23	1.03E-06	0.047083658	0.05622929	0.072440919
24	3.78E-05	0.030332957	0.087293898	0.068397676
25	1.39E-08	0.545414953	0.345294506	0.144957145
26	5.11E-08	0.242656512	0.324250546	0.642235407
27	1.88E-07	0.642760171	0.443289802	0.326809236
28	6.91E-05	0.540971772	0.420596592	0.225055625
29	2.54E-06	0.015592349	0.119144331	0.515361802
30	9.36E-07	0.114445331	0.219627045	0.314279918

CT of EBAMM for $k = 2$ is 0.005s, $k = 3$ is 0.004s and $k = 4$ is 0.002s

Table 2: Numerical Results of Problem 2 using the EBAMM for Partition Numbers $k = 2, 3$ & 4

t	Exact Results	K = 2 Approximate Results	K = 3 Approximate Results	K = 4 Approximate Results
1	0.841470985	0.701540973	0.681540956	0.591540987
2	0.909297427	0.612811033	0.582811011	0.482811068
3	0.141120008	0.822812252	0.603812310	0.563634235
4	0.756802495	0.574546556	0.448019234	0.254191645
5	0.958924275	0.385978222	0.206965411	0.15362444
6	0.279415498	0.864157102	0.677653691	0.522802710
7	0.656986599	0.493208521	0.351129312	0.209302214
8	0.989358247	0.731076323	0.622372432	0.525521115
9	0.412118485	0.705611511	0.611385423	0.40067211
10	0.544021111	0.560867123	0.345333234	0.234624551
11	0.999990207	0.60262429	0.401103534	0.311410324
12	0.536572918	0.457187682	0.225671125	0.133045145
13	0.420167037	0.51146356	0.412359672	0.344904234
14	0.990607356	0.738120611	0.691233553	0.501678012

15	0.65028784	0.612614281	0.51026856	0.412680135
16	0.287903317	0.619131112	0.539225144	0.233680126
17	0.961397492	0.674060333	0.465946268	0.376609215
18	0.750987247	0.752081538	0.614759527	0.52263010
19	0.14987721	0.839199445	0.617032428	0.530903457
20	0.912945251	0.754741852	0.610492726	0.417663433
21	0.836655639	0.685744435	0.47214313	0.207689311
22	0.008851309	0.756333472	0.461426523	0.35427817
23	0.846220404	0.609189194	0.534997123	0.345982358
24	0.905578362	0.780312461	0.576858425	0.48398467
25	0.13235175	0.808378142	0.68492422	0.59205041
26	0.76255845	0.756717432	0.592263564	0.41238923
27	0.956375928	0.725328521	0.60187411	0.521482432
28	0.270905788	0.614209455	0.525587413	0.31752334
29	0.663633884	0.637151812	0.569404271	0.442694945
30	0.988031624	0.835114551	0.648320476	0.511657926

CT of EBAMM for $k = 2$ is 0.006s, $k = 3$ is 0.005s and $k = 4$ is 0.002s.

Discussion

The evaluation of the absolute differences of exact results and numerical results for each step number of the derived discrete schemes (12), (13) and (14) of EBAMM gives the Absolute Instability Errors (AIEs) which are presented in graphical forms. The Minimum Absolute Instability Errors (MAIEs) of this method are compared with other Minimum Absolute Instability Errors established in (Evelyn, 2000; Bahar, 2019; Osu et al., 2021) to ascertain the advantage and superiority of the proposed method of this study over other existing methods in literature. The compared Minimum Absolute Instability Errors (MAIEs) are also presented in graphical forms.

Absolute instability errors (AIEs)

The results are summarized accordingly in the tables 3 and 4 below;

Table 3: Absolute Instability Errors of Problem 1 using the EBAMM for $k = 2, 3$ & 4.

t	K = 2 Absolute Instability Errors	K = 3 Absolute Instability Errors	K = 4 Absolute Instability Errors
1	0.221955492	0.321916131	0.421842538
2	0.439937376	0.539967507	0.740073482
3	0.910903785	0.810832272	0.624729638
4	0.228168882	0.501170972	0.518342685
5	0.272884826	0.597665194	0.473639363
6	0.832099028	0.819393835	0.700916112
7	0.451485495	0.281563128	0.081601726
8	0.638293255	0.715495796	0.904301102
9	0.668237131	0.617652981	0.608185535
10	0.952536726	0.608760879	0.400538346
11	0.34932294	0.512571558	0.708418337
12	0.632102464	0.588553933	0.531800148

13	0.73309976	0.000037608	0.412448524
14	0.219567906	0.518924882	0.617559441
15	0.005710278	0.317848001	0.432734597
16	0.665821316	0.481674897	0.31510459
17	0.339390203	0.570728376	0.838535902
18	0.339902277	0.221223955	0.000008972
19	0.428575713	0.167505794	0.166858498
20	0.221899309	0.359836466	0.459055825
21	0.112880363	0.456792165	0.312159509
22	0.087267917	0.071061821	0.06411621
23	0.011348228	0.05622826	0.072439889
24	0.030295157	0.087256098	0.068359876
25	0.545414939	0.345294492	0.144957131
26	0.242656461	0.324250495	0.642235356
27	0.642759983	0.443289614	0.326809048
28	0.540902672	0.420527492	0.228405461
29	0.015589809	0.119141791	0.515359262
30	0.114444395	0.219626109	0.314278982

Table 4: Absolute Instability Errors of Problem 2 using the EBAMM for $k = 2, 3$ & 4

t	K= 2 Absolute Instability Errors	K=3 Absolute Instability Errors	K= 4 Absolute Instability Errors
1	0.139930012	0.159930029	0.249929998
2	0.296486394	0.326486416	0.426486359
3	0.681692244	0.462692302	0.422514227
4	0.182255939	0.308783261	0.50261085
5	0.572946053	0.751958864	0.805299835
6	0.584741604	0.398238193	0.243387212
7	0.163778078	0.405857287	0.447684385
8	0.258281924	0.366985815	0.463837132
9	0.193493026	0.299266938	0.211446375
10	0.016846012	0.198687877	0.30939656
11	0.397365917	0.598886673	0.688579883
12	0.079385236	0.310901793	0.403527773
13	0.091296523	0.007807365	0.085262803
14	0.252486745	0.299373803	0.488929344

15	0.037673559	0.14001928	0.237607705
16	0.231227795	0.351321827	0.154223191
17	0.287337159	0.495451224	0.584788277
18	0.001094291	0.13622772	0.228357147
19	0.589322235	0.567155218	0.481026247
20	0.158203399	0.302452525	0.495281818
21	0.150911204	0.000039309	0.628966328
22	0.647482163	0.352575214	0.245426861
23	0.23703121	0.311223281	0.500238046
24	0.000469901	0.328719937	0.000001942
25	0.276026392	0.305257247	0.45969866
26	0.005841018	0.270294886	0.35016922
27	0.231047407	0.354501818	0.434893496
28	0.443303667	0.354681625	0.246617552
29	0.026482072	0.094229613	0.320938939
30	0.152917073	0.439711148	0.476373698

The Absolute Instability Errors (AIEs) obtained from the above numerical solutions of some examples of the modeled equations of this study represent consumers' buying behavior under economic instability in Nigeria

Graphical display of absolute instability errors

The graphs of absolute instability errors (AIEs) of EBAMM for Problems 1 and 2 above in tables 1 to 2 are plotted using the softwares of R and R – studio and presented as;

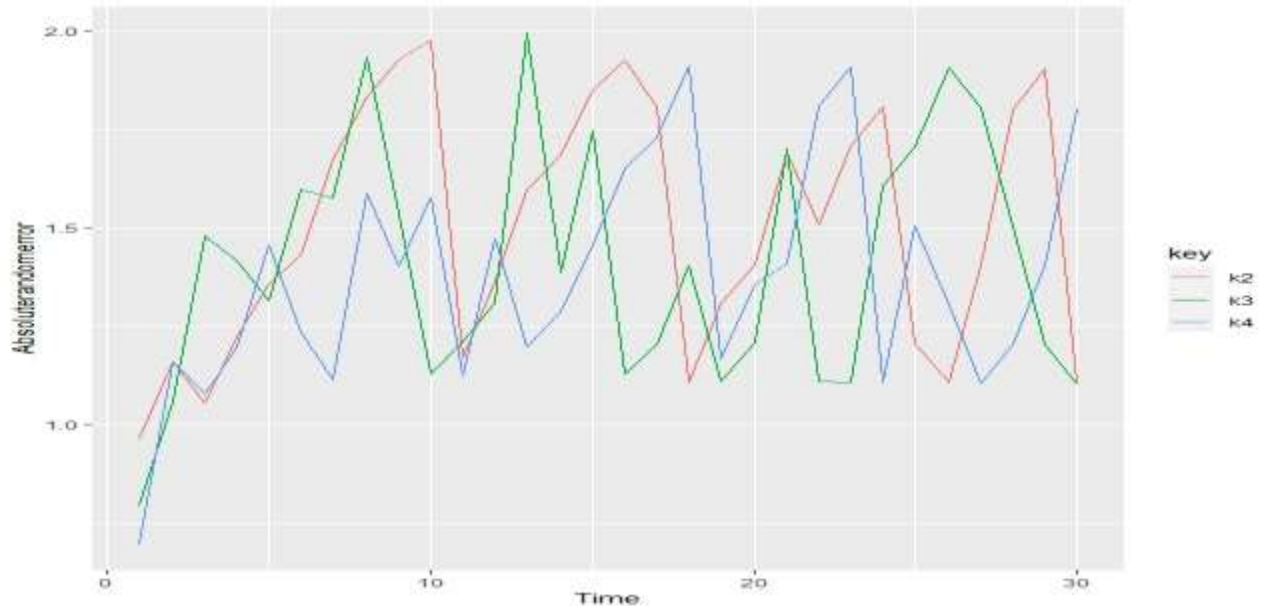


Figure 7: Absolute Instability Errors (AIEs) for Problem 1 using EBAMM (coloured-lines) against time variations of consumers' buying behavior fluctuations measured in months. The coloured -lines represent the volatility display of the method for step numbers $k = 2, 3$ and 4 with different Absolute Instability Errors (AIEs) representing the volatility adverse effects of economic instability on consumers' buying behaviour in Nigeria.

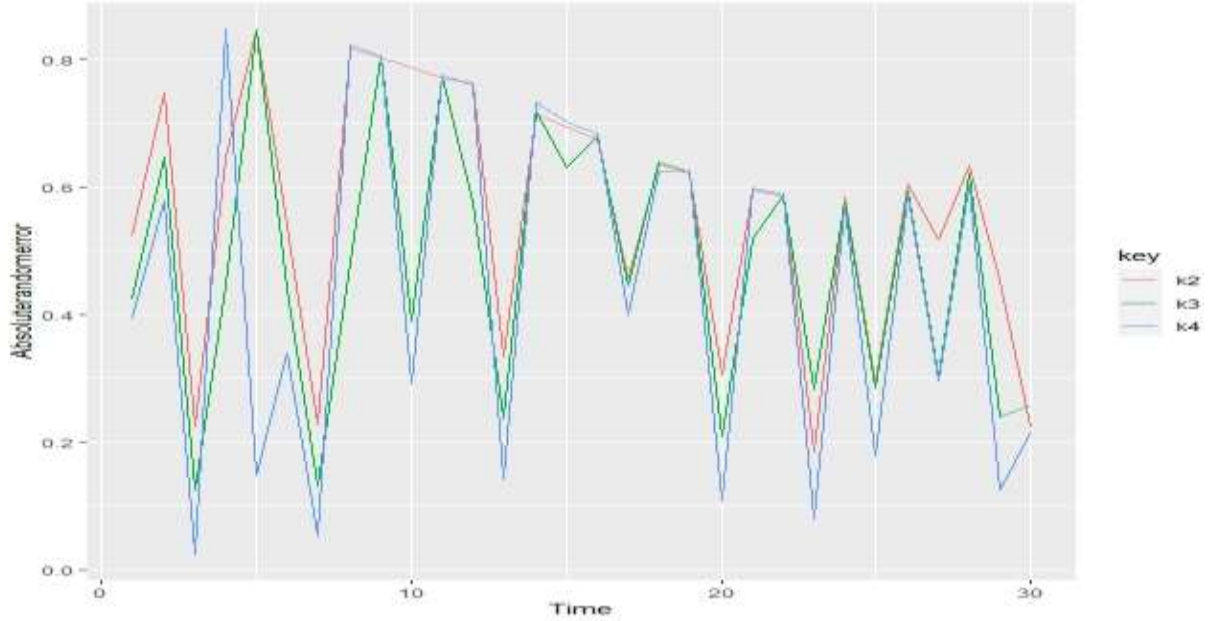


Figure 8: Absolute Instability Errors (AIEs) for Problem 2 using EBAMM (coloured-lines) against time variations of consumers' buying behavior fluctuations measured in months. The coloured-lines represent the volatility display of the method for step numbers $k = 2, 3$ and 4 with different Absolute Instability Errors (AIEs) representing the volatility adverse effects of economic instability on consumers' buying behaviour in Nigeria.

Comparison of results

Comparing the Minimum Absolute Instability Errors (MAIEs) of the method with other Absolute Instability Errors established in methods in (Evelyn, 2000; Bahar, 2019; Osu et al., 2021), the advantage of the proposed method over other existing methods are established and presented below:

Table 5: Comparing the Minimum Absolute Instability Errors (MAIEs) of EBAMM for $k = 2, 3$ and 4 with (Evelyn, 2000; Bahar, 2019; Osu et al., 2021) for fixed partition size $d = 0.01$ using Problem 1

Numerical Method	Compared MAIEs with (Evelyn, 2000; Bahar, 2019; Osu et al., 2021)
EBAMM MAIE $k = 2$	5.71E-03
EBAMM MAIE $k = 3$	3.76E-05
EBAMM MAIE $k = 4$	8.97E-08
ECSSEMM MAIE $k = 2$ Evelyn, (2000)	5.76E-01
ECSSEMM MAIE $k = 3$ Evelyn, (2000)	8.17E-01
ECSSEMM MAIE $k = 4$ Evelyn, (2000)	1.62E-01
EEMM MAIE $k = 2$ Bahar, (2019)	1.84E+00
EEMM MAIE $k = 3$ Bahar, (2019)	3.47E-01
EEMM MAIE $k = 4$ Bahar, (2019)	8.73E-01
EBSM MAIE $k = 2$ Osu et al. (2021)	9.04E-01
EBSM MAIE $k = 3$ Osu et al. (2021)	8.05E-01
EBSM MAIE $k = 4$ Osu et al. (2021)	8.06E-01

Table 6: Comparing the Minimum Absolute Instability Errors (MAIEs) of EBAMM for $k = 2, 3$ and 4 with (Evelyn, 2000; Bahar, 2019; Osu et al., 2021) for fixed partition size $d = 0.01$ using Problem 2

Numerical Method	Compared MAIEs with (Evelyn, 2000; Bahar, 2019;
------------------	--

Osu et al., 2021)	
EBAMM MAIE k = 2	4.69E-04
EBAMM MAIE k = 3	3.93E-05
EBAMM MAIE k = 4	1.94E-06
ECSSEMM MAIE k = 2 Evelyn, (2000)	5.76E-01
ECSSEMM MAIE k = 3 Evelyn, (2000)	6.17E-01
ECSSEMM MAIE k = 4 Evelyn, (2000)	4.62E-01
EEMM MAIE k = 2 Bahar, (2019)	2.84E+00
EEMM MAIE k = 3 Bahar, (2019)	3.47E-01
EEMM MAIE k = 4 Bahar, (2019)	8.73E-01
EBSM MAIE k = 2 Osu et al. (2021)	9.04E-01
EBSM MAIE k = 3 Osu et al. (2021)	8.05E-01
EBSM MAIE k = 4 Osu et al. (2021)	9.06E-01

Graphical display for compared results

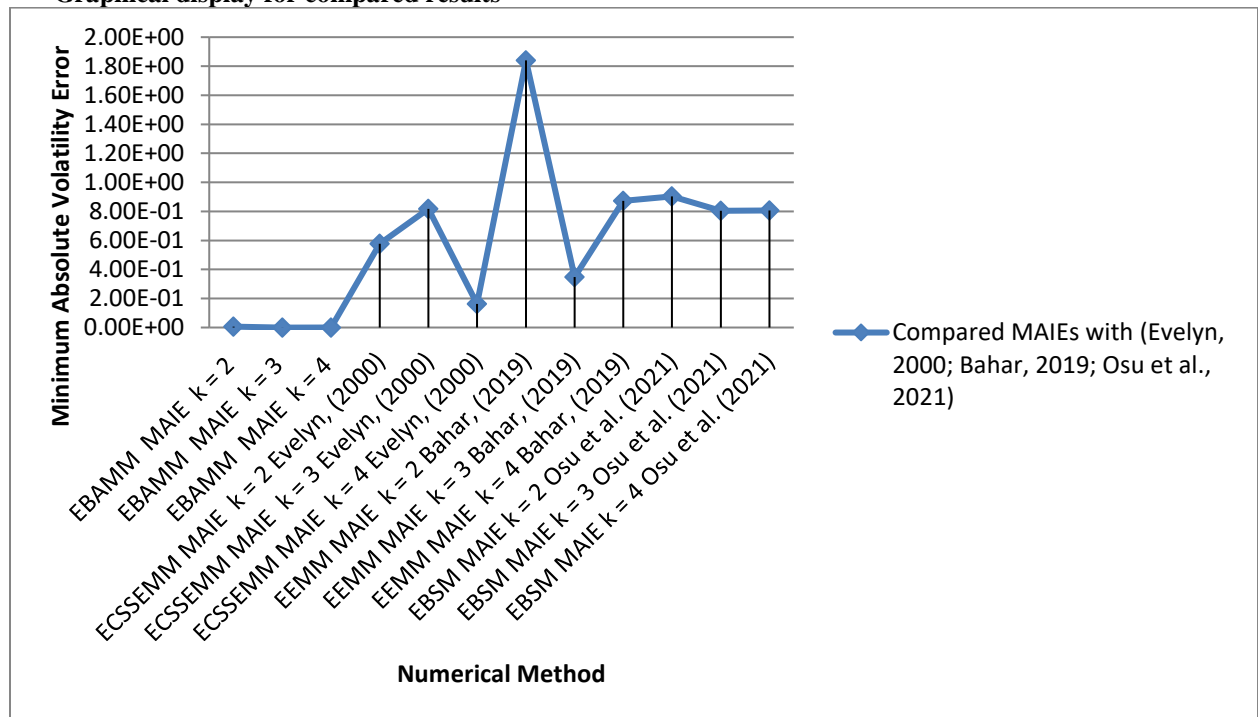


Figure 7: Compared Minimum Absolute Instability Errors (MAIEs) of EBAMM for k = 2, 3 and 4 with Minimum Absolute Instability Errors (MAIEs) of (Evelyn, 2000; Bahar, 2019; Osu et al., 2021) for Problem 1

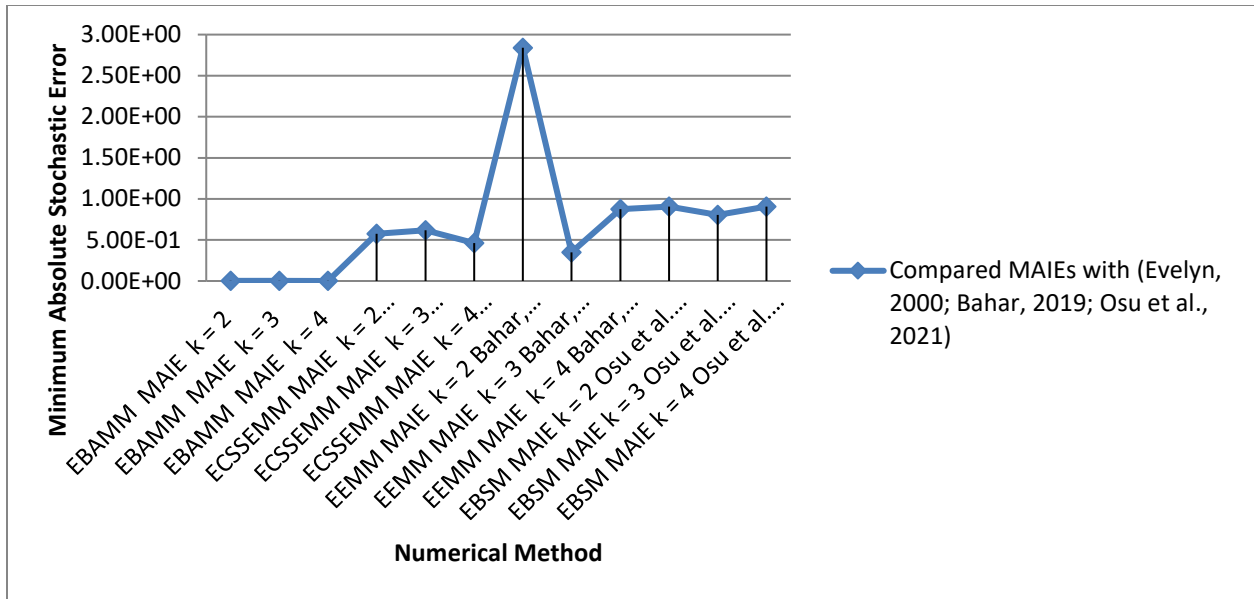


Figure 8: Compared Minimum Absolute Instability Errors (MAIEs) of EBAMM for $k = 2, 3$ and 4 with Minimum Absolute Instability Errors (MAIEs) of (Evelyn, 2000; Bahar, 2019; Osu et al., 2021) for Problem 2

The Absolute Instability Errors (MAIEs) obtained after the application of the method in evaluating the modeled equation as computed and presented in tables and figures above shows the instability fluctuations of consumers' buying behavior under economic instability in Nigeria.

Conclusion

This study has demonstrated that Extended Block Adams Moulton Methods (EBAMM) is suitable for evaluating some Econometric Instability Time-Delay Differential Equation (EITDDE). In tables 1 to 4 and figures 7 to 8, the Absolute Instability Errors (MAIEs) of step number $k = 4$ of EBAMM produced better and faster numerical results than the partition numbers $k = 3$ and 2 by giving the Least Minimum Absolute Instability Errors (LMAIEs) at a Lower Computational Time (LCT). Comparing the Minimum Absolute Instability Errors (MAIEs) of this method with established methods in literature as shown in tables 5 to 6 and figures 7 to 8, the schemes of EBAMM performed more accurate by giving Least Minimum Absolute Instability Errors (LMAIEs) than other methods that applied interpolation methods in computing the delay term and the instability term. The lower the Absolute Instability Error (AIE), the lower the volatility effects of economic instability on consumers' buying behaviour in Nigeria.

Recommendations

Following the Least Minimum Absolute Instability Errors (LMAIEs) obtained in solving some examples of EITDDE with the proposed method; this study recommends that;

- the Nigerian government should adopt more robust and consistent monetary policies and diversify its economy by investing in non-oil sectors such as agriculture, technology, and manufacturing in order to create a more stable and resilient economic environment and to boost consumers' buying behaviour.
- More research should be carried-out using the proposed method for partition numbers $k = 5, 6, 7, \dots$ in obtaining the computational results of the modeled equation.

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