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SOLUTION OF STIFF ORDINARY DIFFERENTIAL EQUATIONS BY ABOODH TRANSFORM METHOD

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Abstract
The stiff differential equation occurs in almost every field of science. These systems are encountered in electric circuits, mathematical biology, chemical reaction process, meteorology, mechanics and vibrations. The use of normal numerical approaches to analyse and predict such systems requires more time and memory, and getting an exact solution is utterly uneconomical and unreliable. When dealing with a stiff system using numerical approaches, stability problem emerges. In getting over this restriction, the Aboodh transform is suggested as one of the convenient tools to obtain closed-form solutions for linear and nonlinear stiff ordinary differential equations due to it mathematical simplicity. In this paper, we considered some examples to demonstrate the simplicity and potency of the Aboodh transform method in providing an exact solution to the linear stiff ordinary differential equation and comparing the answer with the Laplace transform method.

Keywords: Aboodh transform, closed form, Laplace transform, Numerical techniques, exact solution.

Introduction
The stiff differential equation occurs in almost every field and in real–world problems. These systems are encountered in electric circuits, mathematical biology, chemical reaction process, meteorology, mechanics and vibrations. Stiff initial value problems for ordinary differential equations arise in Fluid mechanics elasticity, electrical networks, chemical reactions and many other areas of physical importance (Joseph & Marley, 1977). A Stiff equation is a differential equation for which certain numerical methods for solving the equation are numerically unstable unless the step size is taken to be extremely small. It has proven difficult to formulate a precise definition of Stiffness, but the main idea is that the equation includes some terms that can lead to rapid variation in the solution (Ghassan, 2019). Many numerical and analytical methods have been developed, modified and applied to Stiff ordinary differential equations. These techniques include Variational Iteration Method (Olayiwola , 2018), the decomposition method (Mahood et al., 2005), the Modified Homotopy perturbation method (Aminikhah, 2011), A-stable Block Method (ASBM) (Muhammed Izzat et al., 2014), Backward Euler method (BEM) (Sumithra & Tamilselvan, 2015), Additive Runge Kutta (Cooper & Ali, 1983), Patches Method (Brydon & Marder , 1998), Implicit Trapezoidal Method (Sumithra & Tamilselvan, 2015), Laplace homotopy analysis method (Chong , Lem & Wong, 2015) and many more modified methods. Analyzing and solving the stiff ODE with conventional numerical techniques require more time and memory combined with an uneconomical and uncertain accurate solution, hence the need for methods to take care of the setbacks of the conventional methods. Therefore, the motivation of this paper is to present the Aboodh transform as an effective and efficient method for solving linear stiff ordinary differential equations in no time.

Materials and Methods
Some important functions in the Aboodh and Laplace transform

Table 1: The Aboodh and Laplace Transform of Functions.

Functions	Aboodh Transform	Laplace Transform
$F(t)$	$A[F(t)]$	$L[F(t)]$
$F(1)$	$\frac{1}{v^2}$	$\frac{1}{S}$
$[F(t)]$	$\frac{1}{v^3}$	$\frac{1}{S^3}$
$F(t^n)$	$\frac{n!}{v^{n+1}}$	$\frac{n!}{S^{n+1}}$
$F(e^{bt})$	$\frac{1}{v^2 - bv}$	$\frac{1}{S-b}$
$F(\sin bt)$	$\frac{b}{v^3 + b^2 v}$	$\frac{b}{S^2 + b^2}$
$F(\cos bt)$	$\frac{1}{v^2 + b^2}$	$\frac{S}{S^2 + b^2}$
$F[y'(t)]$	$vA(y) - y(0)$	$SL(y) - y(0)$
$F[y''(t)]$	$v^2 A(y) - \frac{y'(0)}{v} - y(0)$	$SL(y) - Sy(0) - y'(0)$
$F[y^n(t)]$	$v^n A(y) - \sum_{k=0}^{n-1} \frac{y^k(0)}{v^{2-n+k}}$	$S^n L(y) - S^{n-1} y(0) - \ldots - S^{n-2} y'(0)$ $- S^{n-3} y''(0) - y^{n-1}(0)$

Application of Aboodh Transform

Test 1: Let us consider the first-order differential equation of the form

$$y' = ay + bx^p + m, \qquad y(0) = c, \qquad 0 \le x \le n.$$

where n is an integer (i.e. positive integer) and a, b, c, m, p are constants.

Taking the Aboodh transform, we have

$$A(y') = A(ay) + A(bx^n) + A(m)$$

$$vA(y) - \frac{y(0)}{v} = aA(y) + b\left(\frac{n!}{v^{n+2}}\right) + A(m)$$

$$vA(y) - \frac{c}{v} = aA(y) + \frac{bn!}{v^{n+2}} + A(m)$$

$$A(y)[v - a] = \frac{c}{v} + \frac{bn!}{v^{n+2}} + A(m)$$

$$A(y)[v - a] = \frac{cv^{n+1} + bn! + A(m)v^{n+2}}{v^{n+2}}$$

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$$A(y)=\frac{cv^{n+1}+bn!+A(m)v^{n+2}}{[v-a]v^{n+2}} \tag{1}$$

By taking the inverse Aboodh transform, we arrived at the desired result

Test 2: Let us consider the first-order differential equation of the form

$$y'=ay+b\sin kx+d\cos qx, \qquad y(0)=c, \qquad 0\le x\le n.$$

where n is an integer (i.e. positive integer) and a,b,c,d,q are constants.

Taking the Aboodh transform, we have

$$A(y')=A(ay)+A(b\sin kx)+A(d\cos qx)$$

$$vA(y)-\frac{y(0)}{v}=aA(y)+\frac{bk}{v^3+b^2v}+\frac{d}{v^2+b^2}$$

$$vA(y)-\frac{c}{v}=aA(y)+\frac{bk}{v^3+b^2v}+\frac{d}{v^2+b^2}$$

$$A(y)[v-a]=\frac{c}{v}+\frac{bk}{v^3+b^2v}+\frac{d}{v^2+b^2}$$

$$A(y)=\frac{cv^2+cb^2+bk+dv}{v^4-av^3+b^2v^2-ab^2v} \tag{2}$$

Application of Laplace Transform

Test 1: Let us consider the first-order differential equation of the form

$$y'=ay+bx^p+m, \qquad y(0)=c, \qquad 0\le x\le n.$$

where is n an integer (i.e. positive integer) and a,b,c,m,p are constants.

Taking the Laplace transform we have

$$L(y')=L(ay)+L(bx^n)+L(m)$$

$$SL(y)-y(0)=aL(y)+b\left(\frac{p!}{S^{p+1}}\right)+L(m)$$

$$L(y)[S-a]=c+\frac{bp!}{S^{p+1}}+L(m)$$

$$L(y)=\frac{cS^{p+1}+bp!+S^{p+1}L(m)}{S^{p+1}[S-a]} \tag{3}$$

Test 2: Let us consider the first-order differential equation of the form

$$y'=ay+b\sin kx+d\cos qx, \qquad y(0)=c, \qquad 0\le x\le n.$$

where is n an integer (i.e. positive integer) and a,b,c,d,q are constants.

Taking the Laplace transform, we have

$$L(y')=L(ay)+L(b\sin kx)+L(d\cos qx)$$

$$SL(y)-y(0)=aL(y)+\frac{bk}{S^2+b^2}+\frac{dS}{S^2+b^2}$$

$$SL(y)-c=aL(y)+\frac{bk}{S^2+b^2}+\frac{dS}{S^2+b^2}$$

$$L(y)[S-a]=\frac{c[S^2+b^2]+bk+dS}{S^2+b^2}$$

$$L(y)=\frac{c[S^2+b^2]+bk+dS}{[S-a][S^2+b^2]} \tag{4}$$

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Result

Example 1

Solve $y' = -100\left(y - x^3\right) + 3x^2 \qquad y(0) = 0$

(I) Using Aboodh transform

Recall equation (1)

$$A(y) = \frac{cv^{n+1} + bn! + A(m)v^{n+2}}{[v-a]v^{n+2}}$$

$a = -100 \qquad b = 100 \qquad c = 0 \qquad p = 3 \quad m = 3x^2, \; A\left(3x^2\right) = \frac{6}{v^4}$

substituting the above values, we have

$$A(y) = \frac{6v + 600}{v^6 + 100v^5} = \frac{6}{v^5}$$

by taking inverse Aboodh transform of *A(y)*, the solution gives

$$y(x) = x^3$$

(II) Using Laplace transform

Recall equation (3)

$$L(y) = \frac{cS^{p+1} + bp! + S^{p+1}L(m)}{S^{p+1}[S-a]}$$

$a = -100, \; b = 100, \quad c = 0, \quad p = 3, \; m = 3x^2, L\left(3x^2\right) = \frac{6}{S^4}$

Substituting the above values, we have

$$L(y) = \frac{6S + 600}{S^5 + 100S^4} = \frac{6}{S^4}$$

Taking the inverse Laplace transform of *L(y)*, the result gives

$$y(x) = x^3$$

Example 2

Solve $y' = -100(y - x) + 1 \qquad y(0) = 1$

$$y' = -100y + 100x + 1$$

(I) Using Aboodh transform

$$A(y) = \frac{cv^{n+1} + bn! + A(m)v^{n+2}}{[v-a]v^{n+2}}$$

$a = -100, \; b = 100, \quad c = 1, \quad p = 1, \; m = 1,$

Substituting the above values, we have

$$A(y) = \frac{v^2 + v + 100}{v^4 + 100v^3} = \frac{1}{v^2 + 100v} + \frac{1}{v^3}$$

By taking the inverse Aboodh transform of *A(y)*, the result gives

$$y(x) = x + e^{-100x}$$

(II) Using Laplace transform

$$L(y) = \frac{cS^{p+1} + bp! + S^{p+1}L(m)}{S^{p+1}[S-a]}$$

$a = -100, \; b = 100, \quad c = 1, \quad p = 1, \; m = 1,$

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Substituting the above values, we have

$$L(y)=\frac{S^2+S+100}{S^3+100S^2}=\frac{1}{S+100}+\frac{1}{S^2}$$

Taking the inverse Laplace transform of *L(y)*, we have

$$y(x)=x+e^{-100x}$$

Example 3

Solve $y'=-20y+20\sin x+\cos x \qquad y(0)=1$

(I) Using Aboodh transform

Recall equation (2)

$$A(y)=\frac{cv^2+cb^2+bk+dv}{v^4-av^3+b^2v^2-ab^2v}$$

$$a=-20,\ b=20,\ c=1,\ \ d=1,\ \ q=1,k=1$$

Substituting the above values, we have

$$A(y)=\frac{v^2+v+21}{v^4+20v^3+v^2+20v}=\frac{1}{v^2+v}+\frac{1}{v^2+20v}$$

By taking inverse Aboodh transform

$$y(x)=\sin x+e^{-20x}$$

(II) Using Laplace transform

Recall equation (4)

$$L(y)=\frac{c\left[S^2+b^2\right]+bk+dS}{\left[S-a\right]\left[S^2+b^2\right]}$$

$$a=-20,\ b=20,\ c=1,\ \ d=1,\ \ q=1,\ k=1$$

Substituting the above values, we have

$$L(y)=\frac{S^2+S+21}{S^3+20S^2+S+20}=\frac{1}{S^2+1}+\frac{1}{S+20}$$

Taking the inverse Laplace transform of *L(y)*, we have

$$y(x)=\sin x+e^{-20x}$$

Conclusion

In this paper, we made use of the Aboodh transform method in a convenient way similar to the Laplace transform to solve stiff differential equations with constant coefficients. Aboodh transform is effective in providing exact solutions to boundary value problems for stiff ordinary differential equations. The method has proven to be effective in providing exact solutions to stiff ordinary differential equations.

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