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Application of the Hermite Series Kernel Density Estimator in Identifying the Confrontational Hazards of Maritime Piracy on Nigeria's Economic Growth

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Abstract
Maritime piracy prevalence is a global phenomenon that anchors mostly on the illegal lifting of crude oil in the Nigeria waters. It is an illegal act of depredation, committed for isolated ends by the crews of private sea vessels which has degraded the economic growth of the Nigeria economy. This research aims to examine the hazardous effects of piracy in the Niger Delta region (NDR) using novel nonparametric density estimation methods on the nation's economy. The maritime piracy data showed that there are five-hundred and ninety-four (594) menace incidence in the NDR that negatively affected the annual gross domestic product (GDP) and annual incomes in million Dollars in the Nigeria economy within the period of 2002-2024. The positive economic growth index was recorded in 2002 and negative growth rates recorded in 2016 and 2020 by both model estimators. The kernel density estimator (KDE) and Hermite Series kernel density estimator ($HSeKDE$) Mathematical model estimators' visualizations of both economic growth rates and maritime piracy incidence hinged on the model estimators smoothing parameter and the kernel functions. In contrast to the KDE model visualizations, the $HSeKDE$ estimator through the Gaussian kernel and others captured the multiple undulating patterns of piracy incidence persistence and rippled economic growth rates. This was possible with the aid of the smoothing parameter and kernel functions applied in the model estimator to ascertain the influx in the Nigeria economy. Also, the asymptotic mean integrated squared error ($AMISE$) is the error criterion whose least value of 0.00000125586 and 0.0000460329 were obtained for maritime piracy incidence, while that of economic growth rate is 0.0000010743 and 0.000039378 using the Epanechnikov and Gaussian kernels. These $AMISE$ values obtained from the sea theft incidence affirmed its occurrence in the Nigeria seas while that of the economic growth rates affirmed the indemnities of the incessant sea theft in the Nigeria waters have adversely affected the country's economic growth rate stability. Therefore, we recommend that fund appropriation should be allocated to foster the training of personnel and purchase of modern robotic security equipments to halt all forms of sea theft in the Niger Delta region (NDR).

Keywords: Maritime Piracy, Nonparametric Density Estimation, Hermite Series Kernel Density Estimator, Economic Growth Rate.

Introduction

Maritime piracy is an illegal act of violence of depredation committed for private ends by the crew of a private ship or aircraft carried out on any high seas within the jurisdiction of any State (Yusuf & Mohd, 2022). The major area of its occurrence is the South-South geopolitical zone of the Country which is the heart of the Niger Delta region (NDR). The predominant source of investment in Nigeria economy is from the maritime sector due to abundant natural mineral deposits. And one of such deposit is the crude oil found in the states of this region which has positioned the country as a global icon in the international communities of trade and investment (Umezurike & Ekiyor, 2023).

The golden nature of the crude oil deposit in the sea beds in this region which should have made Nigeria economy becomes very robust, is being confronted with the biggest threat of maritime piracy (Ikechuku, 2019). In this regard, the security of the waterways is paramount for the maritime sector and coastal communities in the Niger Delta region because the waterways are a platform used for the transportation of goods and services due to the provision of resources that support the livelihood (Ezeozue, 2021). Unfortunately, maritime piracy in the region has turned into a social problem which is affecting virtually everyone. Maritime piracy activities include: kidnapping, sea robbery, pipelines vandalism, maritime cult clashes and attacks and lifting of crude oil illegally.

These forms of maritime piracy activities have their share of economic challenges to the Nigeria economic stability (Nwalozie, 2020). The federal government has made several efforts to annihilates the rising trends of maritime piracy activities through several agencies and security organizations. This effort brought the enacting of the suppress piracy menaces and other maritime offences (SPOMO) Act in 2019, Amnesty programmes, Niger Delta Development Commission (NDDC), the military joint task force (JTF), Nigeria maritime and safety agency (NIMASA) and national inland waterways authority (NIWA) to check the excesses of these maritime scoundrels in the Niger Delta region (Egobueze & Ajieh, 2024; Ezeozue, 2019). These efforts by the government to curb any form of maritime security threats continue unabated and still impinge on the economic growth.

Maritime piracy can be viewed from two perspectives: theoretical and conceptual perspectives to have a holistic comprehension of all it entails. The theoretical perceived it as a structural conflict theory (SCT) proposed by Karl Marx in the 18th century in discussing maritime piracy issues. It argued that there is constant competition inherent in every society over scarce and limited resources that made certain members of the societal to violate laws to make a living (Akangbou, 2023). It explicitly explained the exploitation of the working class in capitalist societies, factors of production, bourgeoisie, proletariat, false consciousness and alienation as the crux of societal conflicts. It analyzes gender imbalances, the functioning of societal structures, relationship between individuals and societies as the causes of conflict leading to theft and other vices in the societies (Denton & Harris, 2019b).

Then, the conceptual dimension of maritime piracy is a field of discipline which is synonymous with robbery at sea, seizure, hijacking, violation of enacted laws, kidnapping crewmembers for ransom, theft of cargo and mineral resources, sabotage and other violence against ships or maritime infrastructure that are committed at sea and unauthorized duplication of goods protected by intellectual property law (Abubakar, 2023; Ehiane, 2025). This is the dimension that has become a persistent issue throughout the history of Nigeria. It has become a momentous problem in recent years, especially along coastline of the NDR, which is regarded as the hub of piracy activities.

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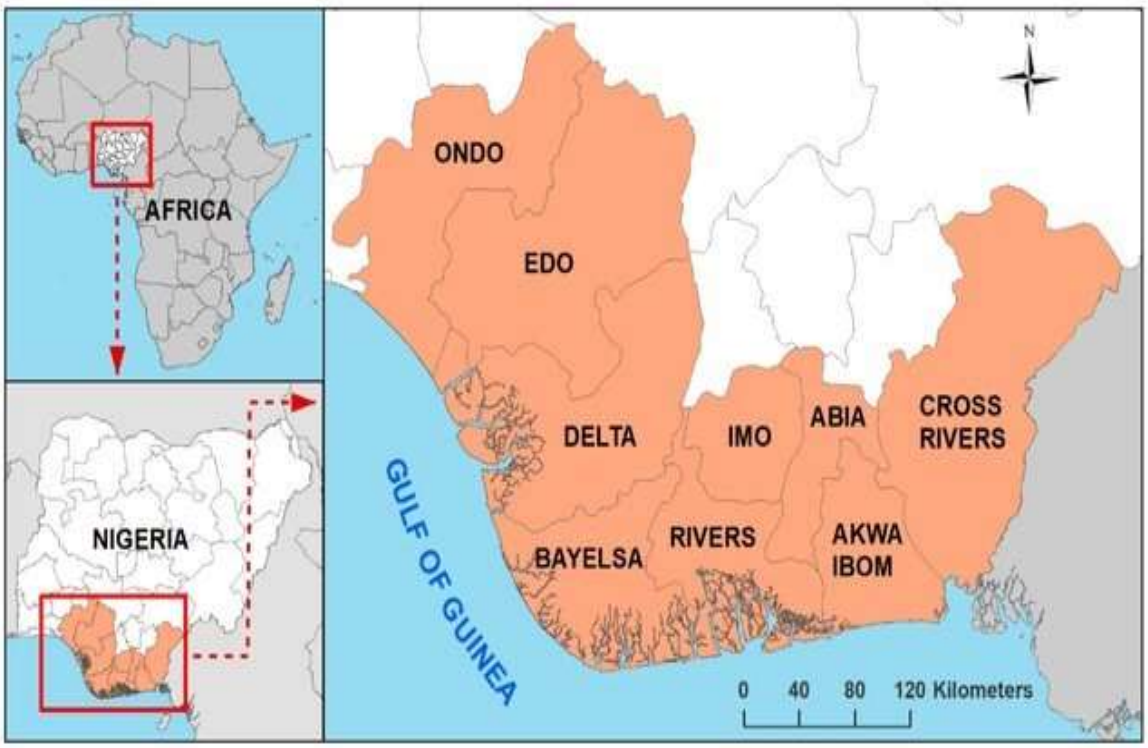

Figure 1: Map of NDR of Nigeria showing the Nine States as adopted by Ebhuoma et al., 2020.

The reckless nature of this menace is the distortion of international commerce, maritime security and seafarer safety. It is through the trade hubs that Nigeria is hoped to attain the economic stability through revenues made from crude oil and all foreign exchange revenues (Abimiku et al., 2023; Ikechukwu et al., 2022). The Niger Delta region has an estimated population of forty-four million, one hundred and twelve thousand, nine hundred and eight (NPCNBS, 2022) with huge role in the national economy development.

Thus, maritime piracy activities and its challenges have impacted the Nigeria economy negatively and has plunged the country into adverse poverty, lack of production infrastructures, lack of basic amenities, lack of developmental plans and conflicts in most coastal line communities as a result of illegal exploration of resources in the region.

Aim and Objectives of the Study

The aim of the study is the application of Hermite Series Kernel Density Estimator $(HSeKDE)$ for identifying the confrontational effects of Maritime Piracy on Economic Growth in Nigeria. The specific objective is to: deduce the Mathematical model of the Hermite Series Kernel Density Estimator $(HSeKDE)$; determine the smoothing parameter and Statistical quality of the derived Mathematical model and the computation of the proposed Mathematical model discrepancies using the maritime piracy and economic growth data.

Methods and Materials

Data Collection

The researchers explored mainly secondary sources of data. They were obtained from relevant establishments like journals, articles, magazines, internets publications and government gazettes.

Overviews of Nonparametric Estimation Methods

The nonparametric estimation method applies the probability density function in the description of the distribution of a random variable X and determining the probabilities associated with the random variable (Fleming & Calabrese, 2017). And it provides a fascinating alternative to parametric estimation methods that employs the underlying distribution. There are several methods that applies this method but the usefulness of kernel density estimator due to its Statistically effectiveness has made it gained an exceptional stride in smoothing techniques. The origin of kernel density estimation and its applications have made it a competitor in estimating class of distribution densities with its univariate compact proposed by (Lawal et al., 2024; Hernandez et al., 2020) as a weighting function given by:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right) \quad (1)$$

where $h > 0$ is the bandwidth which controls the amount of smoothing applied to the set sample observation X_i, n is the sample size, x is the data point of evaluation and $K(.)$ is the kernel function. The kernel functions must satisfy the axioms in Equation (2) to ensure they are probability density function (Nur'eni et al., 2021; Ejakpovi et al., 2025).

$$\int K(x)dx = 1, \int xK(x)dx = 0 \; and \; \int x^2K(x)dx < \infty \quad (2)$$

The issue surrounding the applications of the kernel density estimator is the choice of the kernel function and the selection of appropriate smoothing parameter to regulates the degree of smoothness of the estimated probability density function (pdf). The significance of the choice of the kernel function help in the visualizing features of the underlying distribution of the sampled observations while novel smoothing schemes are being developed (Lawal et al., 2024; Silverman, 2018). The effectiveness of the kernel density estimation involves the crucial selection of the smoothing parameters for if its value is too large it will lead to over smoothing of the distribution and when chosen to be very small, it will lead to a noisy distribution.

Mathematical Model Formulation

The essentials behind the method hinges on the bias reduction techniques amidst other several approaches and it stems from the kernel density derivative estimator $(KDDE)$ consisting of the r^{th} differentials of $\hat{f}(x)$ (Gramike, 2018; Ejakpovi et al., 2019). It has the $K^{(r)}$ as the r^{th} differential order of the kernel function K with at least r non-zero derivatives. This possession of several differential of the kernel registered with the limiting case of the second order Gaussian function and incorporates the Hermite Series Polynomial, $H_r(x)$ of r^{th} differential orders (Siloko & Ojobor, 2023; Celeghini et al., 2022). The model is obtained by removing the asymptotic bias of the $KDDE$ from the KDE with $s = 1$ in the $(s + 2)$ times continuously derivatives of the distributional function of $\hat{f}^{(s+2)}(x)$ (Ejakpovi et al., 2025). It generates the new density estimator, called the Hermite Series Kernel Density Estimator $(HSeKDE)$ as:

$$\tilde{f}_{HSeKDE}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{X_i - x}{h}\right)\left[1 - \frac{3I_2}{2h}\left(\left\{\frac{X_i - x}{h}\right\}^2 - 1\right)\right] \quad (3)$$

where $I_2 = \int x^2K(x)dx$, n is the sample size of the random variable X, x is at which value the function is estimated, $K(.)$ is the kernel function that satisfies the axioms in Equation (2) and h is the smoothing parameter also called bandwidth.

Performance Metrics of Kernel Density Estimations

The measure of performance of KDE, is the asymptotic mean integrated squared error $(AMISE)$ with the ease of Mathematical tractability and inclusion of dimensionality in the criterion function. It consists of the integrated variance and integrated squared bias elements with the smoothing parameter playing a significant role in both components. The $AMISE$ is the global precision error function and when the KDE in Equation (1) is considered, the $AMISE$ expression is:

$$AMISE_{\beta}\left(\hat{f}(x)\right) = \frac{R(K)}{nh} + h^{2p}\left(\frac{\mu_p(K)}{p!}\right)^2 R\left(f^{(p)}\right) \quad (4)$$

where $\mu_p(K) = \int x^p K(x)dx$, $R(K) = \int K^2(x)dx$ and $R\left(f^{(p)}\right)$ in Equation (4) represent the second moment of the kernel, the roughness of the kernel and the measure of the unknown pdf curvature respectively with p as the second derivative order of the underlying distribution (Henderson & Parmeter, 2015). The smoothing parameter that will yield the minimum $AMISE$ is obtained from Equation (4) and called the optimal smoothing parameter as:

$$h_{\beta} = \left(\frac{R(K)(p!)^2}{2p\mu_p^2(K)R(f^p)}\right)^{\frac{1}{2p+1}} X\, n^{\frac{-1}{2p+1}} \quad (5)$$

Moreover, the $AMISE$ of the proposed Mathematical model estimator called the $HSeKDE$ in Equation (3) is achieved from the combination of its integrated variance and integrated squared bias expressions after neglecting higher order terms. The precision measure of the $HSeKDE$ is obtained after some differential evaluations and algebraic simplifications as:

$$AMISE_{\alpha}\left(\tilde{f}(x)\right) = \frac{3\delta_0}{2}\left(\int_{-\infty}^{\infty} K^2(x)dx\right)\left(\int_{-\infty}^{\infty}(f''(x))^2dx\right)^{\frac{9}{14}} I_2^{\frac{13}{7}} n^{\frac{-11}{7}} \quad (6)$$

where $I_2 = \int_{-\infty}^{\infty} x^2K(x)dx < \mu_2(K)$; $\delta_0 = \sigma_3(\|f''\|_2^2)^{\frac{6}{7}} + \sigma_4(\|K\|_2^2)^{\frac{3}{2}}$ and the variables of the δ_0 denoted as $\|f''\|_2^2 = \int_{-\infty}^{\infty}(f''(x))^2dx < \infty$, $\|K\|_2^2 = \int_{-\infty}^{\infty} K^2(x)dx$. Its $AMISE$ depends on the sample size n besides other factors like the kernel function, estimate of the underlying distributional function of the data and the standard deviation of the data. The smoothing parameter that will yield the minimum $AMISE$ of the $HSeKDE$ is obtained from the minimization of differential equation of the $AMISE$ expression and solving yields the smoothing parameter expression as:

$$h_{\alpha} = \left(\frac{8\left(\int_{-\infty}^{\infty} K^2(x)dx\right)^2}{3I_2^4\left(\int_{-\infty}^{\infty}(f''(x))^2dx\right)^3}\right)^{\frac{1}{14}} n^{\frac{-1}{7}} \quad (7)$$

The smoothing parameter depends on the kernel function parameters, sample size and the second derivatives pdf being estimated whose influence reveals the vital features of data during visualization when the estimator is implemented.

Results

The results evidently showed the gross domestic product (GDP) has two major growth phases with the first instant of increase that begins from 2002 to 2008 whose annual revenue (income) increased to the tune of N2.2 million Dollars at the end of 2008. Thereafter, there was a sharp decline in GDP in 2009 and persisted till 2010 that led to the fall of annual revenue to N 1.9 million Dollars. The second rising phase begins in 2011 to 2014 with an accrue annual GDP of N574.18 million Dollars at the end of 2014. The annual revenue income in this period also experienced an increase of N3.2 million Dollars in 2014. And between 2010 and 2014, there was also a notable increase in the country's performance in revenue (income) of an estimated amount of N3.2 million Dollars in 2014 before an observed immediate decline in GDP and annual revenue (income) around 2014. This decline in GDP persisted till 2017 with estimated amount of N375.75 million Dollars and 1.9 million Dollars respectively.

In 2015 when a decline was recorded, the nation's performance started developing an undulating pattern of increase and a total collapse resulting to GDP of N440.83 million Dollars and 2.1 million Dollars in revenue (income) in 2021. Though, there are evidence of potential rise between 2021 and 2022, the indices cannot be equated to the first increase phase witnessed in 2002 to 2008 and eventually the fell to N162.55 million Dollars in GDP and N1.6 million Dollars in revenue (income) in 2024 respectively as seen in the Figure 2.

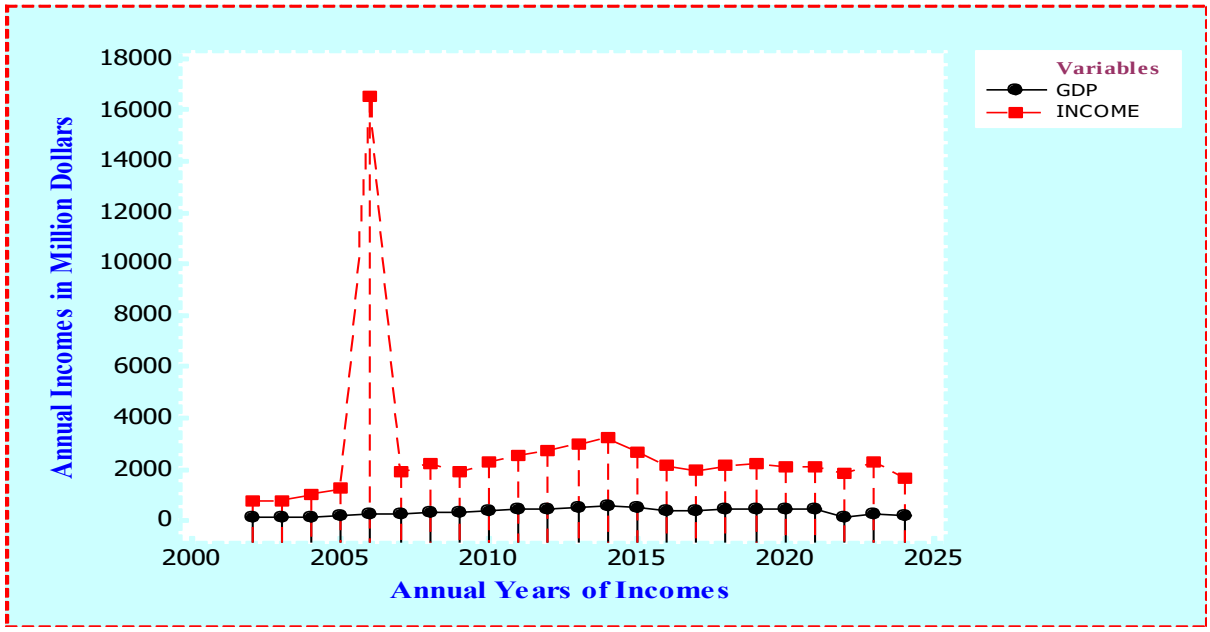

Figure 2: Plot of Estimates of GDP and Annual Incomes in Years

The influence of the maritime piracy activities for these periods have affected the nation's economy with reference to GDP and economic growth rate. These effects can be ascertained with the asymptotic mean integrated squared error ($AMISE$) derived from the KDE and $HSeKDE$ estimators. In any of these estimators, the precision criterion value of the $AMISE$ that produced the smallest value is the better precision to the true situation under consideration. Thus, with the aid of the derived $AMISE$ expressions for both estimators in Equation (4) and (6), obtaining the error values of the observation data becomes certain.

In the effects of both estimators, the kernel estimates of maritime piracy incidence are the number of piracy theft that took place in the shore of Nigeria seas. The kernel estimates indicated that the data were multimodal with the application of KDE and $HSeKDE$, which clearly depicted the undulating nature of the Nigeria GDP values and economic growth rate from 2002 till 2024. The kernel estimator's visualization of the maritime piracy on both the Nigeria economy index is possible through the application of the kernel functions and the smoothing parameters. In the graphical display of each visualization, there are regions of bumps and valleys that depict high and low rate of sea theft activities.

The bumps regions of the sea theft activities that posed a major confrontation to the Nigeria economy were captured by the Epanechnikov kernel. It was observed at 11 and 30 of the KDE and 12 and 29 for the $HSeKDE$ estimators' visualizations, represented the highest incidence of crude lifted illegally from the NDR. The probabilities of the bumps of the various estimates lie between 0 and 0.04 for the piracy incidence and 0 and 0.07 for the economic growth respectively. In Figure 3, the visualization of piracy activities at the inception of the country's booming economy indicated a gradual rise and fall trends in the NDR by both estimators. In Figure 4, the hazards of this sea theft effects on the economic growth rate were not detected by the KDE estimator but the $HSeKDE$ captured the low

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rising nature of the economy. The graphs of the kernel density estimate with the Epanechnikov functions is shown below.

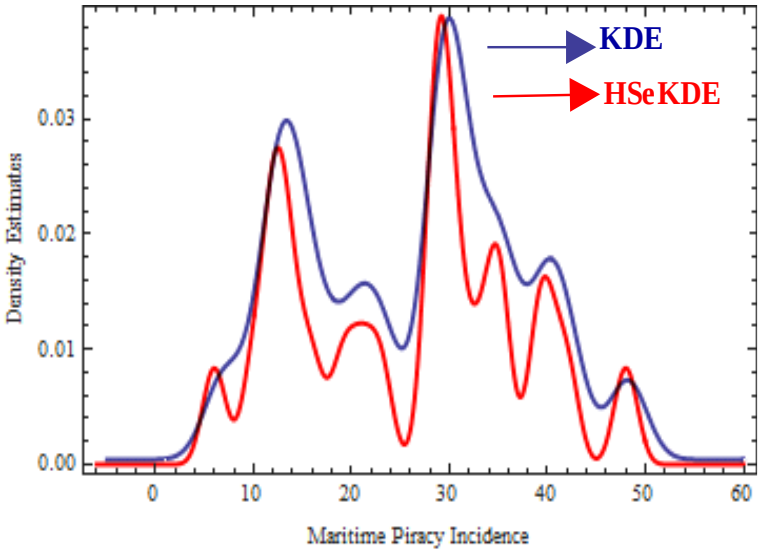

Figure 3: Epanechnikov Kernel Estimates of Maritime Piracy Incidence

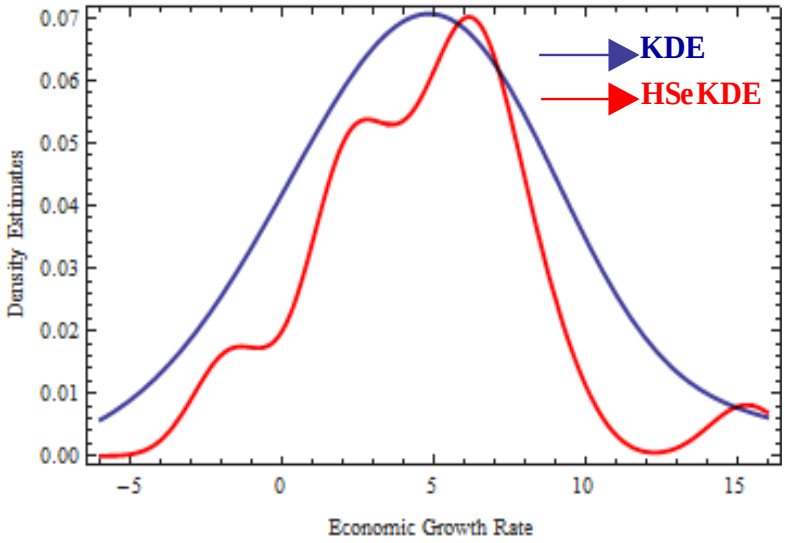

Figure 4: Epanechnikov Kernel Estimates of Economic Growth Rate

The piracy hostility to the Nigeria economy begin to deepened as bumps region captured by the Biweight kernel was observed at 11 and 30 for both KDE and $HSeKDE$ estimators. The probabilities of the peaks of the various estimates lie between 0 and 0.038 for the piracy incidence and 0 and 0.064 for the economic growth rate respectively. In Figure 5, captured the multiple visualization of sea theft activities in the face of a struggling performance by both estimators, implying that right from 2002, there has been several illegal activities in the Nigeria seas in a rise and fall trends in the NDR. Also, the $HSeKDE$ estimator captured a constant period where the sea theft was unchecked and became a normality before it falls, suggesting the worse moment of piracy activities in our

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seas. Then, in Figure 6, the dangers of this effect on the economic growth rate were detected in a unimodal pattern by the KDE estimator affirming its effect to a particular period, but the $HSeKDE$ estimator captured a multimodal pattern effect on the economy. The graphs of the kernel density estimate with the Biweight function is shown below.

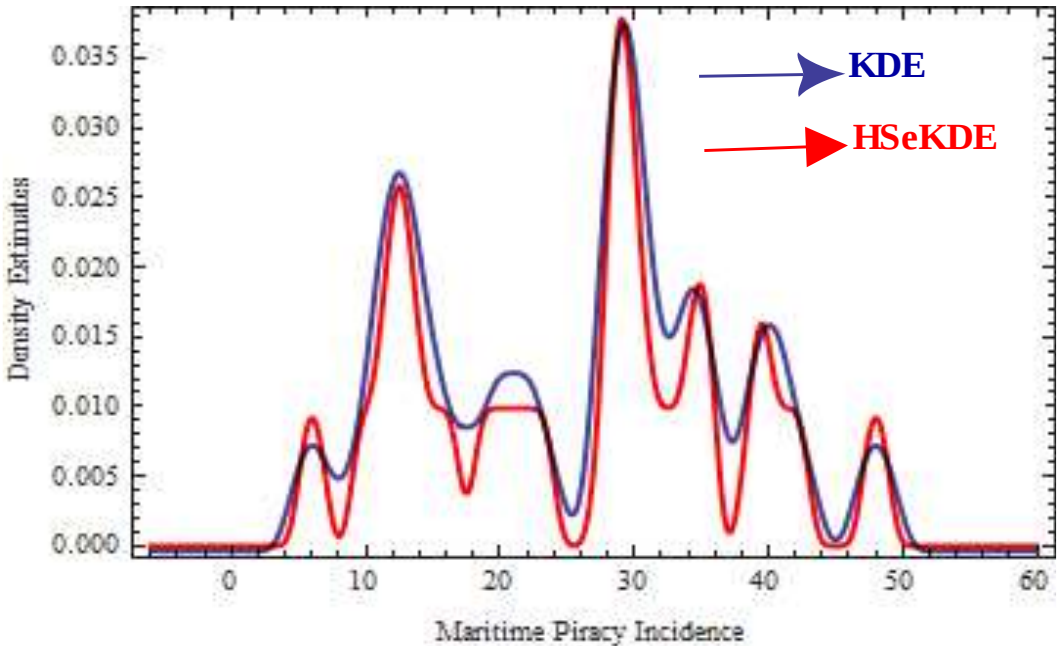

Figure 5: Biweight Kernel Estimates of Maritime Piracy Incidence

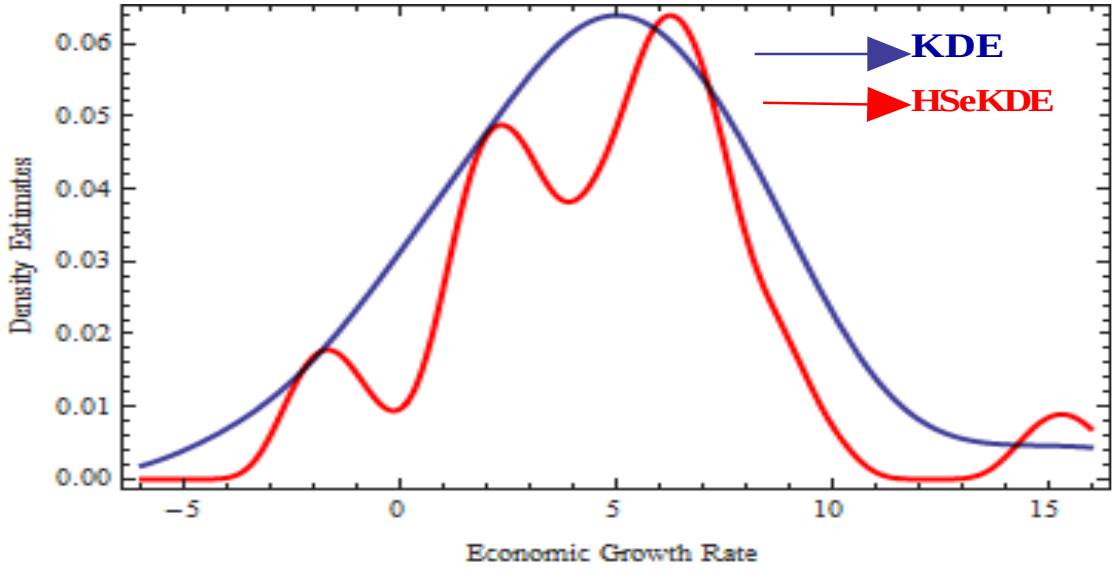

Figure 6: Biweight Kernel Estimates of Economic Growth Rate

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The dangers of piracy to the Nigeria economy begin to get worse as bumps regions captured by another kernel function, called the Triweight kernel occurred at 11 and 30 for both KDE and $HSeKDE$ estimators. It shows a conventionality in the occurrence rate of illegal crude oil lifting from the NDR with the $HSeKDE$ estimator indicating a spike extension within these bumps. The probabilities of the bumps of the various estimates lie between 0 and 0.025 for the piracy incidence and 0 and 0.060 for the economic growth rate respectively. In Figure 7, the multiple visualizations of sea theft activities by both estimators since 2002 led to several illegal sea activities. Then, in Figure 8, the adverse effects of the sea water loots on the economic growth rate were detected in a unimodal pattern by the KDE estimator affirming its effect to a particular period. But the $HSeKDE$ estimator revealed a multimodality pattern effect on the nation's economy. The graphs of the kernel density estimates with the Triweight kernel is shown below.

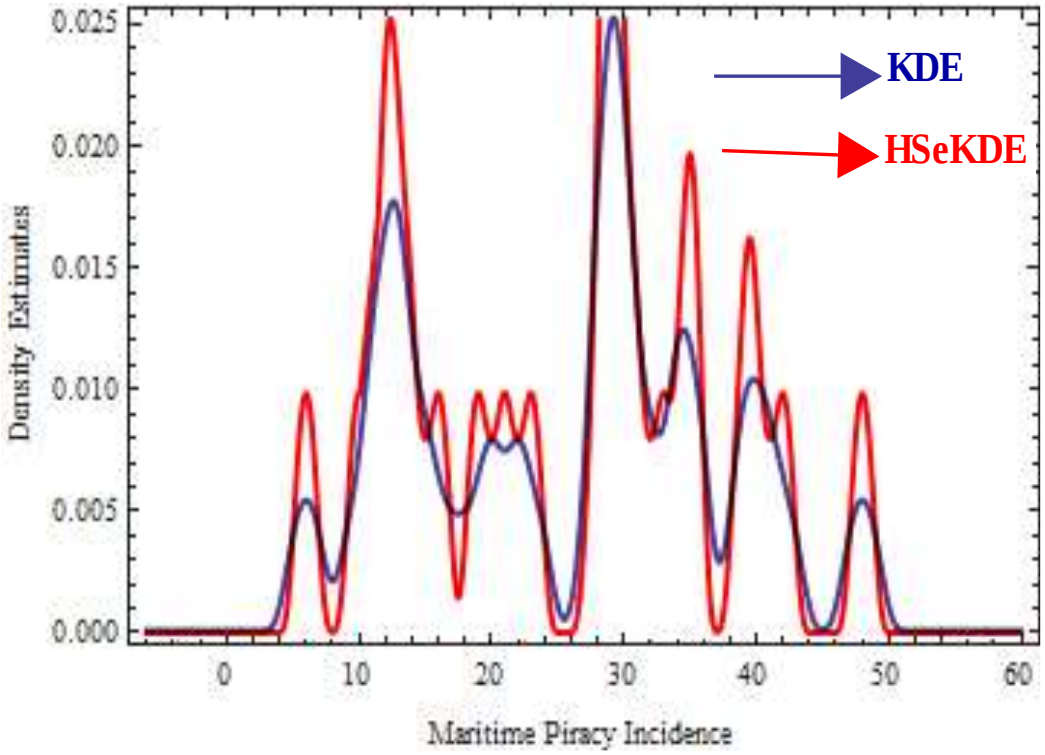

Figure 7: Triweight Kernel Estimates of Maritime Piracy Incidence

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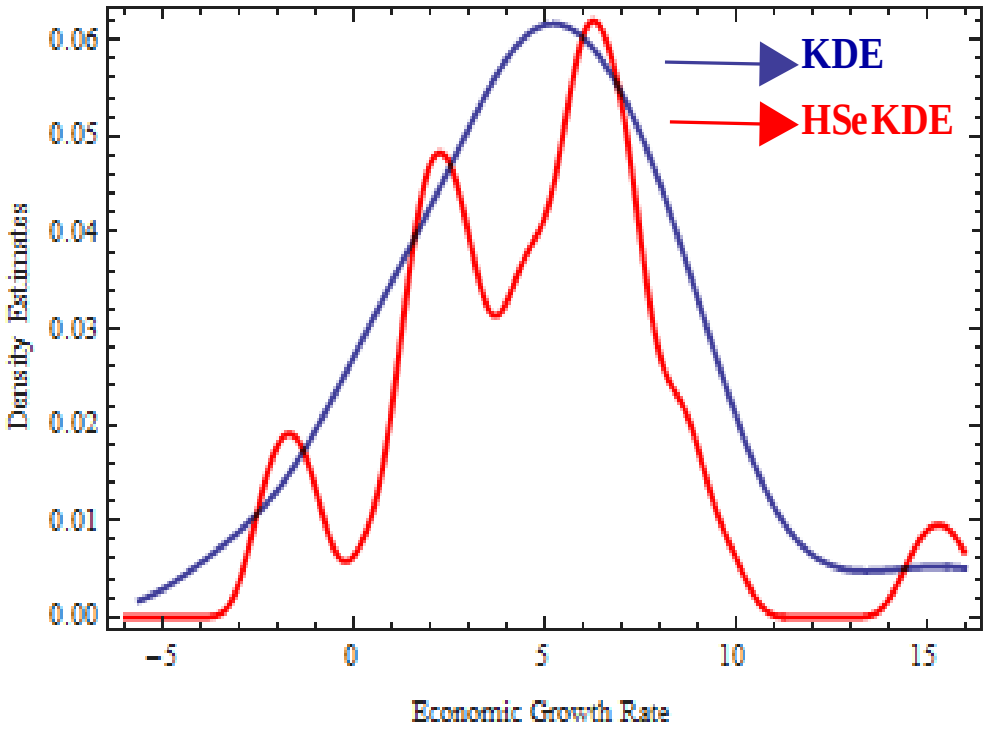

Figure 8: Triweight Kernel Estimates of Economic Growth Rate

Lastly, as the sea theft phenomenon persists into the future of Nigeria existence, with evidence of it being unabated soon, we applied an infinity kernel function, called the Gaussian kernel. This kernel function captured the bumps menace at 10 and 29 for the KDE while the $HSeKDE$ estimator observed the bumps regions at 11 and 30. The visualizations show both estimators captured almost the same rate of illegal crude oil lifting from the NDR. The probabilities of the bumps of the various kernel density estimates lie between 0 and 0.112 for the piracy incidence and 0 and 0.17 for the economic growth rate respectively. In Figure 9, the multiple visualizations of sea theft activities were revealed by both estimators at its inception year, 2002 with an excavated nature of the menace in the NDR. Then, in Figure 10, the adverse effects of the piracy menace on the economic growth rate were detected in a unimodal pattern by the KDE estimator affirming only a period of positive growth. But the $HSeKDE$ estimator captured a multimodal pattern of economic growth rates that is galloping in nature. The graphs of the kernel density estimates with the Gaussian kernel is shown below.

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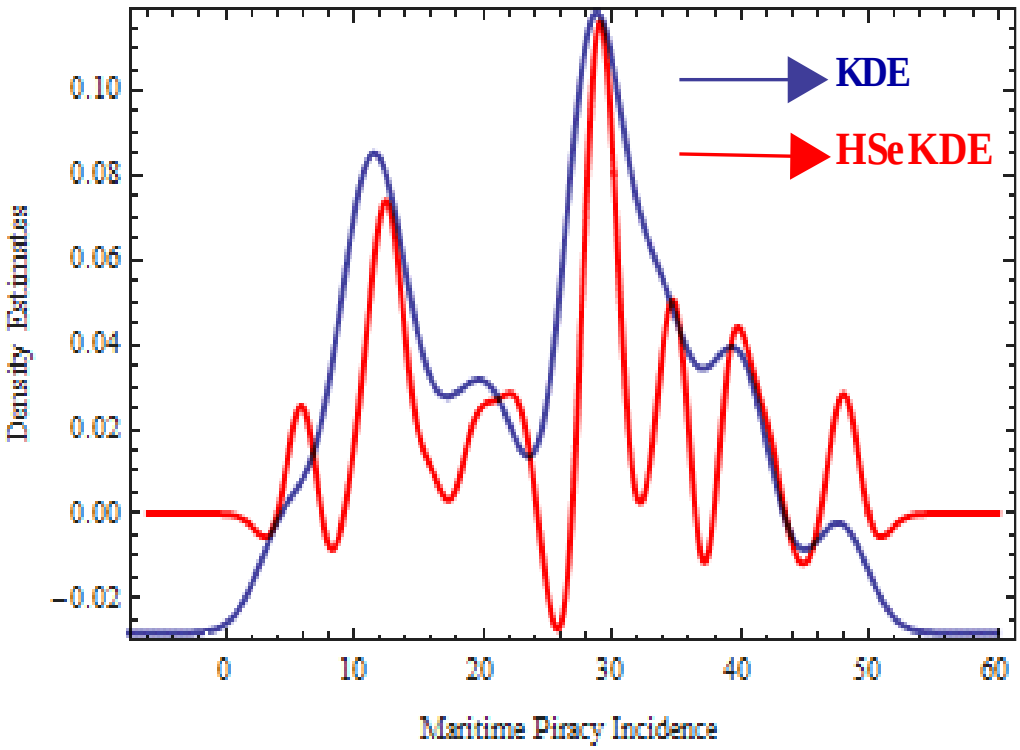

Figure 9: Gaussian Kernel Estimates of Maritime Piracy Incidence

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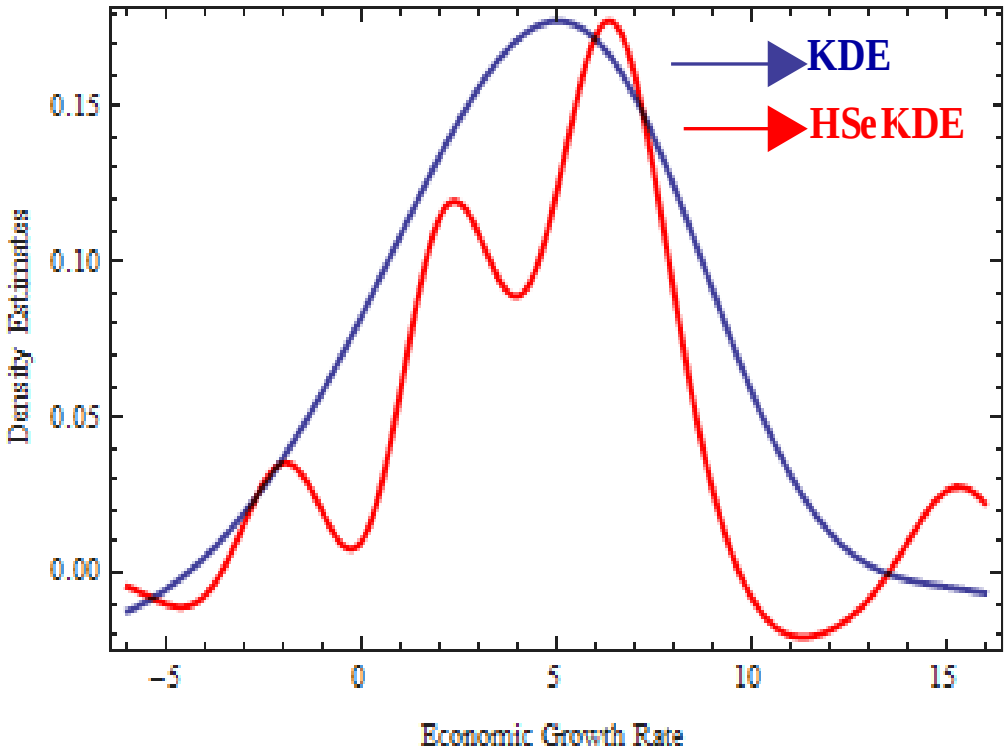

Figure 10: Gaussian Kernel Estimates of Economic Growth Rate

In Figures 3-10, the kernel estimates of the data with the kernels are controlled by the optimal smoothing parameters whose values helped in the visualization of sea theft incidence and economic growth rate by both estimators. The values of the smoothing parameter detected the influx of the piracy incidence during the period of positive and negative economic growth trends. And significantly, it resulted to the Statistical properties of both the KDE and $HSeKDE$ shown in Table 1.0 and Table 2.0 below

Table 1.0. Estimators, Smoothing parameter, Statistical Qualities and *AMISE* Values

S/N	*ESTIMATORS*	*Kernels*	h_β	$\int Bias^2(\hat{f}(x))dx$	$\int Var(\hat{f}(x))dx$	$AMISE_\beta(\hat{f}(x))$
1.	***KDE***	Epanechnikov	2.77893	0.00118570	0.00180436	0.00299006
		Biweight	3.02468	0.00118337	0.00181318	0.00299656
		Triweight	3.22314	0.00117583	0.00182381	0.00299964
		Gaussian	2.09422	0.00152291	0.00469274	0.00621565
2.	***HSeKDE***	***Kernels***	h_α	$\int Bias^2(\tilde{f}(x))dx$	$\int Var(\tilde{f}(x)))dx$	$AMISE_\alpha(\tilde{f}(x))$
		Epanechnikov	1.86752	4.77531E-10	1.25538E-06	1.25586E-06
		Biweight	1.98405	1.47954E-10	1.65580E-06	1.65595E-06
		Triweight	2.07619	6.18520E-11	4.39202E-05	4.39203E-05
		Gaussian	1.52585	1.53009E-09	4.60314E-05	4.60329E-05

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The smoothing parameters revealed the presence of piracy activities on the Nigeria waters through kernel estimators. It is evidential from Table 1.0 that as the kernel function increases, the value of the smoothing parameter is also on the increase as well as the asymptotic mean integrated squared error ($AMISE$). In comparison, the $HSeKDE$ estimator is having the least $AMISE$ values for its computed value vividly shows it outperformed the other kernel estimator. This implies that its potentials of detecting swiftly the threats of the piracy menace as it achieved the smallest error of 0.00000125586 and 0.0000460329 with the Epanechnikov and Gaussian kernels.

Table 2.0. Estimators, Smoothing parameter, Statistical Qualities and $AMISE$ Values

S/N	$ESTIMATORS$	$Kernels$	h_β	$\int Bias^2(\hat{f}(x))dx$	$\int Var(\hat{f}(x))dx$	$AMISE_\beta(\hat{f}(x))$
1.	KDE	Epanechnikov	4.65854	0.00139995	0.00559982	0.00699977
		Biweight	5.51892	0.00125675	0.00640669	0.00766344
		Triweight	6.26689	0.00108812	0.00741982	0.00850794
		Gaussian	2.64570	0.00279977	0.01533770	0.01813750
2.	$HSeKDE$	$Kernels$	h_α	$\int Bias^2(\tilde{f}(x))dx$	$\int Var(\tilde{f}(x)))dx$	$AMISE_\alpha(\tilde{f}(x))$
		Epanechnikov	1.91676	3.98004E-10	1.07390E-06	1.07430E-06
		Biweight	2.03636	1.23314E-10	1.43126E-06	1.43138E-06
		Triweight	2.13093	5.15513E-11	4.19649E-05	4.19650E-05
		Gaussian	1.56608	1.06289E-09	3.93769E-05	3.93780E-05

The smoothing parameters revealed the continuous perils in the Nigeria economy as a result of incessant piracy activities on the Nigeria waters through kernel estimators. The Table 2.0 revealed that as the kernel function increases, the value of the smoothing parameter is also on the increase as well as the asymptotic mean integrated squared error ($AMISE$). In comparison, the $HSeKDE$ estimator is having the least $AMISE$ values for its computed values vividly show it outperformed the other kernel estimator. The sharp detection by the $HSeKDE$ of the worse state of the economy as it achieved the smallest error of 0.00000107430 and 0.0000393780 with the Epanechnikov and Gaussian kernels are products of its strides.

Discussion

The main source of the Nigeria economy is derived from the NDR which is rich in crude oil. The presence of sea theft in the region through the activities of maritime piracy has eventually delve the nation into an epileptic economy through the illegal crude lifting. This act has created the most challenging ecological issues in the NDR. And it has impaired the gross domestic product (GDP) and revenue (income) of the country, thereby promoting economic instability and poverty among Nigeria citizens.

In the years were there are recorded estimated increase in GDP, evidently showed that the sea theft activities were very minimal. These are the seasons were all the economies of the nation were gainfully engaged and yielded positive returns for stable economic growth. In the periods were there are recorded estimated decline in the GDP, affirmed the persistence sea theft activities. It soars high on the Nigeria waters thereby making resources not to be gainfully engaged in commerce through foreign exchange and as such resulted to negative impact on the economic growth.

This undulating nature of the Nigeria economic growth rate since its inception in 2002 was unmasked by the nonparametric Mathematical models. The models' estimators visualized the persistence nature of sea theft and its effects on the economic growth rates with the aid of the smoothing parameter and kernel functions. The Epanechnikov function attained a point of convergence in both model estimators affirming the worse economy of the country in 2024. Also, the Biweight function in the $HSeKDE$ showed three periods that Nigeria experienced economic growth and thereafter it crashed. This is the direct consequence of rampant piracy activities that cripples the economic growth rate stability till 2024.

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The trends continuous when the Triweight function was applied in the $HSeKDE$ model with a significant evident of spike extensions in all the areas of bumps. This portrayed the severity of the menace, characterized with massive illegal sea theft in the NDR and marred with worse economic growth depletions from 2002 till 2024. Thus, in an attempt to predict the future of Nigeria economic growth stability in the midst of sea theft hostility, the Gaussian function in the $HSeKDE$ affirmed that the issue has gone very severe. And its rate of occurrence is unimpeded thereby making the Nigeria economic growth rate to deteriorate beyond 2025.

Conclusion

This study employed a novel nonparametric estimation approach to clarifies the magnitudes of maritime piracy activities in the Niger Delta region (NDR) of Nigeria on the gross domestic product (GDP) and Economic growth of the nation's economy. The sourced data revealed the hazards of maritime piracy through the continuing inclination of the maritime piracy incidence that reveals the rising and falling trend of the gross domestic product (GDP) and economic growth rate within the years of investigation. The negative impact of this menace is both localized and national. On the localized basis, it implies that the continued atrocities of sea theft in the NDR is ravaging our economy and making the area unproductive for the country. This will make the region to remain undeveloped without infrastructures, social amenities like water and electricity, high level of unemployment, dearth of small-scale businesses and lack of foreign investors in the region.

Then, on the national basis, the return on investment from the country is very low affecting our foreign exchange in the international market. Also, as the menace persists due to self-aggrandizement of individuals who are involved directly or indirectly in this act of maritime terrorism, the federal government goals to develop the NDR is incapacitated. Therefore, there is an urgent need to develop modern methods to eradicate maritime piracy besides the securing of the water ways and the coastal lines of the NDR.

Recommendations

In this regard, the following recommendations are made:

i. **Deployment of Robotic Securities:** This is the adoption of autonomous robots to monitor and protect the Niger Delta region with the aim of providing continuous surveillance, threat detection and rapid response. It provides a continuous security round the clock and improved efficiency in the areas. This will help to curb the piracy incidence and bring to account those who are involved.

ii. **Reviewing Government Policy:** The government policies on Maritime piracy in Nigeria need an immediate review. The review should focus on prison terms on Nigeria water violators, the vessels lifting crudes in the area, the foreign bodies that enter the Nigeria water, granting of amnesty to culprit etc

iii. **Registration of Sea Vessels:** The government agencies responsible should deregister and register new sea vessels. When security law is breach by a sea vessel, its right into the region should be revoke and deregistered. This will pave way for new investors with new vessels into the region for the purpose of carrying out economic activities that is benefial to the country.

iv. **Maritime Criminal Networks:** The government of Nigeria should crack down the maritime criminal network in the region. The network span from local king-pins to foreign ones around the Gulf of Guinea. This act will deter those who are involved in the menace of sea piracy

v. **Training:** The Nigeria government should make fund appropriation for the training of personnel, purchasing of maritime enforcement equipment and the purchase of state-of-the-art surveillance equipment to bolster the efficacy of maritime counter measures in the NDR.

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