



NUMERICAL TREATMENT OF MANHATTAN DISTANCE

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Abstract

The Manhattan Distance Measure is an important metric that can provide a more robust explanation for variations in cowpea and groundnut yields. Using a well-established ordinary differential equation and the ODE45 numerical simulation method, this study offers a numerical analysis of the Manhattan measure in relation to the variations in the initial biomass of cowpeas and groundnuts over a specified growing season. The results demonstrate that a slight variation in the initial conditions for both crops leads to a larger estimated value of the Manhattan distance, while a relatively larger variation in the same initial conditions results in a smaller estimated value of the Manhattan distance.

Keywords: Numerical treatment, Manhattan, Distance, Measure, Metric

Introduction

According to the National Council of Teachers of Mathematics (NCTM) as cited in Egbo et al. (2019), basic mathematics skills should not be limited to routine computation at the expense of understanding, application, and problem-solving. The council emphasizes that the identification of basic mathematics skills is a dynamic process and should be continually updated to reflect new and changing needs. For mathematics and statistics students, major employability skills include the ability to use mathematics and statistics to develop solutions to practical problems.

In the theory of functional analysis, three types of distance measures between vector spaces and time-dependent data play a significant role in the numerical simulation of the nonlinear continuous dynamical system described by the first-order ordinary differential equation used to model cowpea-groundnut interaction in an agricultural setting. This concept is supported by Maslor (1998), Chidume (1984), Mba et al. (2020a) and Mba et al. (2020b).

The Manhattan distance measures on a vector space are defined as the maximum value of the metric of pairs of time-dependent data, as noted by Mba (2021). When applied in the study of numerical simulation, this functional analysis idea can be used to classify the behavior of solution trajectory data in response to variations in the initial conditions. The research question of how variations in the initial condition affect the expected Manhattan distance measures can be a challenging analytical problem. To address this challenging numerical analysis problem, we propose the implementation of the computationally efficient Ordinary Differential Equation 45 (ODE45) Runge-Kutta numerical method, as suggested by Chidume (1984), Mba et al. (2021), Fair (1983), and Jiri (2010). It is not a common practice in numerical analysis to compute estimated values of the Manhattan distance measures using a numerical method for a large sample size, as we have explored in this study."

Materials and Methods

To achieve the aim and objective of this present study, a system of non-linear first-order ordinary differential equations is indexed by the appropriate initial condition as given by Ekaka-a et al. (2017).

Has been considered and extended. It is an ordinary differential equation because it involves derivatives concerning a single independent variable.

$$\frac{dC(t)}{dt} = \alpha_1 C - \beta_1 C^2 + r_1 CG - k_1 C^2 G \quad (1)$$

$$\frac{dG(t)}{dt} = \alpha_2 G - \beta_2 G^2 + r_2 CG - k_2 CG^2 \quad (2).$$

The present study equations (1) and (2) by including a random perturbation variation, generalized non-linear first-order ordinary differential equations will have the following mathematical structure:

$$\frac{dC(t)}{dt} = \alpha_1 C - \beta_1 C^2 + r_1 CG - k_1 C^2 G + (r_{n1} (rand(1))) \quad (3)$$

$$\frac{dG(t)}{dt} = \alpha_2 G - \beta_2 G^2 + r_2 CG - k_2 C^2 G + (r_{n1} (rand(1))) \quad (4)$$

With the initial conditions, $C(0) = C_0 > 0$ $G(0) = G_0 > 0$

G = Groundnut biomass

C = Cowpea biomass

Where :

$C(t)$ defines the cowpeas biomass at time, t .

$G(t)$ defines the groundnut biomass at time, t . α_1 and α_2 define the intrinsic growth rates of the two interacting legumes of cowpea and groundnut respectively. B_1 and B_2 define the intra-competition co-efficient of the two interacting plant species,

r_1 and r_2 define the inter-competition co-efficient of the two interacting plant species. K_1 and k_2 define the disease-inhibiting factors that affect the yields of cowpea and groundnut respect.

r_{n1} specifies the random noise intensity value of 0.00000001 which is insignificant due to the area of study, Imo state. Due to the magnitude of the sample size of the data which we have used, it makes sense to apply a computationally efficient method of ODE45 to compute the real line metric between the parallel data from which the biggest value of the metric defines the expected Manhattan distance measure due to the initial condition values of 0.04 together at-a-time.

Results

The full results of this study are presented in Tables 1-10 (see appendix).

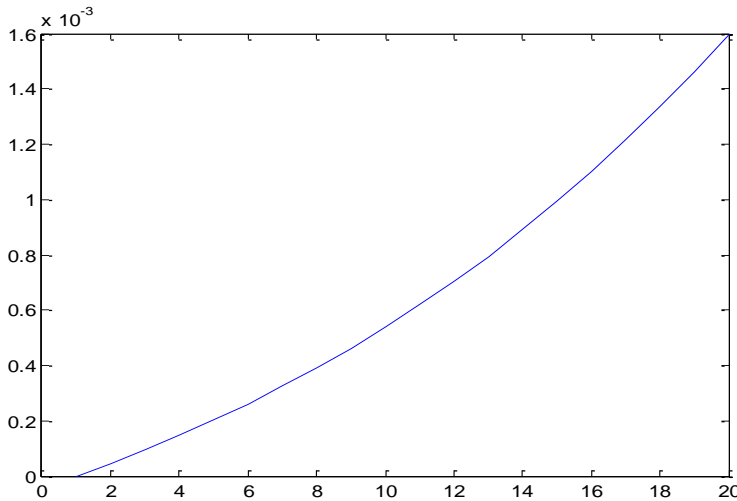


Figure1: Graph of the numerical simulation of the initial condition, (0.04, 0.04), due to fixed initial conditions on the estimated Manhattan distance (EMD) using ODE45

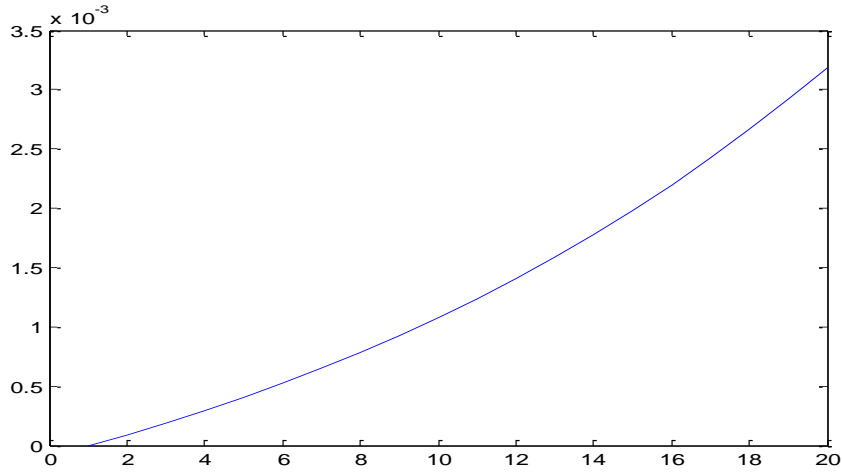


Figure2: Graph of the numerical simulation of the initial condition, (0.04, 0.04), due to 5% variation on the estimated Manhattan distance (EMD) using ODE45

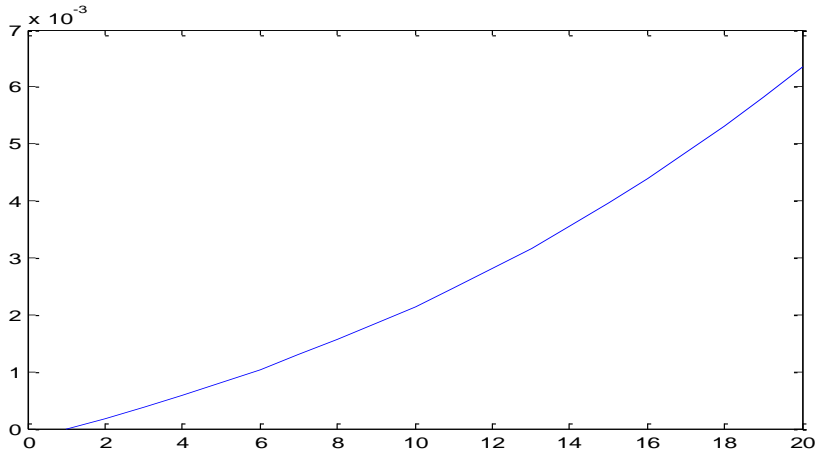


Figure 3: Graph of numerical simulation of the effects of 10% variations of the initial condition (0.04, 0.04) on EMD using ODE45

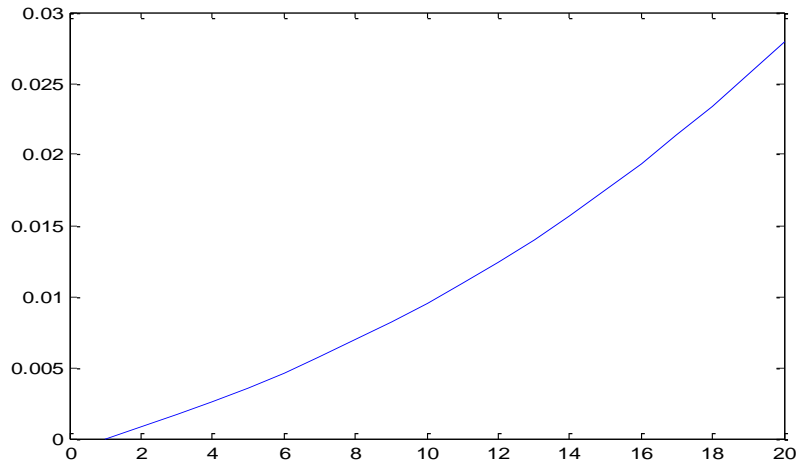


Figure 4: Graph of numerical simulation of the effects of 20% variation of the initial condition (0.04, 0.04) on EMD using ODE45

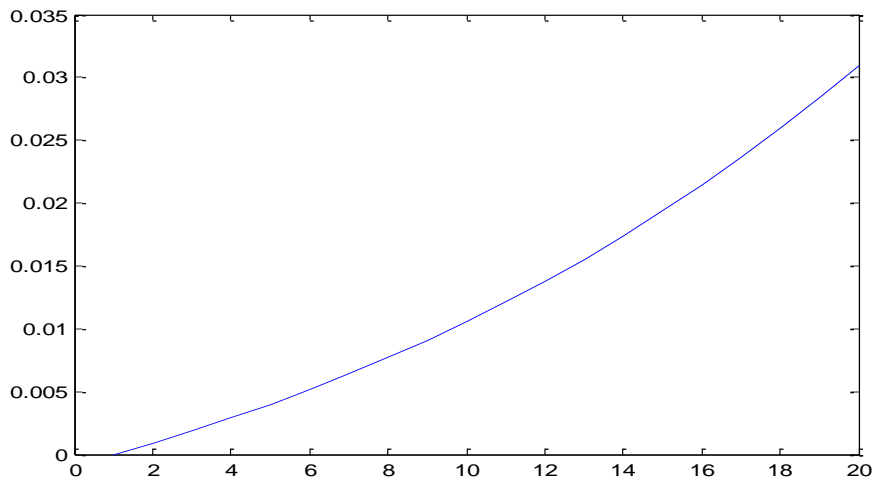


Figure 5: Graph of numerical simulation of the effects of 90% variation of the initial condition (0.04,0.04)on (EMD) using ODE45

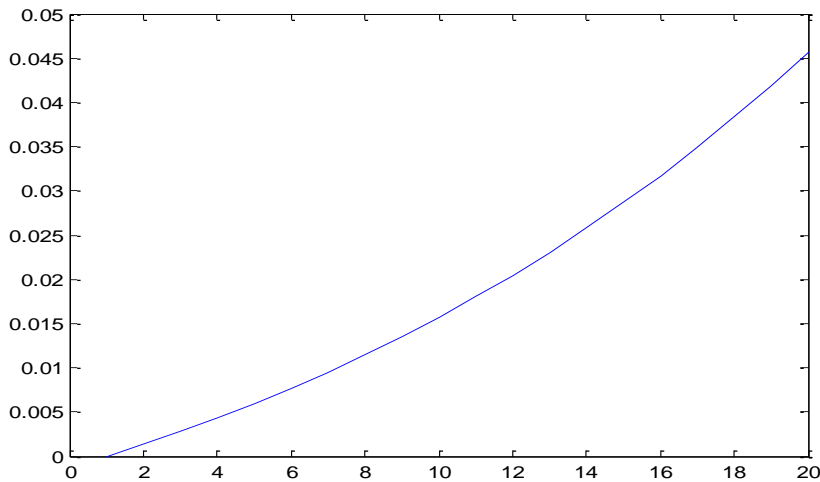


Figure 6: Graph of numerical simulation of the effect of 99.99% variation of the initial condition (0.04,0.04) on EMD using ODE45

Discussion

This present numerical method of computing the effects of varying the initial condition on the Manhattan distance has produced the following important result that we have not seen elsewhere. In the instance when the initial condition is fixed, the estimated computed value of the Manhattan distance is 0.027945099419457 varied by 5% the estimated computed value of the Manhattan distance is 0.0015735467452118 whereas the variation of the same initial condition by 10% has produced a different estimated Manhattan distance value of 0.003189824384137. in the same pattern, the variation of the initial condition by 15%, 20%, 90% ,95%, 99.999%, 110% and 150% have produced six other distinct values estimated Manhattan distance such as 0.004776670665776, 0.006160298859, 0.027945099419457, 0.029448059838065, 0.030942911756800, 0.045646125794108 and 0.033926298375535 to the best of our knowledge, this present results demonstrate that a small variation of the initial condition would lead to a bigger estimated value of the Manhattan distance whereas a relatively bigger decrease variation of the initial condition will lead to a small estimated of the Manhattan distance. Mathematically and numerically, these observations are tractable because a small variation of the initial condition is associated with a bigger disparity between interacting parallel data whereas a relatively bigger variation of the initial condition is associated with a small disparity between interacting parallel data. The figures present a clearer interpretation of our results.

Conclusion

The present observations and results of this numerical simulation study have gone beyond the result of our previous studies to prove how the decreasing variation of the same initial variation boundary affects the estimated Manhattan distance in particular a bigger decreased variation of the initial condition has produced a small magnitude of the Manhattan distance measure. We expect these results to be useful in the other numerical studies of other types of interaction between cowpeas and groundnuts such, as competition, predation, and commensalism which we did not consider in this present study.

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Appendix

Table 1: Numerical simulation of the initial condition, (0.04, 0.04) on the estimated Manhattan distance (EMD) using ODE45

S/N	C(t)	G(t)	m_1m_2	0	0
0	0.0400000000000000	0.0400000000000000		0	0
1.0000000000000000	0.040899083060333	0.041801943711763	0.041801943711763	0.000902860651430	0.000902860651430
2.0000000000000000	0.041818126372863	0.043683999630332	0.043683999630332	0.001865873257470	0.001865873257470
3.0000000000000000	0.042757562086931	0.045649631274126	0.045649631274126	0.002892069187195	0.002892069187195
4.0000000000000000	0.043717831248255	0.047702443409398	0.047702443409398	0.003984612161143	0.003984612161143
5.0000000000000000	0.044699383925556	0.049846185886874	0.049846185886874	0.005146801961319	0.005146801961319
6.0000000000000000	0.045702679409535	0.052084759663656	0.052084759663656	0.006382080254120	0.006382080254120
7.0000000000000000	0.046728186348457	0.054422220926101	0.054422220926101	0.007694034577644	0.007694034577644
8.0000000000000000	0.047776382941642	0.056862787009805	0.056862787009805	0.009086404068162	0.009086404068162
9.0000000000000000	0.048847757090690	0.059410840991430	0.059410840991430	0.010563083900740	0.010563083900740
10.0000000000000000	0.049942806583959	0.062070937343144	0.062070937343144	0.012128130759184	0.012128130759184
11.0000000000000000	0.051062039266046	0.064847807058200	0.064847807058200	0.013785767792154	0.013785767792154
12.0000000000000000	0.052205973211224	0.067746362903554	0.067746362903554	0.015540389692330	0.015540389692330
13.0000000000000000	0.053375136912264	0.070771705091798	0.070771705091798	0.017396568179534	0.017396568179534
14.0000000000000000	0.054570069442256	0.073929126023152	0.073929126023152	0.019359056580896	0.019359056580896
15.0000000000000000	0.055791320662402	0.077224116460137	0.077224116460137	0.021432795797735	0.021432795797735
16.0000000000000000	0.057039451373007	0.080662369677291	0.080662369677291	0.023622918304284	0.023622918304284
17.0000000000000000	0.058315033538481	0.084249788034359	0.084249788034359	0.025934754495878	0.025934754495878
18.0000000000000000	0.059618650429589	0.087992486479377	0.087992486479377	0.028373836049788	0.028373836049788
19.0000000000000000	0.060950896858863	0.091896799205914	0.091896799205914	0.030945902347051	0.030945902347051

Where CB (t) and GB (t) represent cowpea and groundnut biomass respectively while rn1 represents their symmetric property of a metric function.

Table 2: Numerical simulation of the effects of 5% variation of the initial condition (0.04,0.04) on the estimated Manhattan distance, (EMD)using ODE45

S/N	C(t)	G(t)	m ₁	m ₂
0	0.0200000000000000	0.0200000000000000	0	0
1.0000000000000000	0.020452320733933	0.020906579742276	0.000454259008344	0.000454259008344
2.0000000000000000	0.020914808635235	0.021853986268677	0.000939177633442	0.000939177633442
3.0000000000000000	0.021387689456096	0.022844033213260	0.001456343757164	0.001456343757164
4.0000000000000000	0.021871193848844	0.023878612748825	0.002007418899981	0.002007418899981
5.0000000000000000	0.022365557448211	0.024959698200062	0.002594140751851	0.002594140751851
6.0000000000000000	0.022871020992083	0.026089347861354	0.003218326869271	0.003218326869271
7.0000000000000000	0.023387830410285	0.027269707897260	0.003881877486975	0.003881877486975
8.0000000000000000	0.023916236943767	0.028503016229578	0.004586779285811	0.004586779285811
9.0000000000000000	0.024456497243634	0.029791605851016	0.005335108607382	0.005335108607382
10.0000000000000000	0.025008873487009	0.031137908714231	0.006129035227222	0.006129035227222
11.0000000000000000	0.025573633487840	0.032544459523987	0.006970826036147	0.006970826036147
12.0000000000000000	0.026151050808542	0.034013899581319	0.007862848772777	0.007862848772777
13.0000000000000000	0.026741404883248	0.035548981093095	0.008807576209847	0.008807576209847
14.0000000000000000	0.027344981125180	0.037152570946395	0.009807589821215	0.009807589821215
15.0000000000000000	0.027962071062206	0.038827655545443	0.010865584483237	0.010865584483237
16.0000000000000000	0.028592972440658	0.040577344514047	0.011984372073389	0.011984372073389
17.0000000000000000	0.029237989372295	0.042404876038754	0.013166886666459	0.013166886666459
18.0000000000000000	0.029897432436021	0.044313620520089	0.014416188084069	0.014416188084069
19.0000000000000000	0.030571618832653	0.046307086284772	0.015735467452118	0.015735467452118

Table 3: Numerical simulation of the effects of 10% variations of the initial condition (0.04, 0.04) on EMD using ODE45

(t) in Weeks	C(t)	G(t)	m ₁	m ₂
0	0.0040000000000000	0.0040000000000000	0	0
1.0000000000000000	0.004090908926751	0.004182213645148	0.000091304718396	0.000091304718396
2.0000000000000000	0.004183881447471	0.004372716987843	0.000188835540373	0.000188835540373
3.0000000000000000	0.004278964291718	0.004571886133243	0.000292921841525	0.000292921841525
4.0000000000000000	0.004376205244186	0.004780114214093	0.000403908969907	0.000403908969907
5.0000000000000000	0.004475653164394	0.004997812033240	0.000522158868846	0.000522158868846
6.0000000000000000	0.004577358014470	0.005225408966598	0.000648050952128	0.000648050952128
7.0000000000000000	0.004681370880511	0.005463353689306	0.000781982808795	0.000781982808795
8.0000000000000000	0.004787744000266	0.005712115114838	0.000924371114572	0.000924371114572
9.0000000000000000	0.004896530786972	0.005972183236594	0.001075652449622	0.001075652449622
10.0000000000000000	0.005007785856590	0.006244070092873	0.001236284236283	0.001236284236283

11.000000000000000	0.005121565054465	0.006528310743578	0.001406745689113	0.001406745689113
12.000000000000000	0.005237925481958	0.006825464256270	0.001587538774313	0.001587538774313
13.000000000000000	0.005356925526088	0.007136114832641	0.001779189306554	0.001779189306554
14.000000000000000	0.005478624885591	0.007460872817021	0.001982247931429	0.001982247931429
15.000000000000000	0.005603084603528	0.007800375981712	0.002197291378184	0.002197291378184
16.000000000000000	0.005730367092972	0.008155290565948	0.002424923472976	0.002424923472976
17.000000000000000	0.005860536172401	0.008526312723275	0.002665776550874	0.002665776550874
18.000000000000000	0.005993657091383	0.008914169605785	0.002920512514402	0.002920512514402
19.000000000000000	0.006129796567998	0.009319620952135	0.003189824384137	0.003189824384137

Table 4: Numerical simulation of the effect of 15% variation of the initial condition (0.04,0.04) onEMDusing ODE45

(t) in Weeks	C(t)	G(t)	m ₁
0	0.006000000000000	0	0
1.000000000000000	0.006136279986053	0.006273152117868	0.000136872131814
2.000000000000000	0.006275649690900	0.006558715392835	0.000283065701935
3.000000000000000	0.006418178910542	0.006857251453755	0.000439072543213
4.000000000000000	0.006563939009390	0.007169347219434	0.000605408210044
5.000000000000000	0.006713002949183	0.007495615839271	0.000782612890087
6.000000000000000	0.006865445329965	0.007836698020792	0.000971252690827
7.000000000000000	0.007021342421429	0.008193263090613	0.001171920669184
8.000000000000000	0.007180772203698	0.008566010371772	0.001385238168074
9.000000000000000	0.007343814402312	0.008955670411377	0.001611856009065
10.000000000000000	0.007510550528288	0.009363006391656	0.001852455863368
11.000000000000000	0.007681063917250	0.009788815552448	0.002107751635198
12.000000000000000	0.007855439768542	0.010233930627848	0.002378490859306
13.000000000000000	0.008033765188728	0.010699221484109	0.002665456295381
14.000000000000000	0.008216129229795	0.011185596582616	0.002969467352821
15.000000000000000	0.008402622936997	0.011694004845343	0.003291381908346
16.000000000000000	0.008593339386445	0.012225437154384	0.003632097767939
17.000000000000000	0.008788373737021	0.012780928448160	0.003992554711139
18.000000000000000	0.008987823267880	0.013361559277498	0.004373736009618
19.000000000000000	0.009191787433323	0.013968458099099	0.004776670665776

Table 5: Numerical simulation of the effects of 20% variation of the initial condition (0.04, 0.04) on EMD using ODE45

(t) weeks	C(t)	G(t)	m ₁
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m ₂				
0	0.008000000000000	0.008000000000000	0	0
1.000000000000000	0.008181595445623	0.008363978369440	0.000182382923817	0.000182382923817
2.000000000000000	0.008367302959929	0.008744473791429	0.000377170831500	0.000377170831500
3.000000000000000	0.008557215207327	0.009142231740402	0.000585016533075	0.000585016533075
4.000000000000000	0.008751426924470	0.009558031077011	0.000806604152541	0.000806604152541
5.000000000000000	0.008950034957982	0.009992685271715	0.001042650313734	0.001042650313734
6.000000000000000	0.009153138318169	0.010447044139248	0.001293905821078	0.001293905821078
7.000000000000000	0.009360838219900	0.010921995215867	0.001561156995967	0.001561156995967
8.000000000000000	0.009573238135998	0.011418465554431	0.001845227418433	0.001845227418433
9.000000000000000	0.009790443842856	0.011937423315383	0.002146979472527	0.002146979472527
10.000000000000000	0.010012563472827	0.012479879600131	0.002467316127304	0.002467316127304
11.000000000000000	0.010239707565237	0.013046890291579	0.002807182726342	0.002807182726342
12.000000000000000	0.010471989117445	0.013639557913729	0.003167568796284	0.003167568796284
13.000000000000000	0.010709523641563	0.014259033747331	0.003549510105768	0.003549510105768
14.000000000000000	0.010952429214229	0.014906519714993	0.003954090500764	0.003954090500764
15.000000000000000	0.011200826538988	0.015583270786206	0.004382444247219	0.004382444247219
16.000000000000000	0.011454838995151	0.016290596899321	0.004835757904170	0.004835757904170
17.000000000000000	0.011714592705491	0.017029865658167	0.005315272952677	0.005315272952677
18.000000000000000	0.011980216584862	0.017802504314650	0.005822287729787	0.005822287729787
19.000000000000000	0.012251842411724	0.018610002710583	0.006358160298859	0.006358160298859

Table 6: Numerical simulation of the effects of 90% variation of the initial condition (0.04,0.04)on (EMD) using ODE45

(t) weeks	C(t)	G(t)	m ₁	m ₂
0	0.036000000000000	0.036000000000000	0	0
1.000000000000000	0.036810175160316	0.037623767745007	0.000813592584691	0.000813592584691
2.000000000000000	0.037638381648721	0.039319912890693	0.001681531241972	0.001681531241972
3.000000000000000	0.038485011852986	0.041091581802071	0.002606569949085	0.002606569949085
4.000000000000000	0.039350466327563	0.042942050763848	0.003591584436286	0.003591584436286
5.000000000000000	0.040235153914171	0.044874729675066	0.004639575760894	0.004639575760894
6.000000000000000	0.041139491928306	0.046893167749241	0.005753675820935	0.005753675820935
7.000000000000000	0.042063906288581	0.049001057515569	0.006937151226988	0.006937151226988
8.000000000000000	0.043008831698701	0.051202240467715	0.008193408769014	0.008193408769014
9.000000000000000	0.043974711791717	0.053500711547391	0.009525999755674	0.009525999755674

10.000000000000000	0.044961999304290	0.055900624598979	0.010938625294689	0.010938625294689
11.000000000000000	0.045971156238280	0.058406297404408	0.012435141166128	0.012435141166128
12.000000000000000	0.047002654025509	0.061022216827525	0.014019562802016	0.014019562802016
13.000000000000000	0.048056973707720	0.063753044421851	0.015696070714131	0.015696070714131
14.000000000000000	0.049134606091395	0.066603621175907	0.017469015084512	0.017469015084512
15.000000000000000	0.050236051945749	0.069578973664206	0.019342921718456	0.019342921718456
16.000000000000000	0.051361822148439	0.072684318331952	0.021322496183514	0.021322496183514
17.000000000000000	0.052512437899996	0.075925068104443	0.023412630204446	0.023412630204446
18.000000000000000	0.053688430862439	0.079306836176776	0.025618405314337	0.025618405314337
19.000000000000000	0.054890343383957	0.082835442803414	0.027945099419457	0.027945099419457

Table 7: Numerical simulation of the effect of 95% variation of the initial condition (0.04,0.04) on EMD using ODE45

(t) weeks	C(t)	G(t)	m ₁	m ₂
0	0.038000000000000	0.038000000000000	0	0
1.000000000000000	0.038854656892300	0.039712911766505	0.000858254874204	0.000858254874204
2.000000000000000	0.039728311423247	0.041502075946997	0.001773764523750	0.001773764523750
3.000000000000000	0.040621375955316	0.043370798276088	0.002749422320772	0.002749422320772
4.000000000000000	0.041534271386772	0.045322520146389	0.003788248759617	0.003788248759617
5.000000000000000	0.042467427275449	0.047360822380314	0.004893395104866	0.004893395104866
6.000000000000000	0.043421282031682	0.049489431098241	0.006068149066559	0.006068149066559
7.000000000000000	0.044396283050961	0.051712221787881	0.007315938736920	0.007315938736920
8.000000000000000	0.045392886901864	0.054033225096740	0.008640338194877	0.008640338194877
9.000000000000000	0.046411559474000	0.056456631378572	0.010045071904572	0.010045071904572
10.000000000000000	0.047452776157600	0.058986796256989	0.011534020099389	0.011534020099389
11.000000000000000	0.048517022009273	0.061628245716074	0.013111223706801	0.013111223706801
12.000000000000000	0.049604791921336	0.064385681309082	0.014780889387745	0.014780889387745
13.000000000000000	0.050716590806453	0.067263985810487	0.016547395004034	0.016547395004034
14.000000000000000	0.051852933756186	0.070268227971136	0.018415294214950	0.018415294214950
15.000000000000000	0.053014346244175	0.073403668695262	0.020389322451087	0.020389322451087
16.000000000000000	0.054201364274724	0.076675765270670	0.022474400995945	0.022474400995945
17.000000000000000	0.055414534602830	0.080090177976307	0.024675643373476	0.024675643373476
18.000000000000000	0.056654414874874	0.083652773743371	0.026998358868496	0.026998358868496
19.000000000000000	0.057921573858992	0.087369632897056	0.029448059038065	0.029448059038065

Table 8: Numerical simulation of the effect of 99.99% variation of the initial condition (0.04,0.04) on EMD using ODE45

(t) weeks	C(t)	G(t)	m ₁	
0	0.0399960000000000	0.0399960000000000	0	0
1.0000000000000000	0.040894994263449	0.041797765759719	0.000902771496270	0.000902771496270
2.0000000000000000	0.041813946857552	0.043679636021833	0.001865689164281	0.001865689164281
3.0000000000000000	0.042753289892268	0.045645073990775	0.002891784098507	0.002891784098507
4.0000000000000000	0.043713464373212	0.047697684107774	0.003984219734561	0.003984219734561
5.0000000000000000	0.044694920328289	0.049841215887387	0.005146295559098	0.005146295559098
6.0000000000000000	0.045698117006653	0.052079569939102	0.006381452932449	0.006381452932449
7.0000000000000000	0.046723523014286	0.054416802090127	0.007693279075842	0.007693279075842
8.0000000000000000	0.047771616507479	0.056857129305134	0.009085512797655	0.009085512797655
9.0000000000000000	0.048842885344052	0.059404934278012	0.010562048933960	0.010562048933960
10.0000000000000000	0.049937827267821	0.062064771086146	0.012126943818324	0.012126943818324
11.0000000000000000	0.051056950078073	0.064841370315943	0.013784420237870	0.013784420237870
12.0000000000000000	0.052200771802992	0.067739644315384	0.015538872512391	0.015538872512391
13.0000000000000000	0.053369820888481	0.070764692865941	0.017394871977460	0.017394871977460
14.0000000000000000	0.054564636359967	0.073921807924620	0.019357171564653	0.019357171564653
15.0000000000000000	0.055785768030193	0.077216479798662	0.021430711768469	0.021430711768469
16.0000000000000000	0.057033776650201	0.080654401295450	0.023620624645249	0.023620624645249
17.0000000000000000	0.058309234134323	0.084241474295795	0.025932240161472	0.025932240161472
18.0000000000000000	0.059612723702436	0.087983813257371	0.028371089554935	0.028371089554935
19.0000000000000000	0.060944840115354	0.091887751872154	0.030942911756800	0.030942911756800

Table 9: Numerical simulation of the effect of 110% variation of the initial condition (0.04,0.04) on EMD using ODE45

(t) weeks	C(t)	G(t)	m ₁	
0	0.0600000000000000	0.0600000000000000	0	0
1.0000000000000000	0.061340289215091	0.062686100961403	0.001345811746311	0.001345811746311
2.0000000000000000	0.062709962552079	0.065490079639430	0.002780117087351	0.002780117087351
3.0000000000000000	0.064109639836049	0.068416891418860	0.004307251582811	0.004307251582811
4.0000000000000000	0.065539952937675	0.071471680897460	0.005931727959786	0.005931727959786
5.0000000000000000	0.067001545910506	0.074659785729678	0.007658239819172	0.007658239819172

6.000000000000000	0.068495075230261	0.077986743443462	0.009491668213201	0.009491668213201
7.000000000000000	0.070021209940061	0.081458295340690	0.011437085400629	0.011437085400629
8.000000000000000	0.071580631878485	0.085080392858846	0.013499760980361	0.013499760980361
9.000000000000000	0.073174035841759	0.088859201719143	0.015685165877384	0.015685165877384
10.000000000000000	0.074802129795294	0.092801107559562	0.017998977764267	0.017998977764267
11.000000000000000	0.076465635056193	0.096912720353736	0.020447085297543	0.020447085297543
12.000000000000000	0.078165286484925	0.101200879068233	0.023035592583308	0.023035592583308
13.000000000000000	0.079901832689493	0.105672656302762	0.025770823613269	0.025770823613269
14.000000000000000	0.081676036195783	0.110335361748917	0.028659325553134	0.028659325553134
15.000000000000000	0.083488673672748	0.115196546926030	0.031707873253283	0.031707873253283
16.000000000000000	0.085340536081772	0.120264007241883	0.034923471160111	0.034923471160111
17.000000000000000	0.087232428920312	0.125545786616599	0.038313357696287	0.038313357696287
18.000000000000000	0.089165172352376	0.131050177971518	0.041885005619143	0.041885005619143
19.000000000000000	0.091139601461023	0.136785727255131	0.045646125794108	0.045646125794108

Table 10: Numerical simulation of the effect of 150% variation of the initial condition (0.04,0.04) on EMD using ODE45

(t) weeks	C(t)	G(t)	m ₁	m ₂
0	0.044000000000000	0.044000000000000	0	0
1.000000000000000	0.044987768713489	0.045979671409756	0.000991902696268	0.000991902696268
2.000000000000000	0.045997411834699	0.048047129036147	0.002049717201448	0.002049717201448
3.000000000000000	0.047029400525720	0.050206147230586	0.003176746704866	0.003176746704866
4.000000000000000	0.048084215540830	0.052460652330826	0.004376436789996	0.004376436789996
5.000000000000000	0.049162347357825	0.054814726590988	0.005652379233162	0.005652379233162
6.000000000000000	0.050264296387954	0.057272614468885	0.007008318080931	0.007008318080931
7.000000000000000	0.051390573116241	0.059838726812860	0.008448153696619	0.008448153696619
8.000000000000000	0.052541698304929	0.062517646989369	0.009975948684440	0.009975948684440
9.000000000000000	0.053718203150003	0.065314135514273	0.011595932364270	0.011595932364270
10.000000000000000	0.054920629474246	0.068233135835723	0.013312506361478	0.013312506361478
11.000000000000000	0.056149529902765	0.071279779468976	0.015130249566211	0.015130249566211
12.000000000000000	0.057405468043372	0.074459391277238	0.017053923233866	0.017053923233866
13.000000000000000	0.058689018682266	0.077777495112229	0.019088476429963	0.019088476429963
14.000000000000000	0.060000767951010	0.081239818463009	0.021239050511999	0.021239050511999
15.000000000000000	0.061341313541940	0.084852298543562	0.023510985001621	0.023510985001621
16.000000000000000	0.062711264862486	0.088621086206388	0.025909821343902	0.025909821343902
17.000000000000000	0.064111243268364	0.092552552352739	0.028441309084375	0.028441309084375

Numerical treatment of manhattan distance

18.000000000000000 0.065541882207384 0.096653291034884 0.031111408827499 0.031111408827499
19.000000000000000 0.067003827463042 0.100930125838577 0.033926298375535 0.033926298375535