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Mathematical Computation of the Flow of Non-Newtonian Fluid with Heat Generation in a Cylindrical Pipe

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Abstract
In this paper, a mathematical computation of flow of non-Newtonian fluid with heat generation in a cylindrical pipe is undertaken and studied using the third-grade parameter to represent the non-Newtonian parameter. The coupled system of nonlinear ordinary differential equation (ODE) is solved using the traditional perturbation technique. The results show that the third-grade parameter (β) and the magnetic field parameter(M) reduces the flow velocity and the temperature while the Brinkman number enhances the temperature of the cylindrical walls. Some quantitative and numerical result are also presented to strengthen the utility of the method.

Keywords: Computational flow Newtonian-fluid, heat generation, Brinkman number, magnetic field, perturbation technique.

Introduction

The study of non-Newtonian fluid flow in cylindrical pipes has gained significant in diverse engineering and industrial applications, such as polymer processing, biomedical fluid dynamics, and oil extraction (Bird et al., 2006). In Newtonian fluids, the viscosity remains constant irrespective of shear rate, this attributes is quite complex with non-Newtonian fluids that exhibit flow behaviours that require advanced mathematical models for proper analysis (Schowalter, 1978). Specifically, third-grade fluids, a subclass of non-Newtonian fluids, have attracted considerable attention due to their practical relevance in numerous scientific and industrial fields (Fosdick & Rajagopal, 1978). Critical to studying non-Newtonian fluid dynamics is understanding how external influences such as heat generation and magnetic fields influence fluid motion. The interaction of heat and fluid flow plays a crucial role in applications where temperature variations significantly alter the fluid's viscosity and overall behavior (Mukhopadhyay et al., 2013). The presence of heat generation within the fluid may arise from chemical reactions, internal friction, or external sources, all of which contribute to the overall energy balance within the system (Gebhart et al., 1988).

Mathematical modeling of such systems often involves solving coupled nonlinear ordinary differential equations (ODEs), which describe both the momentum and energy transport equations. Traditional analytical techniques such as the perturbation method provide effective means of approximating solutions to these equations, particularly when exact solutions are not feasible (Nayfeh, 2000). The perturbation technique has been widely used in fluid mechanics for handling nonlinear problems where small parameter expansions lead to manageable mathematical expressions (Van Dyke, 1975). Recent studies have demonstrated that the third-grade parameter (β) has a substantial impact on fluid velocity and temperature profiles (Hayat et al., 2007). Similarly, the magnetic field parameter (M) is known to influence flow resistance and energy dissipation, reducing both velocity and temperature in magnetohydrodynamic (MHD) flows (Rana et al., 2016). Moreover, the Brinkman number (Br), which represents the ratio of heat generated by viscous dissipation to heat conducted, plays a crucial role in determining the thermal characteristics of the fluid (Brinkman, 1952). A higher Brinkman number enhances heat transfer, leading to increased temperatures along the cylindrical walls (Mahmood et al., 2015).

This study aims to contribute to the understanding of non-Newtonian fluid flow by employing the third-grade fluid model to analyze the effects of heat generation and magnetic field on flow characteristics in a cylindrical pipe. The application of the perturbation technique to solve the governing equations provides insights into the dynamic interplay of velocity, temperature, and external parameters. Additionally, numerical and quantitative results are presented to validate the computational approach and highlight its effectiveness in handling nonlinear fluid flow problems.

Mathematical Formulations

We consider a steady incompressible non-Newtonian fluid flow, in non- dimensional form of the equations of motion as seen in Farayola, (2017)

$$\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}\right)+\frac{\beta\,\mu}{k}u-\sigma B^2u=\frac{dp}{dz} \tag{2.1}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\mu}{k}\left(\frac{du}{dr}\right)^2+\sigma B_0u^2=0 \tag{2.2}$$

$$\frac{du}{dr}(0)=\frac{dT}{dr}(0), u(a)=0, T(a)=T_0 \tag{2.3}$$

Where u is fluid velocity, T is the absolute temperature, μ is the dynamic viscosity, σ is the electrical conductivity, T_0 is the ambient temperature, 'a' is the radius of the pipe and r is radial distance.
Introducing the following parameters for non-dimensionalization,

$$\bar{r}=\frac{r}{a}, \bar{u}=\frac{u}{u_0}, \theta\frac{T}{T_0} \tag{2.4}$$

Substituting eqn (2.4) into equation (2.1) yields

$$\frac{1}{a\bar{r}}\frac{d}{d(a\bar{r})}\left(\mu a\bar{r}\frac{d(u_0\bar{u})}{d(a\bar{r})}\right)+\frac{\beta\,\mu(u_0\bar{u})}{k}-\sigma B^2{}_0u_0\bar{u}=\frac{dp}{dz}$$

$$\frac{u_o}{a^{2\bar{r}}}\frac{d}{d\bar{r}}\left(\mu\bar{r}\frac{d\bar{u}}{d\bar{r}}\right)+\frac{u_o\beta\mu\bar{u}}{K}-u_o\sigma B_o^2\bar{u}=\frac{dp}{dz}$$

Divide through by $\frac{u_o\mu}{a^2}$, we have

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\bar{u}}{d\bar{r}}\right)+\frac{a^2u_o\mu\beta\bar{u}}{u_o\mu K}-\frac{a^2u_o\sigma B_o^2\bar{u}}{u_o\mu}=\frac{a^2}{u_o\mu}\frac{dp}{dz}$$

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\bar{u}}{d\bar{r}}\right)+\frac{\beta\bar{u}a^2}{K}-\frac{a^2\sigma B_o^2\bar{u}}{\mu}=\frac{a^2}{u_o\mu}\frac{dp}{dz}$$

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\bar{u}}{d\bar{r}}\right)+\delta\beta\bar{u}-M\bar{u}=C$$

Dropping the bars for simplicity,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{d\bar{r}}\right)+\delta\beta u-Mu=-1 \tag{2.5}$$

Where δ is the porosity parameter
β is the material coefficient
Relating to third grade fluid

$$\mathrm{U}(0)=0, \mathrm{u}(1)=0, \theta(0)=0, \theta(1)=0 \tag{2.6}$$

Method of solution

In order to solve equation (2.5), we introduce the perturbation series as

$$u(r)=u_o(r)+\beta u_1(r)+0(\beta^2), \theta(r)+\beta\theta_1(r)+0(\beta^2),$$
$$m=\beta m\,\delta=\beta Q \tag{3.1}$$

Substituting equation (6) into equation (5), we have

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{d\bar{r}}\right)+\delta\beta u-Mu=-1$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{d\bar{r}}(u_o+\beta u_1)\right)+\beta^2Q\,(u_o+\beta u_1)-\beta m\,(u_o+\beta u_1)=-1$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_o}{d\bar{r}}+r\beta\frac{du_1}{dr}\right)+\beta^2Qu_o+\beta^3Qu_1-\beta mu_o-\beta^2mu_1=-1 \tag{3.2}$$

Choosing the order of β from equation

$$\beta^o: \frac{1}{r}\frac{d}{dr}\left(r\frac{du_o}{d\bar{r}}\right)=-1 \tag{3.3}$$

$$\beta: \frac{1}{r}\frac{d}{dr}\left(r\frac{du_1}{d\bar{r}}\right)-Mu_o=0 \tag{3.4}$$

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Solution of Momentum equation of the zeroth order solving equation (2.8) with the condition equation (3.4)(3.2), yields

$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_o}{d\bar{r}}\right) = -1$

Multiply through by r, yields

$$\frac{d}{dr}\left(r\frac{du_o}{d\bar{r}}\right) = -r \qquad (3.5)$$

Integrating equation (3.5)

$r\frac{du_o}{d\bar{r}} = -\frac{r^2}{2} + C$

Divide through by r

$$\frac{du_o}{d\bar{r}} = -\frac{r}{2} + \frac{C}{r} \qquad (3.6)$$

Integrating eqn (2) wrt r, we have

$$u_o = -\frac{r^2}{4} + C\ lnr + D \qquad (3.7)$$

C = 0 for bounded solution

$u_o = -\frac{r^2}{4} + D$

Using the condition (12), we have

$0 = 0 + D \Rightarrow D = 0$

$0 = -\frac{1}{4} + D \Rightarrow D = \frac{1}{4}$

$\therefore\ u_o = -\frac{r^2}{4} + \frac{1}{4}$

$u_o(r) = \frac{1}{4} - \frac{1}{4}r^2$

$$\frac{du_o}{dr} = -\frac{1}{2}r \qquad (3.8)$$

3. Solution of the Momentum equation of order β in order to solve the momentum equation of order β, we substitute for u_o in equation (3.2), and yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_1}{d\bar{r}}\right) - M\left(\frac{1}{4} - \frac{1}{4}r^2\right) = 0 \qquad (3.9)$$

Multiply through by r,

$$\frac{d}{dr}\left(r\frac{du_1}{d\bar{r}}\right) - M\left(\frac{1}{4}r - \frac{1}{4}r^3\right) = 0 \qquad (2.16)$$

Integrating equation (16) wrt r,

$$r\frac{du_1}{d\bar{r}} - M\left(\frac{1}{8}r^2 - \frac{1}{16}r^4\right) + C = 0 \qquad (3.10)$$

Divide eqn (17) through by r

$$\frac{du_1}{d\bar{r}} - M\left(\frac{1}{8}r - \frac{1}{16}r^3\right) + \frac{C}{r} = 0 \qquad (3.11)$$

Integrating equation (18) wrt r,

$u_1 - M\left(\frac{1}{16}r^2 - \frac{1}{64}r^4\right) + Clnr + D = 0$

C = 0, for bounded solution,

$$u_1 - M\left(\frac{1}{16}r^2 - \frac{1}{64}r^4\right) + D = 0 \qquad (3.12)$$

Using eqn (3.2) in eqn (3.12), we obtain

$0 - 0 + D = 0 \Rightarrow D = 0$

$0 - M\left(\frac{1}{16} - \frac{1}{64}\right) + D = 0$

$D = \frac{1}{16} - \frac{1}{64} = \frac{4-1}{64} = \frac{3}{64}$

$$\therefore D = \frac{3}{64}M \qquad (3.14)$$

$u_1 - M\left(\frac{1}{16}r^2 - \frac{1}{64}r^4\right) - D$

$$u_1 - M\left(\frac{1}{16}r^2 - \frac{1}{64}r^4\right) - \frac{3}{64}M \qquad (3.15)$$

$\frac{du_1}{d\bar{r}} = M\left(\frac{1}{8}r - \frac{1}{16}r^3\right)$

$$u(r) = \frac{1}{4} - \frac{1}{4}r^2 + \beta\left(M\left(\frac{1}{16}r^2 - \frac{1}{64}r^4\right) - \frac{3}{64}M\right) \qquad (3.16)$$

3.3 Heat Transfer Analysis

Recall equation (2.2)

$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\mu}{K}\left(\frac{d(u)}{d(r)}\right)^2 + J\beta_o u^2 = 0$

Substituting eqn (4) into eqn (2), yields

$$\frac{1}{a\bar{r}}\frac{d}{d(a\bar{r})}\left(a\bar{r}\frac{d\theta T_0}{d(a\bar{r})}\right)+\frac{\mu}{K}\left(\frac{d(u_o\bar{u})}{d(a\bar{r})}\right)^2+J\beta_o(u_o\bar{u})^2=0$$

$$\frac{aT_0}{a^3}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\theta}{d\bar{r}}\right)+\frac{\mu u_o}{a^2K}\left(\frac{d\bar{u}}{d\bar{r}}\right)^2+J\beta_o{u_o}^2\bar{u}^2=0$$

Divide through by $\frac{T_0}{a^2}$, we have

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\theta}{d\bar{r}}\right)+\frac{a^2}{T_0}\frac{\mu u_o}{a^2K}\left(\frac{d\bar{u}}{d\bar{r}}\right)^2+\frac{a^2}{T_0}J\beta_o{u_o}^2\bar{u}^2=0$$

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\theta}{d\bar{r}}\right)+\frac{\mu u_o}{T_0K}\left(\frac{d\bar{u}}{d\bar{r}}\right)^2+\frac{a^2J\beta_o{u_o}^2\bar{u}^2}{T_0}=0$$

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\theta}{d\bar{r}}\right)+\beta r\left(\frac{d\bar{u}}{d\bar{r}}\right)^2+M\bar{u}=0 \quad (3.17)$$

Dropping the bars in eqn (23), yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)+\beta r\left(\frac{du}{dr}\right)^2+Mu=0 \quad (3.18)$$

In order to solve eqn (24), we substitute eqn (6) into eqn (24) and yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}(\theta_o+\beta\theta_1)\right)+\beta r\left(\frac{d}{dr}(u_o+\beta u_1)\right)^2+\beta M\left(\frac{d}{dr}(u_o+\beta u_1)\right)^2=0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_o}{dr}+\beta\frac{d\theta_1}{dr})\right)+\beta r\left\{\left(\frac{du_o}{dr}\right)^2+2\beta\frac{du_o}{dr}\frac{du_1}{dr}+\beta^2\left(\frac{du_1}{dr}\right)^2\right\}$$

$$\beta M(u_o+2\beta u_ou_1+\beta^2{u_1}^2)=0 \quad (3.19)$$

Choosing the order of β, we have

$$\beta^o:\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+\beta r\left(\frac{du_o}{dr}\right)=0 \quad (3.20)$$

$$\beta:\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+\beta r\frac{du_o}{dr}\frac{du_1}{dr}+M{u_o}^2=0 \quad (3.21)$$

3. Solution of energy equation of the zeroth order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+\beta r\left(\frac{du_o}{dr}\right)^2=0$$

Substituting for $\frac{du_o}{dr}$ in eqn (26), we have

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+\beta r\left(-\frac{1}{2r}\right)^2=0$$

Multiply through by r

$$\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+\beta r\left(\frac{1}{4}r^3\right)=0$$

Integrating eqn (28) wrt, yields

$$\left(\frac{1}{4}-\frac{1}{4}r^2\right)\left(\frac{1}{4}-\frac{1}{4}r^2\right)=\frac{1}{16}-\frac{1}{16}r^2-\frac{1}{16}r^2+\frac{1}{16}r^4=\frac{1}{16}-\frac{1}{8}r^2+\frac{1}{8}r^4$$

$$r\frac{d\theta_0}{dr}+\beta r\left(\frac{1}{16}r^4\right)+C=0$$

Divide through by r

$$\frac{d\theta_0}{dr}+\beta r\left(\frac{1}{16}r^3\right)+\frac{C}{r}=0 \quad (3.22)$$

Integrating eqn (29) wrt r, we have

$$\theta_1+\frac{1}{16}r^4Br+clnr+D=0 \quad (3.23)$$

Using the condition in eqn (30), we have

$\theta_0(0)=0,\ \theta_0(1)=0$

$0+0+0+D=0 \implies D=0$

$C=0, for\ bounded\ solution$

$0+\frac{1}{64}\beta r+D=0$

$$D=-\frac{1}{64}\beta r \quad (3.24)$$

$$\theta_o=\frac{1}{64}r^4Br+\frac{1}{16}Br$$

$$\theta_o=\left(\frac{1}{64}-\frac{1}{64}r^4\right)Br \quad (3.25)$$

Solution of energy equation of order β

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+2\beta r\left(\frac{du_o}{dr}\frac{du_1}{dr}\right)+M{u_o}^2=0$$

Substituting for $\frac{du_o}{dr},\frac{du_1}{dr}$ and u_o in eqn (27), gives

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+2\beta r\left\{\left(-\frac{1}{2}r\right)\left(M\left(\frac{1}{8}r-\frac{1}{16}r^3\right)\right)\right\}+M\left(\frac{1}{4}-\frac{1}{4}r^2\right)^2=0 \quad (3.25)$$

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Expanding eqn (32), gives

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+2\beta r\left(M\left(-\frac{1}{16}r^2+\frac{1}{32}r^4\right)\right)+\ M\left(\frac{1}{16}-\frac{1}{8}r^2+\frac{1}{16}r^4\right)=0 \quad (3.27)$$

Expanding eqn (33) further, yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+\beta r\left(M\left(-\frac{1}{8}r^2+\frac{1}{16}r^4\right)\right)+\ M\left(\frac{1}{16}-\frac{1}{8}r^2+\frac{1}{16}r^4\right)=0 \quad (3.28)$$

Multiplying eqn (34) by r, we have

$$\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+\beta r\left(M\left(-\frac{1}{8}r^3+\frac{1}{16}r^5\right)\right)+\ M\left(\frac{1}{16}r-\frac{1}{8}r^3+\frac{1}{16}r^5\right)=0 \quad (3.29)$$

Integrating eqn (3.37) wrt r, yields

$$r\frac{d\theta_1}{dr}+\beta r\left(M\left(-\frac{1}{32}r^3+\frac{1}{96}r^6\right)\right)+\ M\left(\frac{1}{16}r-\frac{1}{24}r^3+\frac{1}{80}r^6\right)+\ C=0 \quad (3.30)$$

Divide eqn (3.38) through by r, yields

$$r\frac{d\theta_1}{dr}+\beta r\left(M\left(-\frac{1}{32}r^2+\frac{1}{96}r^5\right)\right)+\ M\left(\frac{1}{16}r-\frac{1}{24}r^2+\frac{1}{80}r^5\right)+\ \frac{C}{r}=0 \quad (3.31)$$

Integrating eqn (37) wrt r, yields

$$\theta_1+\ \beta r\left(M\left(-\frac{1}{96}r^3+\frac{1}{576}r^6\right)\right)+\ M\left(\frac{1}{16}r-\frac{1}{72}r^3+\frac{1}{480}r^6\right)$$
$$+Clnr+D=0 \quad (3.32)$$

Applying the condition (6) in eqn (38) gives c = 0, for bounded solution

$$0+Br\ \left(M\left(-\frac{1}{a}\ 0+0+0+0+D=0\ \Rightarrow D=0\right)\right) \quad (3.33)$$

$$0+\beta r\left(M\left(-\frac{1}{96}+\frac{1}{576}\right)\right)+\ M\left(\frac{1}{16}-\frac{1}{72}+\frac{1}{480}\right)+\ D=0$$
$$\beta r\left(-\frac{5}{576}M\right)-\frac{73}{1440}M \quad (3.34)$$

Substituting for value of D in eqn (38), we have

$$\theta_1=\ -\beta r\left(M\left(-\frac{1}{96}r^3+\frac{1}{576}r^6\right)\right)+\ M\left(\frac{1}{16}r-\frac{1}{72}r^3+\frac{1}{480}r^6\right)$$
$$\beta r\left(-\frac{5}{576}M\right)-\frac{73}{1440}M \quad (3.35)$$

$$\theta(r)=\left(\frac{1}{64}-\ \frac{1}{64}r^4\right)Br+\ \beta\left(-Br\left(M\left(-\ \frac{1}{96}r^3+\frac{1}{576}r^6\right)\right)\right)$$
$$+M\left(\frac{1}{16}r=\ \frac{1}{72}r^3+\frac{1}{480}r^6\right)+M\left(\frac{73}{1440}-\ \frac{5}{576}Br\right) \quad (3.36)$$

4.0 Numerical Simulations of Results

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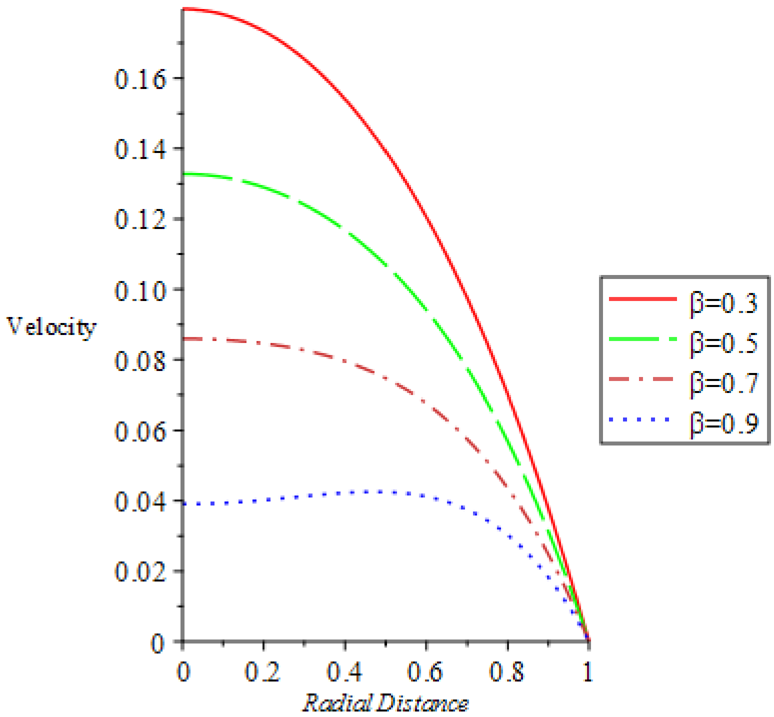

Figure 1: Effects of Third Grade Parameter (β) On Velocity When M=5

Figure 1 shows the effects of third grade parameter on the velocity fluid flow. Results indicate that as the third grade parameter increases, the velocity decreases as seen in the profile

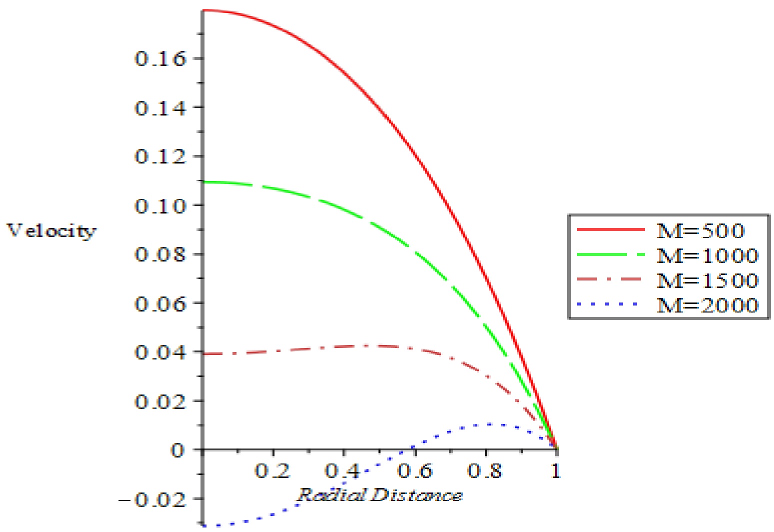

Figure 2: Effects of Magnetic Field Parameter (M) On Velocity When β=0.003

Figure 2 is the effects of magnetic field parameter on the flow regime. It is seen that increase in the magnetic field parameter reduces the flow velocity

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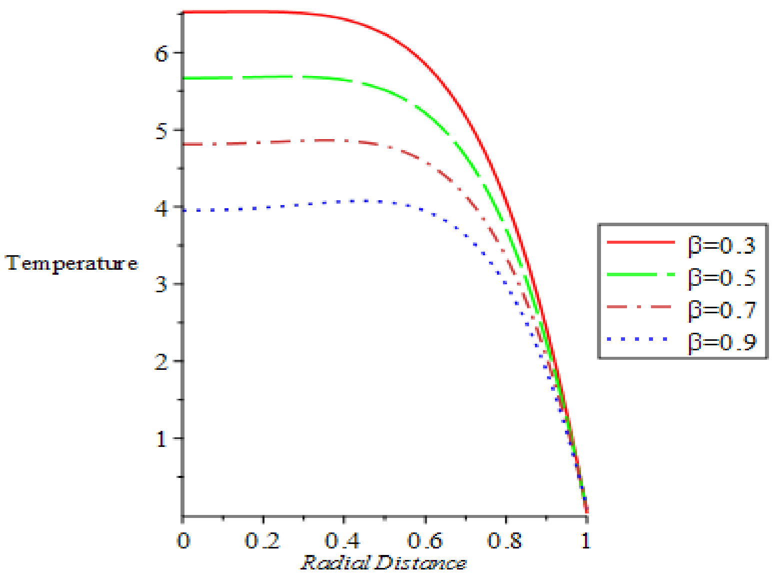

Figure 3: Effects of Third Grade Parameter (β) On Temperature When M=1, $B_r = 500$

Figure 3 shows the effects of third grade parameter on the temperature of the system. It is observed that the parameter (β) has the tendency of reducing the temperature at the cylindrical walls.

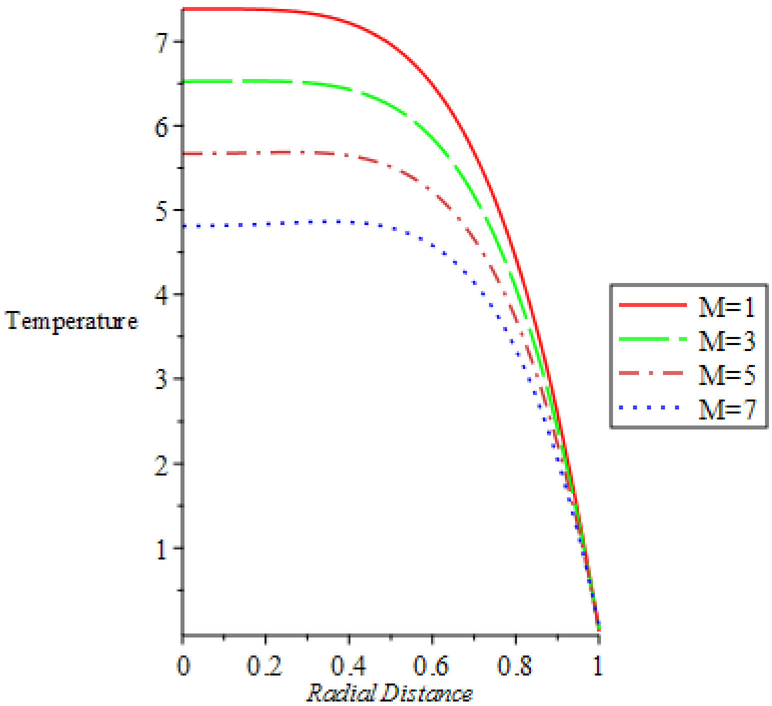

Figure 4: Effects of Magnetic Field Parameter (M) On Temperature When β=0.1, $B_r = 500$

Figure 4 is the temperature profiles for various values of the magnetic field parameters. It is seen that as the magnetic field increases, the temperature of the system decreases.

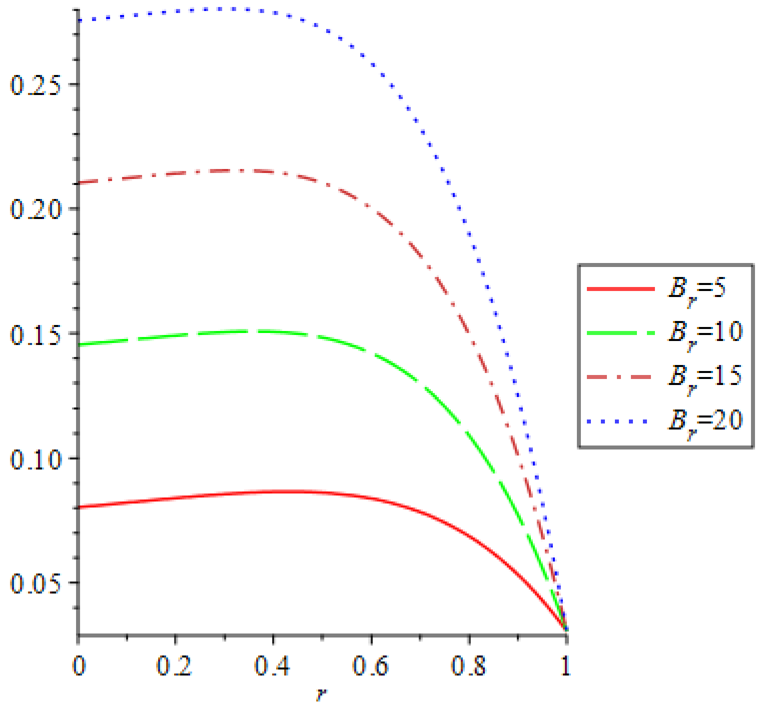

Figure 5: Temperature Profiles For Various Values of The Brinkman Parameter (B_r) When β=0.3, M = 1.

Figure 5 shows the temperature profiles for variation of the Brinkman number. Results show that increase in the Brinkman number, enhances the temperature of the cylindrical pipe.

Discussion

Figure 1 illustrates the effect of the third-grade parameter (β) on velocity when the magnetic field parameter (M) is set to 5. Key observations from the figure includes the fact that as β increases from 0.3 to 0.9, the velocity decreases significantly across the radial distance. The highest velocity is observed for β = 0.3 (solid red line), while the lowest is for β = 0.9 (dotted blue line). The trend confirms that increasing the third-grade parameter (β) leads to a reduction in fluid velocity, which aligns with the behavior of non-Newtonian fluids where higher-order effects introduce more resistance to flow.

Figure 2 shows how the magnetic field strength (M) affects the velocity when the third-grade parameter is fixed at β=0.003. It could be seen from the figure that as M increases (from 500 to 2000), the velocity decreases significantly. The highest velocity occurs when M=500M (solid red line), and the lowest velocity is at M=2000 (dotted blue line). This means that increasing the magnetic field strength slows down the fluid flow, which is a common effect in magneto hydrodynamics (MHD). Over all, higher values of β make the fluid more resistant to motion, reducing its velocity. Stronger magnetic fields (M) also slow down the fluid, likely due to electromagnetic forces opposing the flow. The graph (figures 3) examines how varying the third-grade parameter (β) affects temperature distribution within a fluid when the magnetic field parameter (M) is fixed at 1 and Br=500. The x-axis represents the radial distance, while the y-axis represents temperature. Different curves correspond to different values of β: Red (solid) line: β=0.3 Green (dashed) line: β=0.5 Brown (dash-dot) line: β=0.7 Blue (dotted) line: β=0.9. As the third-grade parameter (β) increases, the overall temperature of the fluid decreases. The highest temperature is observed for β = 0.3 (solid red line), while the lowest temperature occurs at β = 0.9 (dotted blue line). The temperature is relatively stable near the center but drops sharply towards the boundary (radial distance = 1). This indicates that higher values of β enhance heat dissipation, leading to lower temperature levels throughout the fluid. A lower third-grade parameter (β) retains more heat in the fluid, whereas higher β results in a more significant temperature drop. This suggests that increasing β enhances thermal conduction or energy dissipation within the fluid system.

Figure 4 depicts how varying the magnetic field parameter (M) influences temperature distribution within a fluid when the third-grade parameter is fixed at β=0.1. The x-axis represents the radial distance, while the y-axis represents temperature. Different curves correspond to different values of M: Red (solid) line: M=1. Green (dashed) line: M=3 Brown (dash-dot) line: M=5 Blue (dotted) line: M=7 As the magnetic field parameter (M)

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increases, the overall temperature decreases. The highest temperature is observed for M=1 (solid red line), while the lowest temperature occurs at M=7 (dotted blue line). The temperature remains relatively stable near the center but drops significantly as the radial distance approaches 1. A stronger magnetic field leads to enhanced heat dissipation, reducing the fluid temperature. The application of a higher magnetic field (M) creates a magneto hydrodynamic (MHD) effect, which enhances resistance to fluid motion and reduces temperature. This suggests that increasing M enhances thermal conduction and cooling within the system.

This graph (Figure 5) shows how the Brinkman parameter (Br) affects the temperature distribution when the third-grade parameter is fixed at β= 0.3 and the magnetic field parameter is M=1.The x-axis represents radial distance (r), and the y-axis represents temperature. The different curves correspond to different values of Br: Red (solid) line: Br=5, Green (dashed) line: Br=10, Brown (dash-dot) line: Br=15, Blue (dotted) line: Br=20. Observably, Higher Br leads to higher temperature. The smallest Brinkman parameter (Br=5) results in the lowest temperature profile (red line). The largest Brinkman parameter (Br=20) corresponds to the highest temperature profile (blue dotted line). Near the center (r=0), temperature increases with Br and stabilizes. At the boundary (r=1), temperature drops to zero for all cases. Interpretation The Brinkman parameter (Br) is linked to the porosity effects in fluid flow. Increasing Br leads to greater thermal diffusion, enhancing heat transfer within the system. This suggests that higher porosity increases heat retention, raising the overall temperature.

Conclusion Mathematical computation of flow of non-Newtonian fluid with heat generation in cylindrical pipe has been considered in this study aided by the formulation of a coupled system of non-linear differential equation. The analysis of temperature profiles under varying magnetic field strength (M), Brinkman parameter (Br), and other flow parameters reveals that increasing M reduces temperature due to enhanced resistive effects, while higher Br enhances heat retention by increasing thermal diffusion. Additionally, the influence of third-grade fluid parameters highlights the complex interplay between viscosity and heat transfer. These findings provide insight into optimizing thermal management in magneto hydrodynamic (MHD) and porous media flows. Future research could extend this study by incorporating variable thermal conductivity, non-Newtonian effects, or considering time-dependent boundary conditions for a more comprehensive understanding of real-world applications.

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