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A Constrained Multi-Item Inventory Model under Probabilistic Demand Using Linear Programming Techniques

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Abstract
This study involves inventory management model to obtain ordering policy that maximizes profits of several items with constraints under probabilistic demand. The goal is to obtain an optimal EOQ, in order to maximize profit for each of the state of demand for the items. The decision to order more units of items or not to order additional units are given as the decision variables which are represented by zero (0) and one (1), these decisions are made at the beginning of each planning period using 2-period dynamic programming technique (DP). In order to handle the constraints given by the management, the Linear programming technique (LP) is used. The application of the model was demonstrated by a numerical illustration.

Keywords: Economic Order Quantity (EOQ), Dynamic Programming (DP), Inventory Management, Probabilistic Demand, Linear Programming (LP), Multiple Item.

Introduction

The EOQ models can be classified into single-item and multiple-item inventory model. The classical EOQ models were deterministic and single-item models but the multiple-item EOQ models are more realistic and more applicable than the single-item model because it reflects the complexity of managing inventories of multiple products which involves varying demand patterns, lead times and costs. In real-world scenarios, businesses typically deal with a diverse range of products, each requiring different inventory management strategies to optimize the factors like stock outs, carrying costs and setup costs.

The EOQ is a fundamental concept in inventory management that determines the optimal order quantity a company should buy in order to minimize the total inventory costs which includes the carrying costs and the setup costs. In inventory management, EOQ models are classified into deterministic and stochastic inventory model based on the nature of demand involved. The stochastic inventory model refers to the demand that is not predictable, that is, it incorporates variations of the demand and uncertain lead time which makes it more realistic than the deterministic model. The deterministic inventory model refers to a type of demand that is predictable and constant over time, there are no fluctuations or uncertainty involved in the demand pattern, the demand remains stable and does not vary from one period to another.

When inventory consists of multiple items without constraints, the EOQ can be found separately but when the inventory consists of several items with some constraints or limitations, then it is not possible to consider the EOQ of each item separately because of the interactions that may exist between the items. In that case, Linear programming techniques can be used to handle the constraints. Addressing such problem, we first of all solve the problem by ignoring the limitations and then later consider the effect of the limitations.

Literature Review

Kaspi and Rosenblatt (1991) provided a method for a joint replenishment problem using the basic cycle approach. The solution involves the comparison of possible values between the minimum and maximum cycle times. The method shows that the solution procedure is superior to the known algorithms for solving the multiple item problems.

Ben-Daya and Raouf (1993) studied a single period multiple-item inventory problem by considering budgetary and space constraints. The proposed model was simple to implement and allows easy sensitivity analysis in order to note the parameter changes.

Mubiru (2013) developed an economic order quantity model for multiple items under probabilistic demand under periodic review inventory system. The states of demand were represented by Markov chain in which decision variables are represented as to ordering additional products (z=1) and not ordering additional products (z=0) using dynamic programming (DP) technique. The optimal ordering policies and the corresponding total inventory costs for each of the items were obtained for both the favorable and unfavorable states across the planning horizons.

Purohit and Rathore (2012) studied an inventory management model for perishable products with instantaneous production and supply by considering deterministic demand with zero lead time and uniform demand. The Kuhn-Tucker theorem was used to obtain total inventory costs for the item.

Hariga et al. (2007) proposed an optimization model for profit maximization for retailers with constraints on shelf space and storage. The items to be displayed in the store, the locations and the order quantities were considered as decision variables for the model. The mixed integer and non-linear program were the model methodology which was solved using the LINGO software.

Yu and Li (2000) developed a stochastic optimization model that deals with decision making that involves uncertainty supply chain management. The proposed method transformed the model into a linear programming model with number of scenarios and total constraints respectively.

Zhang and Wang (2011) considered a multi-item EOQ model to determine an optimal policy for inventory problem which involves the analyses of the structural properties. They introduced a simple algorithm for generating the optimal solution which was represented by numerical example to demonstrate the application of the model.

Fergany (2016) proposed a multi-item model with single-source model which considers varying mixture shortage cost under two restrictions. The model analyzed how the firm can obtain an optimal order quantity and reorder point for each item in order to minimize the expected total cost.

Sutrisno and Widowati (2016) proposed a mathematical model in stochastic dynamic optimization form to determine the optimal strategy for an integrated single-item inventory control problem and supplier selection problem where the demand and purchasing cost parameters were random. The proposed model was used to calculate the optimal product volume purchased from the optimal supplier so that the inventory level would be located at some point as close as possible to the reference point with minimal cost. They used stochastic dynamic programming to solve the problem and gave several numerical experiments to evaluate the model.

Özen et al. (2012) considered a single-stage inventory model facing non-stationary stochastic demand of the customers in a finite planning horizon. The replenishment times were to be determined at the beginning of the horizon while decisions on the exact replenishment quantities could be deferred until the replenishment time. They considered dynamic fixed-ordering and linear end-of-period holding cost, as well as dynamic penalty costs, or service level. They proved that the optimal ordering policy is a base stock policy for both penalty cost and service level constrained models.

Gutierrez-Alcoba et al. (2017) studied a single-item, single-stocking location and non-stationary stochastic lot sizing problem for a perishable product. They considered fixed and proportional ordering cost, holding cost and penalty cost. The item featured a limited shelf life and therefore they considered a variable cost of disposal. The authors derived exact analytical expressions to determine the expected value of the inventory at different stages. They also discussed a good approximation for the case in which the shelf-life was limited. To tackle the problem, they introduced two new heuristics that extended Silver's heuristic and compared them to an optimal stochastic dynamic programming policy by a numerical study which demonstrated the effectiveness of the approach.

Model Development

An inventory system of multiple items under probabilistic demand was considered. The model includes two states, that is the favorable state (f) and unfavorable state (u) with z as a binary decision variable which is to order or not to order. The maximum expected profits are obtained for each planning horizon. The optimal policy specifies how much to order at each period, depending on the current inventory level and demand.

Model Notations

$i, j =$ States of demand

$n, N =$ Stages going from n=1 to N=2.

w = Number designating the wth item for m = 1, 2…W.

$D^z(\text{w}) =$ Demand matrix for the wth item

$I^z(\text{w}) =$ Inventory matrix for the wth item

$P^z(\text{w}) =$ Profit matrix for the wth item

$P_s(\text{w}) =$ Selling price per unit for the wth item

$P_{ij}(\text{w}) =$ Profit in state i, j for the wth item

$Q^z(\text{w}) =$ Demand transition matrix for the wth item

$Q_{ij}^z(\text{w}) =$ Demand transition probability in state i, j for wth item

$I_{ij}^z(\text{w}) =$ Quantity of inventory in state i, j for wth item

$D_{ij}^z(\text{w}) =$ Quantity demanded in state i, j for wth item

$O_i^z(\text{w}) =$ Economic order quantity for the wth item

$e_i^z(\text{w}) =$ Expected total profits for the wth item

$a_i^z(\text{w}) =$ Accumulated total profits for the wth item

$g_n(i, \text{w}) =$ Optimal expected total inventory cost for the wth item

$C_h(\text{w}) =$ Holding cost per unit for the wth item

$C_o(\text{w}) =$ Ordering cost per unit for the wth item

$C_s(\text{w}) =$ Shortage cost per unit for the wth item

$C_p(\text{w}) =$ Cost price per unit for the wth item

We modelled the change in demand by using the Markov Chain approach, given transition matrix and demand matrix for the wth item as represented below:

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$$Q_{ij}^{z}(\mathrm{w}) = \begin{pmatrix} Q_{ff}^{z}(\mathrm{w}) & Q_{fu}^{z}(\mathrm{w}) \\ Q_{uf}^{z}(\mathrm{w}) & Q_{uu}^{z}(\mathrm{w}) \end{pmatrix}$$

$$D_{ij}^{z}(\mathrm{w}) = \begin{pmatrix} D_{ff}^{z}(\mathrm{w}) & D_{fu}^{z}(\mathrm{w}) \\ D_{uf}^{z}(\mathrm{w}) & D_{uu}^{z}(\mathrm{w}) \end{pmatrix}$$

The inventory and profit matrices are represented as follows

$$I_{ij}^{z}(\mathrm{w}) = \begin{pmatrix} I_{ff}^{z}(\mathrm{w}) & I_{fu}^{z}(\mathrm{w}) \\ I_{uf}^{z}(\mathrm{w}) & I_{uu}^{z}(\mathrm{w}) \end{pmatrix}$$

$$P_{ij}^{z}(\mathrm{w}) = \begin{pmatrix} P_{ff}^{z}(\mathrm{w}) & P_{fu}^{z}(\mathrm{w}) \\ P_{uf}^{z}(\mathrm{w}) & P_{uu}^{z}(\mathrm{w}) \end{pmatrix}$$

Demand Transition Matrix, Profit Matrix and EOQ for the wth item

For the ordering policy $z \in \{0,1\}$, the demand transition probability from state i to j can be represented as the quantity demanded when the demand is in state i and then transiting to state j divided by the total quantity demanded over all states, as represented in the following equation.

$$Q_{ij}^{z}(\mathrm{w}) = \frac{D_{ij}^{z}(\mathrm{w})}{[D_{if}^{z}(\mathrm{w}) + D_{iu}^{z}(\mathrm{w})]};\ i \in \{f,u\};\ z \in \{0,1\} \tag{1}$$

For the case when demand is greater than the inventory at hand, the profit matrix P^{z} can be computed as follows:

Profit = selling price – cost price

$$P_{ij}(\mathrm{w}) = P_{s}(\mathrm{w}) - C_{p}(\mathrm{w}) \tag{2}$$

Therefore,

$$P^{z}(\mathrm{m}) = P_{ij}(\mathrm{w})\{(D^{z}(\mathrm{w}) - (C_{o}(\mathrm{w}) + C_{h}(\mathrm{w})\}I^{z}(\mathrm{w}) - \{C_{o}(\mathrm{w}) + C_{s}(\mathrm{w})\}[D^{z}(\mathrm{w}) - I^{z}(\mathrm{w})] \tag{3}$$

For the case when the demand is less than or equal to the inventory at hand, then we have:

$$P^{z}(\mathrm{w}) = P_{ij}(\mathrm{w})D_{ij}^{z}(\mathrm{w}) - \{C_{o}(\mathrm{w}) + C_{h}(\mathrm{w})\}\,\mathrm{D}_{ij}^{z}(\mathrm{w})$$

$$\Rightarrow$$

$$P^{z}(\mathrm{w}) = \begin{cases} P_{ij}(\mathrm{w})D_{ij}^{z}(\mathrm{w}) - \{C_{o}(\mathrm{w}) + C_{h}(\mathrm{w})\}\mathrm{I}_{ij}^{z}(\mathrm{w}) - \{C_{o}(\mathrm{w}) + C_{s}(\mathrm{w})\}[D_{ij}^{z}(\mathrm{w}) - I_{ij}^{z}(\mathrm{w})] & if \quad D_{ij}^{z}(\mathrm{w}) > I_{ij}^{z}(\mathrm{w}) \\ P_{ij}(\mathrm{w})D_{ij}^{z}(\mathrm{w}) - \{C_{o}(\mathrm{w}) + C_{h}(\mathrm{w})\}\mathrm{D}_{ij}^{z} & if \quad D_{ij}^{z}(\mathrm{w}) \le I_{ij}^{z}(\mathrm{w}) \end{cases}$$

$$\forall \quad i, j \in \{f,u\};\ z \in \{0,1\};\ w = 1,2,3,\ldots \mathrm{W} \tag{4}$$

To justify the above Equation, $D_{ij}^{z}(\mathrm{w}) - I_{ij}^{z}(\mathrm{w})$ units of items must be ordered to meet up with the excess demand of customers. The extra units ordered will attract shortage cost and carrying cost but for the situation where demand is less than or equal to inventory at hand, then no addional units of item will be ordered. The EOQ can be computed using the following equation:

$$O_i^z(\mathrm{w}) = \{D_{if}^z(\mathrm{w}) - I_{if}^z(\mathrm{w})\} + \{D_{iu}^z(\mathrm{w}) - I_{iu}^z(\mathrm{w})\};\ D_{ij}^z(\mathrm{w}) > I_{ij}^z(\mathrm{w}) \quad (5)$$

$$i, j \in \{f, u\},\ z \in \{0,1\}$$

Otherwise, $O_i^z = 0$

The EOQ for the two planning horizons

We consider a two-period (N=2) planning horizon for the multi-item inventory system in this section.

First planning horizon

The ordering policy favorable demand is

$$z = \begin{cases} 1, & if\ e_f^1(\mathrm{w}) > e_f^0(\mathrm{w}) \\ 0, & if\ e_f^1(\mathrm{w}) \le e_f^0(\mathrm{w}) \end{cases} \quad (6)$$

The associated total profit and EOQ are as follows:

$$g_1(\mathrm{f}, \mathrm{w}) = \begin{cases} e_f^1(\mathrm{w}), & if\ z = 1 \\ e_f^0(\mathrm{w}), & if\ z = 0 \end{cases} \quad (7)$$

and

$$O_f^z(\mathrm{w}) = \begin{cases} \{D_{ff}^1(\mathrm{w}) - I_{ff}^1(\mathrm{w})\} + \{D_{fu}^1(\mathrm{w}) - I_{fu}^1(\mathrm{w})\}; & if\ z = 1 \\ 0; & if\ z = 0 \end{cases};\ D_{ij}^z(\mathrm{w}) > I_{ij}^z(\mathrm{w}) \quad (8)$$

Also the ordering policy unfavorable demand is as follows:

$$z = \begin{cases} 1, & if\ e_u^1(\mathrm{w}) > e_u^0(\mathrm{w}) \\ 0, & if\ e_u^1(\mathrm{w}) \le e_u^0(\mathrm{w}) \end{cases} \quad (9)$$

The total profits and EOQ are represented as follows:

$$g_1(u, \mathrm{w}) = \begin{cases} e_u^1(\mathrm{w}), & if\ z = 1 \\ e_u^0(\mathrm{w}), & if\ z = 0 \end{cases} \quad (10)$$

and

$$O_u^z(\mathrm{w}) = \begin{cases} \{D_{uf}^1(\mathrm{w}) - I_{uf}^1(\mathrm{w})\} + \{D_{uu}^1(\mathrm{w}) - I_{uu}^1(\mathrm{w})\}; & if\ z = 1 \\ 0; & if\ z = 0 \end{cases};\ D_{ij}^z(\mathrm{w}) > I_{ij}^z(\mathrm{w}) \quad (11)$$

Second planning horizon

By considering the recursive equation (1) for the dynamic programming problem, $a_i^z(\mathrm{w})$ represents the accumulated total profit for item w. at the end of period 1 which is as a result of decisions made during that period. The accumulated profit can be computed using the following equation:

$$a_i^z(\mathrm{w}) = e_i^z(\mathrm{w}) + Q_{if}^z(\mathrm{w}) g_1(f, \mathrm{w}) + Q_{iu}^z(\mathrm{w}) g_1(u, \mathrm{w}) \quad (12)$$

The ordering policy when the demand is favorable is as follows:

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$$z = \begin{cases} 1, & if\ a_f^1(\mathrm{w}) > a_f^0(\mathrm{w}) \\ 0, & if\ a_f^1(\mathrm{w}) \le a_f^0(\mathrm{w}) \end{cases}$$

The total profits and EOQ are:

$$g_2(f,\mathrm{w}) = \begin{cases} a_f^1(\mathrm{w}), & if\ z = 1 \\ a_f^0(\mathrm{w}), & if\ z = 0 \end{cases} \tag{13}$$

and

$$O_f^z(\mathrm{w}) = \begin{cases} \{D_{ff}^1(\mathrm{w}) - I_{ff}^1(\mathrm{w}) + \{D_{fu}^1(\mathrm{w}) - I_{fu}^1(\mathrm{w})\}; & if\ z = 1 \\ 0; \ if\ z = 0 \end{cases} ;\ D_{ij}^z(\mathrm{w}) > I_{ij}^z(\mathrm{w}); \tag{14}$$

The ordering policy when demand is unfavorable is

$$z = \begin{cases} 1, & if\ a_u^1(\mathrm{w}) > a_u^0(\mathrm{w}) \\ 0, & if\ a_u^1(\mathrm{w}) \le a_u^0(\mathrm{w}) \end{cases} \tag{15}$$

The total profit and EOQ are

$$g_2(u,\mathrm{w}) = \begin{cases} a_u^1(\mathrm{w}), & if\ z = 1 \\ a_u^0(\mathrm{w}), & if\ z = 0 \end{cases} \tag{16}$$

and $$O_u^z(\mathrm{w}) = \begin{cases} \{D_{uf}^1(\mathrm{w}) - I_{uf}^1(\mathrm{w}) + \{D_{uu}^1(\mathrm{w}) - I_{uu}^1(\mathrm{w})\}; & if\ z = 1 \\ 0; \ if\ z = 0 \end{cases} ;\ D_{ij}^z(\mathrm{w}) > I_{ij}^z(\mathrm{w})$$

Finding the Appropriate EOQ's using the Constraints

After obtaining the EOQ's for w = 1, 2 … W, i.e. $O_1, O_2 \ldots O_w$, we then adjust them using the constraints. The aim is to obtain the optimal quantity ordered for each item while considering the constraints which may include the Budget constraint, Warehouse constraint, Inventory level constraint and Order Quantity constraint. The constraints are represented as follows:

i. Warehouse Constraint:

$$\sum_{w=1}^{W} f_w O_w \le V$$

where $f_w =$ storage space for wth item

V = space limit for all items

ii. Capital Constraint:

$$\sum_{w=1}^{W} C_w O_w \le F$$

where $C_w =$ purchase price for wth item

F = Investment limit for all items

iii. Average Inventory level constraint:

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$$\frac{1}{2}\sum_{w=1}^{W} O_w \le M$$

where M = Upper limit for average number of items in inventory

iv. Order Quantity constraint:

$$\sum_{w=1}^{W} O_w \le A$$

where A is the maximum total order quantity for all items

Steps for the solution of the multi-item EOQ model

The following steps are followed in finding the appropriate EOQ for constrained multi-item inventory model:

STEP I: Compute the unconstrained EOQ's using equation (12) above.

STEP II: Check to see if the unconstrained EOQ's satisfy the constraints. If they do, then step 1 gives the optimal solution otherwise go to STEP III.

STEP III: Apply the given constraints using linear programming technique to determine the constrained optimal values of the problem as follows:

$$\text{Maximize } P_1O_1 + P_2O_2 + \ldots + P_WO_W$$

$$\text{Subject to } a_{11}O_1 + a_{12}O_2 + \ldots + a_{1n}O_W \le = \ge b_{1,}$$

$$a_{21}O_1 + a_{22}O_2 + \ldots + a_{2n}O_W \le = \ge b_{2,}$$

$$\vdots$$

$$a_{K1}O_1 + a_{K2}O_2 + \ldots + a_{Kn}O_W \le = \ge b_{K,}$$

$$O_1, O_2, \ldots, O_W \ge 0.$$

where $P_1, P_2, \ldots, P_W$ represent the profit coefficients per unit of output of the order quantity $O_1, O_2, \ldots, O_W$, a_{ij} is the amount of resource i needed to order one unit of j and b is the amount of resource i available for $i = 1, \ldots, K$ and $j = 1, \ldots, W$. It is required that the constraints $b_1, b_2, \ldots, b_K$ be non-negative. The decision maker may also give additional constrains by setting limits on the individual order quantities. For example, $O_1 \ge S_1$; where S_1 is the minimum order quantity for i. These constraints can be generally represented as: $O_1 \ge S_1, O_2 \ge S_2, \ldots, O_W \ge S_W$

Numerical Example

A company needs to decide the EOQ for three items (1, 2, and 3) whose demands are stochastic. The aim is to maximize the total profit while considering order quantity constraint, warehouse constraint and capital constraint. The profits per unit of the items are $800, $600, and $800 respectively. Management requires minimum order quantities for the three items to be, 6units, 0 unit and, 6units respectively. The total available capital for purchasing inventory is $10,000 and the total warehouse space available is sufficient for 600 units of item. The following matrices describes the demand and inventory for the three items:

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ITEM 1:

The selling price and the associated total inventory costs for item 1 are given as: The unit selling price $P_s(1)$ is \$1500, the ordering cost $C_o(1)$ is \$500, the holding cost $C_h(1)$ is \$50, the shortage cost $C_s(1)$ is \$40 and the cost price $C_p(1)$ is \$700.

For w=1, z=1

$$D^1(1) = \begin{pmatrix} 23 & 20 \\ 18 & 09 \end{pmatrix}$$

$$I^1(1) = \begin{pmatrix} 20 & 16 \\ 09 & 14 \end{pmatrix}$$

For w=1, z=0

$$D^0(1) = \begin{pmatrix} 20 & 18 \\ 16 & 05 \end{pmatrix}$$

$$I^0(1) = \begin{pmatrix} 18 & 20 \\ 28 & 09 \end{pmatrix}$$

ITEM 2:

The selling price and the associated total inventory costs for item 2 are given as: The unit selling price $P_s(2)$ is \$1400, the ordering cost $C_o(2)$ is \$400, the holding cost $C_h(2)$ is \$100, the shortage cost $C_s(2)$ is \$150 and the cost price $C_p(2)$ is \$800.

For w=2, z=1

$$D^1(2) = \begin{pmatrix} 20 & 14 \\ 18 & 12 \end{pmatrix}$$

$$I^1(2) = \begin{pmatrix} 17 & 12 \\ 15 & 08 \end{pmatrix}$$

For w=2, z=0

$$D^0(2) = \begin{pmatrix} 24 & 21 \\ 28 & 14 \end{pmatrix}$$

$$I^0(2) = \begin{pmatrix} 20 & 17 \\ 16 & 15 \end{pmatrix}$$

ITEM 3:

The selling price and the total inventory costs for item 3 are given as: The unit selling price $P_s(3)$ is \$1500, the ordering cost $C_o(3)$ is \$500, the holding cost $C_h(3)$ is \$100, the shortage cost $C_s(3)$ is \$100 and the cost price $C_p(3)$ is \$700.

For w=3, z=1

$$D^1(3) = \begin{pmatrix} 25 & 21 \\ 18 & 14 \end{pmatrix}$$

$$I^1(3) = \begin{pmatrix} 22 & 19 \\ 14 & 10 \end{pmatrix}$$

For w=3, z=0

$$D^0(3) = \begin{pmatrix} 14 & 20 \\ 24 & 18 \end{pmatrix}$$

$$I^0(3) = \begin{pmatrix} 20 & 14 \\ 19 & 20 \end{pmatrix}$$

The Model Parameters

For w=1, z=1

$$P_{ij}(1) = P_s(1) - C_p(1)$$

$$P_{ij}(1) = 1500 - 700 = 800$$

The demand and profit matrix can be obtained using equations (7) and (11) as follows:

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$$Q^1(1) = \begin{pmatrix} \frac{23}{43} & \frac{20}{43} \\ \frac{18}{27} & \frac{09}{27} \end{pmatrix} = \begin{pmatrix} 0.535 & 0.465 \\ 0.667 & 0.333 \end{pmatrix}$$

$$P^1(1) = 800\begin{pmatrix} 23 & 20 \\ 18 & 09 \end{pmatrix} - 550\begin{pmatrix} 20 & 16 \\ 09 & 14 \end{pmatrix} - 540\begin{pmatrix} 03 & 04 \\ 09 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 18400 & 16000 \\ 14400 & 7200 \end{pmatrix} - \begin{pmatrix} 11000 & 4400 \\ 4950 & 7700 \end{pmatrix} - \begin{pmatrix} 1620 & 1080 \\ 4860 & -2700 \end{pmatrix}$$

$$\therefore \ P^1(1) = \begin{pmatrix} 5780 & 5040 \\ 4590 & 2200 \end{pmatrix}$$

For w=1, z=0

$$Q^0(1) = \begin{pmatrix} \frac{20}{38} & \frac{18}{38} \\ \frac{16}{21} & \frac{05}{21} \end{pmatrix} = \begin{pmatrix} 0.526 & 0.474 \\ 0.762 & 0.238 \end{pmatrix}$$

$$P^0(1) = 800\begin{pmatrix} 20 & 18 \\ 16 & 05 \end{pmatrix} - 550\begin{pmatrix} 20 & 18 \\ 16 & 05 \end{pmatrix}$$

$$= \begin{pmatrix} 16000 & 14400 \\ 12800 & 4000 \end{pmatrix} - \begin{pmatrix} 11000 & 9900 \\ 8800 & 2750 \end{pmatrix}$$

$$\therefore \ P^0(1) = \begin{pmatrix} 5000 & 4500 \\ 4000 & 1250 \end{pmatrix}$$

The expected total profit for w = 1

The matrices $Q^1(1)$ and $P^1(1)$ are combined in the following equation to obtain the accumulated profits for z=1, w =1:

Using equation $a^1(1) = Q^1(1)[\mathrm{P}^1(1)]^T$

$$a^1(1) = \begin{pmatrix} 0.535 & 0.465 \\ 0.667 & 0.333 \end{pmatrix}\begin{pmatrix} 5780 & 5040 \\ 4590 & 2200 \end{pmatrix}^T$$

$$= \begin{pmatrix} 0.535 & 0.465 \\ 0.667 & 0.333 \end{pmatrix}\begin{pmatrix} 5780 & 4590 \\ 5040 & 2200 \end{pmatrix}$$

$$a^1_{1,f}(1) = 5780(0.535) + 5040(0.465) = 5,436$$

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$$a_{1,u}^{1}(1) = 4590(0.667) + 2200(0.333) = 3,794$$

The matrices $Q^0(1)$ and $P^0(1)$ combined to obtain profits when additional unit of items are ordered, for z=0, w =1

Using equation $a^0(1) = Q^0(1)[\mathrm{P}^0(1)]^T$

$$a^0(1) = \begin{pmatrix} 0.526 & 0.474 \\ 0.762 & 0.238 \end{pmatrix} \begin{pmatrix} 5000 & 4500 \\ 4000 & 1250 \end{pmatrix}^T$$

$$= \begin{pmatrix} 0.526 & 0.474 \\ 0.762 & 0.238 \end{pmatrix} \begin{pmatrix} 5000 & 4000 \\ 4500 & 1250 \end{pmatrix}$$

$$a_{1,f}^{0}(1) = 5000(0.526) + 4500(0.474) = 4,763$$

$$a_{1,u}^{0}(1) = 4000(0.762) + 1250(0.238) = 3,346$$

The optimal ordering policy and EOQ for item 1 (w= 1)

For the first planning horizon, it shows that z =1 is the optimal ordering policy for the favorable state with $5,436 which is greater than $4,763 with associated profit of $5,436 and EOQ of 7units.

Also, for the unfavorable state, z = 1 is the optimal ordering policy with $3,794 which is greater than $3,346 with associated profit of $3,794 and EOQ of 7units.

For the second planning horizon, the accumulated profits for the favorable and unfavorable states can be obtained using equation (19)

For the favorable state, we have:

$$a_{2,f}^{1}(1) = 5436 + 5436(0.535) + 3794(0.465) = 10,109$$

$$a_{2,f}^{0}(1) = 4763 + 5436(0.526) + 3794(0.474) = 9,421$$

The above result shows that z =1 is the optimal ordering policy for the favorable state with $10,109 as the associated profit which is greater $9,421 and EOQ of 7 units.

For the unfavorable state, we have:

$$a_{2,u}^{1}(1) = 3794 + 5436(0.667) + 3794(0.333) = 8,683$$

$$a_{2,u}^{0}(1) = 3346 + 5436(0.762) + 3794(0.238) = 8,391$$

The above result shows that z =1 is the optimal ordering policy for the unfavorable state with $8,683 as the profit which is greater $8,391 and EOQ of 7unit.

Table 1: Profits and EOQ's for item 1

FIRST PLANNING	Z=1	Z=0
$a_{1,f}^{z}(1)$	$5,436	$4,763
$a_{1,u}^{z}(1)$	$3,794	$3,346
$o_{f}^{z}(1)$	7	0
$o_{u}^{z}(1)$	7	0
SECOND PLANNING	Z=1	Z=0
$a_{2,f}^{z}(1)$	$10,109	$9,421
$a_{2,u}^{z}(1)$	$8,683	$8,391

Similar computations were carried out for items 2 and 3 and the results obtained are shown in the following tables:

Table 2: Profits and EOQ's for item 2

FIRST PLANNING	Z=1	Z=0
$a_{1,f}^{z}(2)$	$1,623	$2,260
$a_{1,\mathrm{u}}^{z}(2)$	$1,390	$2,334
$o_{1,f}^{z}(2)$	0	0
$o_{1,u}^{z}(2)$	0	0
SECOND PLANNING	Z=1	Z=0
$a_{2,f}^{z}(2)$	$3,914	$4,555
$a_{2,\mathrm{u}}^{z}(2)$	$3,680	$4,619

Table 3: Profits and EOQ's for item 3

FIRST PLANNING	Z=1	Z=0
$a_{1,f}^{z}(3)$	$4,634	$3,506
$a_{1,\mathrm{u}}^{z}(3)$	$3,250	$4,285
$o_{f}^{z}(3)$	5	0
$o_{u}^{z}(3)$	0	0
SECOND PLANNING	Z=1	Z=0
$a_{2,f}^{z}(3)$	$9,109	$7,935
$a_{2,\mathrm{u}}^{z}(3)$	$7,732	$8,769

From the above, the EOQ for the items are:

$$O_1 = 7units$$

$$O_2 = 0$$

$$O_3 = 5units$$

The above unconstrained order quantities do not satisfy the constraints of the company as such, the LP method will be applied as shown below:

The Objective Function:

Maximize $800O_1 + 600O_2 + 800O_3$

Constraints:

1. Capital Constraint:

   $$700O_1 + 800O_2 + 700O_3 \leq 10,000$$

2. Order Quantity Constraints:

   $$\mathrm{O}_1, \mathrm{O}_3 \geq 6,\ O_2 = 0$$

3. Warehouse Space Constraint:

   $$O_1 + O_3 \leq 600$$

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4. Non-negativity and integer constraints:

$$O_1, O_2, O_3 \geq 0$$

Formulation: The complete LP problem can be formulated as follows:

Maximize $800O_1 + 800O_3$

Subject to: $700O_1 + 700O_3 \leq 10,000$

$$\mathrm{O}_1, \mathrm{O}_3 \geq 6$$

$$O_1 + O_2 + O_3 \leq 600$$

$$O_1, O_2, O_3 \geq 0$$

The above problem has been solved using the Matlab software. The optimal order quantities obtained are:

$$O_1 = 6 units$$

$$O_2 = 0 units$$

$$O_3 = 8 units$$

To obtain the optimal profit, we substitute the values into the objective function as follows:

$800(6) + 600(0) + 800(8) = 11,200$ which gives ₦11,200 as the optimal profit.

Conclusion and Recommendation

This work has analyzed a constrained multi-item EOQ model under stochastic demand using dynamic programming approach and linear programming to deal with the constraints. We considered three items in inventory under the constraints of capital, order quantity and warehouse, with each item having different holding cost, ordering cost, shortage cost, cost price and selling price respectively where the demand for each item over the planning horizon was classified as favorable state or unfavorable state. We obtained the EOQ for the three items using the dynamic programming approach which we realized did not satisfy the constraints given by the management and then we applied the LP method to incorporate the constraints in order to obtain an optimal profit. Future research can be carried out by increasing the number of items and constraints using other optimization techniques like the value iteration method, policy iteration method, and geometric programming method e.t.c.

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