



A SIMULATION STUDY OF PREDATOR-PREY INTERACTION MODEL WITH CHANGING CLIMATIC FACTORS IN AN AQUATIC ENVIRONMENT

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Abstract

The predator-prey interaction model has been a long-standing model application in the theory of mathematical modelling of interacting biological processes. While the phenomenon of climate change is not new, the quantification of the effect of climate change on predator and prey populations in an aquatic environment is a challenging scientific problem. The study have applied the method of a numerical simulation to tackle this problem and found out that in the event of a climate change, the predator dominantly enjoys a biodiversity gain whereas the prey is vulnerable to more instances of biodiversity loss.

Keywords: Biodiversity gain, Environmental, Analytical method, Effect of Climate, Mathematical modelling

Introduction

Mathematical modelling is simply the process of interpreting problems from an application area into mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application (Neumaier, 2004; Nwagor, 2020). The application of mathematical modelling covers a large field of science, technology, engineering, health, business and every sector of human activity (Nwagor & Lawson-Jack, 2020; Mba et al., 2020a; Mba et al., 2020b; James et al., 2020). A predator is an organism that feeds on another organism. The prey is the organism which the predator feeds on. Predation is a biological interaction where one organism, the predator kills and feeds on another organism, its prey. The dynamic relationships between the species and their complex properties are at the heart of many ecological and biological processes. One such relationship is the dynamic relationship between "a predator and their prey which has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance". Nature can provide some degree of protection to a given number of prey populations by providing refuges. Such refugia can help in prolonging prey-predator interactions by reducing the chance of extinction due to predation and damp prey-predator oscillations (Ashine & Gebru, 2017). Tyson and Lutscher (2016) developed and analyzed a two-season model for the great horned owl

(*Buboviginalis*) and the snowshoe hare (*Lepus americanus*). They studied the qualitative behaviour of this predator-prey model community as the summer season length changes. Their results illustrate that the impacts of global change on local ecosystems can be driven by internal system dynamics and can potentially have catastrophic consequences. Dymnikov (2015) opined that numerical simulation of global climate models is the major method for examining the changes in contemporary climate under the impact of anthropogenic influences. General circulation models of the atmosphere and ocean are the basis of global models of climate.

Climate change is a long-term shift in global or regional climate patterns. Often climate change refers specifically to the rise in global temperatures from the mid-20th century to the present. Climate change affects ecological communities through its impact on the physiological performance of individuals. Effects of climate change are increasingly observed in the biosphere. The accelerating change prompts the need for assessments of future changes. Ecological communities are particularly vulnerable, but their responses to climate change are also notoriously difficult to predict. Attention so far has focused on predicting the responses of specific populations through the direct impact of changed temperature on their physiology or through the bio-climatic envelope and species distribution models (Zhang et al., 2017).

Predator-prey interactions are characterized by five stages, and the likelihood of prey being consumed is determined by the frequency of encounters and interactions. Therefore, climate change may reduce prey survival either by increasing encounter rates or by increasing the probability of any stage in the interaction. Most systems, however, consist of multiple predator species, and climate may alter the relative importance of different predators to prey mortality rates through differential influences at any stage in the interaction. This alteration may in turn influence the future population dynamics of the prey. However, these complexities remain largely unexplored, because of the difficulties of simultaneously monitoring detailed changes in environmental conditions and cause-specific mortality of the target species (Creel & Christianson, 2008). According to Broitman et al. (2009), habitat temperature is often assumed to serve as an effective proxy for organism body temperature when making predictions of species distributions under future climate change. However, the determinants of body temperature are complex and organisms in identical microhabitats can occupy radically different thermal niches. This can have major implications for our understanding of how thermal stress modulates predator-prey relationships under field conditions.

Climate change can redistribute the strength of ecological interactions between predator and prey species. Higher temperatures and a more stable climate with less seasonal variability lead to more intense predation pressure. However, the increased climate instability that accompanies ongoing climate change, especially in tropical regions, will lead to an overall decline in predation pressure in the tropics. In contrast, predation pressure will rise in some temperate regions (Moon, 2010). Climate has far-reaching impacts on biological systems. Survival and reproduction depend on how well-adapted individuals are to local climate patterns. Climate change can disrupt the match between organisms and their local environment, reducing survival and reproduction and causing subsequent impacts on populations or species' distributions across geographic regions. Climate change may benefit some species and cause extinction for others. Predicting future biological impacts of climate change remains a formidable challenge for science (McCarty et al., 2009).

Aim of the Study

This study aims to construct a mathematical model that is indexed by a semi-stochastic analysis which can be used to predict the effect of climate change on predators and prey over time in an aquatic environment.

Objectives of the Study

The key objectives of this study outlined are as follows:

1. to predict and obtain instances of the effect of climate change on predators using the method of ordinary differential equations of order 45 (ODE45).
2. to forecast and get instances of the effect of climate change on prey using the method of ODE45.

Methodology

Mathematical Formulations

Following (Zill, 1997), Consider the Lotka-Velterra predator-prey model defined by

$$\frac{dx}{dt} = -0.1x + 0.02xy \quad (1)$$

$$\frac{dy}{dt} = 0.2y - 0.025xy \quad (2)$$

where the populations $x(t)$ (predator) and $y(t)$ (prey) are measured in the thousands.

Model Assumptions

In this study, we have considered a predator-prey interaction model with two fundamental assumptions:

1. The prey population contributes to enhancing the growth of the predator population in the aquatic environment.
2. The predator population contributes to inhibiting the growth of the prey population in the aquatic environment.

Existence of Steady state solution

The rates of change for steady state solution $\frac{d}{dt} = 0$,

$$\frac{dx}{dt} = -0.1x + 0.02xy = 0$$

$$\frac{dy}{dt} = -0.2y + 0.025xy = 0$$

from equation (1), we have that

$$-0.1x + 0.02xy = 0$$

$$x(-0.1 + 0.02y) = 0$$

$$x = 0$$

$$-0.1 + 0.02y = 0 \quad \text{Implies that} \quad 0.02y = 0. \quad y = \frac{0.1}{0.02}$$

$$y = 5 \tag{3}$$

At a steady state solution $x = 0$ and $y = 5$ which gives (0,5). This implies that at a steady state solution, the predator population goes into extinction whereas 5,000 of the prey population survived.

From equation (2), we have that

$$0.2y - 0.025xy = 0$$

$$y(0.2 - 0.025x) = 0$$

$$y = 0$$

$$0.2 - 0.025x = 0$$

$$0.025x = 0.2$$

$$x = \frac{0.2}{0.025}$$

$$x = 8 \tag{4}$$

At a steady state solution $x = 8$ and $y = 0$ which gives (8,0). This implies that at a steady state solution 8,000 of the predator population survives whereas the prey population goes into extinction.

Numerical Analysis

In order to calculate the effect of climate change on biological species in an aquatic environment, we have applied the following formula

$$BG(\%) = \left(\frac{X_{(t)new} - X_{(t)old}}{X_{(t)old}} \right) (100) \tag{5}$$

Provided $X_{(t)}$ old is not equal to zero where $X_{(t)}$ specifies the size of predators at time, t. The notation BG denotes biodiversity gain.

$$BL(\%) = \left(\frac{y_{(t)new} - y_{(t)old}}{y_{(t)old}} \right) (100) \tag{6}$$

Provided $y_{(t)}$ is not equal to zero where $y_{(t)}$ specifies the size of prey at a time, t. The notation BL denotes biodiversity loss.

Results

We present and discuss the result that we have obtained on the application of our method of analysis which has been defined earlier.

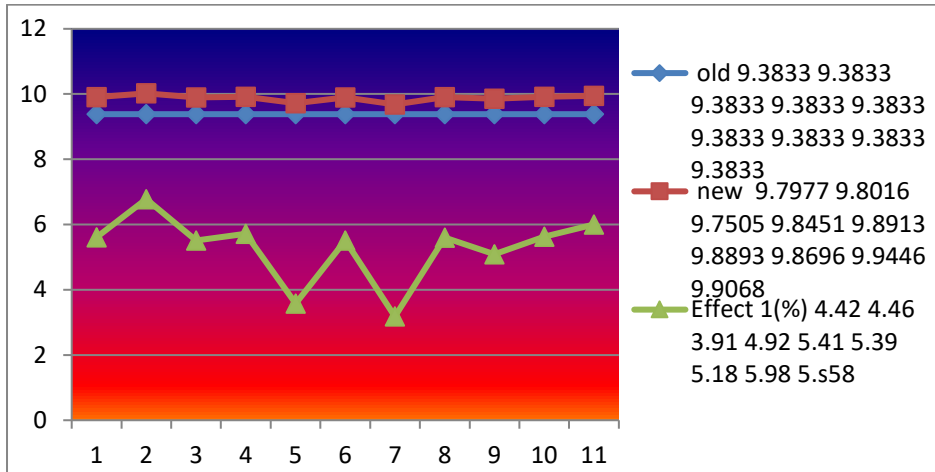


Figure 1: Quantifying the effect of climate change on predators due to a 0.02 random ecological Perturbation: Scenario One (1)

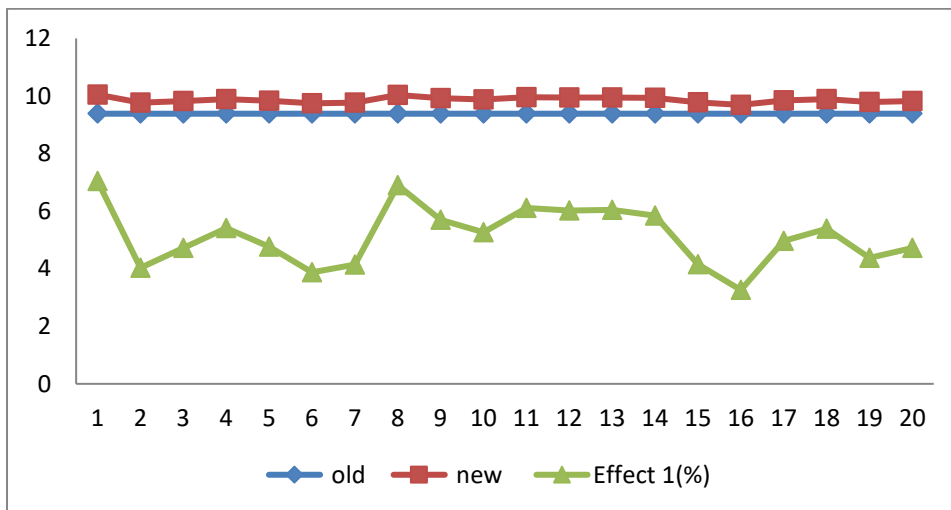


Figure 2: Quantifying the effect of climate change on predators due to a 0.02 random ecological Perturbation: Scenario Two (2)

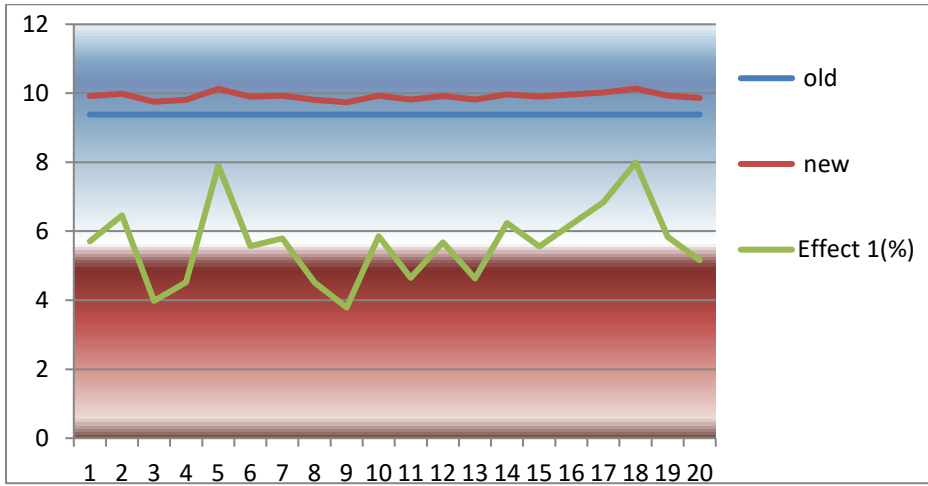


Table 3: Quantifying the effect of climate change on predators due to a 0.02 random ecological perturbation: Scenario Three (3)

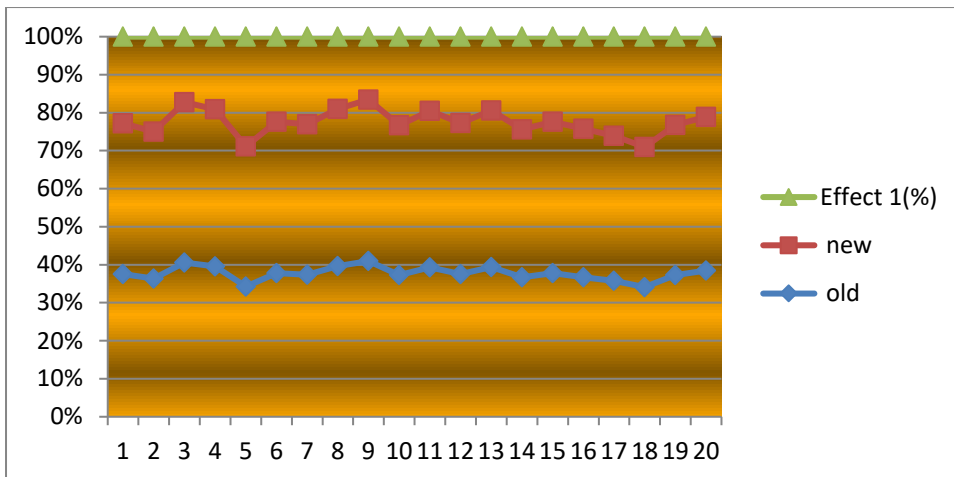


Table 4: Quantifying the effect of climate change on predators due to a 0.02 random ecological perturbation Scenario Four (4)

Discussion

Looking at Figure 1, describing the impact of a 0.02 random ecological perturbation on the predator, the study shows that the population size of predators is 9.3833 otherwise called $x(t)$ old. Whereas, $x(t)$ new which measures the size of the predator population to the 0.02 random ecological perturbation the population size of the predator fluctuates. In this scenario, It is observed that the random ecological perturbation due to the effect of climate change generally indicates biodiversity gain as represented by effect 1(%). For effect 1(%) in Table 1, the minimum biodiversity gain is 3.18 while the maximum biodiversity gain is 6.00.

Similarly in Figure 2, due to a 0.02 random ecological perturbation on the predator, It is also observed that the population size of predators remains the same which gives 9.3833 otherwise called $x(t)$ old. Whereas, $x(t)$ new which measures the size of the predator population due to the 0.02 random ecological perturbation the population size of the predator changes. In the second scenario, we have observed that the minimum biodiversity gain is 3.79 while the maximum biodiversity gain is 7.99.

In summary, the overall minimum biodiversity gain in all scenarios of the predator population due to a 0.02 ecological perturbation is 3.18. Whereas, the overall maximum biodiversity gain in all four scenarios of the predator population is 9.16.

Conclusion

In the application area of using the process of mathematical modelling to study the predator-prey interaction model, the analytical method of finding the steady state solution of a specified predator-prey interaction is a popular mathematical construction in the body of knowledge but an additional application of a computationally efficient Matlab numerical simulation gives this present study an advantage over similar methods of analysis the proposed research problems in an alternative method of analysis. The key results of this study show that more instances of biodiversity gain due to the effect of a climate change of 0.02 random environmental perturbations on the predators in an aquatic environment.

Recommendations

Based on the finding of the application of the Lotka – Volterra predator-prey model to describe the dynamics of the interactions between predators and prey, the listed needs further analysing of population dynamic of the effects of climate change on predators and prey

1. Using Mathematical modelling to solve predator and prey numbers which undergo cycles with predator numbers at the peak before prey numbers
2. Also, analytical methods to solve predator and prey numbers which undergo cycles with prey numbers at the peak before predator numbers.

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Appendices

Table 1: Quantifying the effect of climate change on predators due to a 0.02 random ecological perturbation: Scenario One (1)

$x(t)$ old	$x(t)$ new	Effect 1(%)
9.3833	9.7977	4.42
9.3833	9.8016	4.46
9.3833	9.7505	3.91
9.3833	9.8451	4.92
9.3833	9.8913	5.41
9.3833	9.8893	5.39
9.3833	9.8696	5.18
9.3833	9.9446	5.98
9.3833	9.9068	5.58
9.3833	9.9094	5.61
9.3833	10.0211	6.78
9.3833	9.9001	5.51
9.3833	9.9196	5.72
9.3833	9.7190	3.58
9.3833	9.9003	5.51
9.3833	9.6817	3.18
9.3833	9.9088	5.60
9.3833	9.8611	5.09
9.3833	9.9119	5.63
9.3833	9.9464	6.00

Table2: Quantifying the effect of climate change on predators due to a 0.02 random ecological perturbation: Scenario Two (2)

$x(t)$ old	$x(t)$ new	Effect 1(%)
9.3833	10.0442	7.04
9.3833	9.7612	4.03
9.3833	9.8262	4.72
9.3833	9.8906	5.41
9.3833	9.8303	4.76
9.3833	9.7465	3.87
9.3833	9.7716	4.14
9.3833	10.031	6.90
9.3833	9.9179	5.70
9.3833	9.8766	5.26
9.3833	9.9567	6.11
9.3833	9.9480	6.02
9.3833	9.9498	6.04
9.3833	9.9313	5.84
9.3833	9.7724	4.15
9.3833	9.6890	3.26
9.3833	9.8483	4.96
9.3833	9.8891	5.39
9.3833	9.7932	4.37
9.3833	9.8265	4.72

Table 3: Quantifying the effect of climate change on predators due to a 0.02 random ecological perturbation: Scenario Three (3)

$x(t)$ old	$x(t)$ new	Effect 1(%)
9.3833	9.9193	5.71
9.3833	9.9894	6.46
9.3833	9.7569	3.98
9.3833	9.8078	4.52
9.3833	10.126	7.91
9.3833	9.9056	5.57
9.3833	9.9270	5.79
9.3833	9.8064	4.51
9.3833	9.7389	3.79
9.3833	9.9332	5.86
9.3833	9.8192	4.65
9.3833	9.9166	5.68
9.3833	9.8178	4.63
9.3833	9.9692	6.24
9.3833	9.9047	5.56
9.3833	9.9654	6.20
9.3833	10.025	6.84
9.3833	10.133	7.99
9.3833	9.9313	5.84
9.3833	9.8670	5.16

Table 4: Quantifying the effect of climate change on predators due to a 0.02 random ecological perturbation: Scenario Four (4)

$x(t)$ old	$x(t)$ new	Effect 1(%)
9.3833	9.9434	5.97
9.3833	10.0382	6.97
9.3833	10.2429	9.16
9.3833	9.9113	5.63
9.3833	9.8601	5.08
9.3833	9.7962	4.40
9.3833	9.8044	4.49
9.3833	9.8493	4.97
9.3833	9.7106	3.49
9.3833	9.9076	5.59
9.3833	9.8399	4.87
9.3833	9.9975	6.55
9.3833	10.046	7.06
9.3833	9.8519	4.99
9.3833	9.9224	5.75
9.3833	9.8239	4.70
9.3833	9.9098	5.61
9.3833	9.7997	4.44
9.3833	9.9708	6.26
9.3833	10.001	6.58