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OPTIMIZATION OF PRODUCTION COST OF PALM OIL

[*1]Obeten, B., [2]Biu, E.O., & [3]George, N.R.
[1&3]Department of Mathematics/Statistics, Ignatius Ajuru University of Education, Port Harcourt, Nigeria.
[2]Department of Mathematics University of Port Harcourt, Choba Port Harcourt Nigeria

***Corresponding author email:** brightobeten86@gmail.com

Abstract
Production systems focus not only on providing enough product to supply the market but also on delivering the right product at the right price while minimizing the cost of production and maximising profit. This research was carried out to investigate the optimal solution that will minimize the production planning cost and provide a satisfactory supply of the products demanded by the customer within the specified time. Data analysis was done with "Excel Solver" to solve the linear programming model formulated by applying the simplex algorithm. With the monthly production capacity of 115 units for regular-time production and 25 units during overtime, the result of the analysis shows that the company cannot meet the monthly estimated units of palm oil required during regular time and thus should consider overtime production if it must meet the monthly units' requirement. The optimal solution indicates that for the best production planning to be obtained, the company must engage in both regular time and overtime production to meet demand specifications for the first three months, while regular time production is enough for the company to meet its estimated demand for the last three months.

Keywords: Optimization, Palm Oil, Production, Linear Programming, Excel Solver

Introduction
The basic focus of most organizations is to provide goods and services, or more generally, to fulfil the needs of customers and, meanwhile, lower the costs of these goods and services to enhance the competitive strength of the company (Hartley, 1988). This is accomplished through the use of an effective and efficient production system, which can be defined as the set of resources and procedures involved in converting raw materials into products and delivering them to customers (Adeniyi et al., 2014). An enterprise that provides better products at a lower cost than its competitors can make more profit than these competitors (Behrman & Gopalan 2005). One approach to reducing production costs involves improving the control of the production system (Agboola, 1979). Optimization techniques are mostly used by managers to make valid decisions (Pereira 2013).

The technique of allocating limited resources in a production system to enhance optimal production now known as operational research was first studied as a strategy and tactical measure during World War II. This study of strategic and tactical problems of air and land defence which was used during the war to maximize the efficiency of available resources resulted in the formulation of linear programming models and marked a historical advancement at the time (Joseph et al., 2013). Afterwards, there was an overwhelming interest from experts in various fields, especially mathematicians, managers of industries, and economists who began to explore the application of linear programming in solving real-life problems (Reeves et al 1979). With its invention, real-world problems which can be represented accurately by mathematical equations known as the linear program could be solved using the best feasible solution (Dotson, 2018). Unfortunately, most of the real-world problems are usually complex and dynamic (Gomes, 2014). However, few complex real-world problems can be expressed perfectly in terms of a set of linear functions (Usoro, 1974). Linear programs also give meaningful realistic views of many real-life problems, especially if a little brilliance is applied in the mathematical formulation of the problem (Sung & Rhee, 1987).

Gabriela et al. (2017) applied linear programming to maximize profit in water production. In their work three decision variables were considered (Sachet bag of water, 50cl pack of water, and 75cl pack of water), using three raw materials (production cost, production time and Demand or Sales). The result revealed the best possible production mix that will maximize Viclibo Ventur's profit from water production (Zeven, 1965).

Telsang (2005) used linear programming in the oil sector to find the optimal production process for maximum profit. Dongni-Li et al. (2013) used linear programming for the profit efficiency of small U.S. banks. Ihsan and & Burckaan (1996) used linear programming for maximization of profit in a product-mix company (Umoh, 1998). Schoneveld (2014) applied linear programming for profit optimization at the bank.

Methodology

The method of analysis used in this work to obtain a feasible solution for the linear programming model is the simplex method. Simplex method is an approach to solving linear programming models using slack variables, tableaus, and pivot variables as a means to finding the optimal solution to an optimization problem. To solve the linear programming model using the simplex method the following steps are necessary:

- Transform the linear model to Standard form
- Introduce slack variables
- Create the tableau
- Pivot variables
    - Create new tableau
    - Check for optimality
    - Identify optimal values

Assumptions

Before we get too focused on solving linear programs, it is important to review some theories. For instance, several assumptions are implicit in linear programming problems. These assumptions are:

1. Proportionality: The contribution of any variable to the objective function or constraints is proportional to that variable. This implies no discounts or economies of scale. For example, the value of 8x1 is twice the value of 4x1, no more or less.
2. Additivity: The contribution of any variable to the objective function or constraints is independent of the values of the other variables.
3. Divisibility Decision variables can be fractions. However, by using a special technique called integer programming, we can bypass this condition.
4. Certainty: This assumption is also called the deterministic assumption. This means that all parameters (all coefficients in the objective function and the constraints) are known with certainty. Realistically, however, coefficients and parameters are often the result of guesswork and approximation. The effect of changing these numbers can be determined with sensitivity analysis.

Since this work uses a Solver engine from Excel, we only show how to set up a Simplex method.

Setting Up A Simplex Tableau

Consider the following three variables linear programming problem.

$Minimize\ Z = c_1x_1 + c_2x_2 + \cdots + c_n$

Subject to:

$$a11x1 + a12x2 + \cdots + a1nxn \geq b1$$
$$a21x1 + a22x2 + \cdots + a2nxn \geq b2$$
$$\cdots$$
$$ap1x1 + ap2x2 + \cdots + apnxn \geq bn$$
$$x_j \geq 0, = 1, 2, n \text{ (non-negativity constraints)}$$

where c_j = the objective function coefficient corresponding to the jth variable

x_j = the jth decision variable

a_{pn}= the coefficient on x_j in constraint j, and

b_j = the right-hand-side coefficient on constraint j.

A set of x_j satisfying all the constraints is called the feasible point and the set of all such points is called the feasible region, or else not all constraints would be satisfied. The above linear programming model can be written in standard form as:

$minimize\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx$

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Subject to:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n - s_1 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n - s_2 = b_2$$

$$\ldots$$

$$a_{p1}x_1 + a_{pn}x_n + a_{pn}x_n - s_n = b_n$$

$x_j \geq 0, = 1, 2, n$ (non-negativity constraints)

Table 1: Simplex Tableau

Row	Basis	x_1	x_2	…	x_n	s_1	s_2	…	s_n	RHS
R_0	$-Z$	c_1	c_2	…	c_n	0	0	…	0	0
R_1	s_1	a_{11}	a_{12}	…	a_{1n}	1	0	…	0	b_1
R_2	s_2	a_{21}	a_{22}	…	a_{2n}	0	1	…	0	b_2
…	…	…	…	…	…	…	…	…	…	…
R_n	s_n	a_{p1}	a_{p2}	…	a_{pn}	0	0	…	1	b_n

Results

The data used for this research is presented in Table 1 below

Table 1

Month	January	February	March	April	May	June
Demand	130	140	135	110	100	100

Table 2 shows monthly estimated demand for the first 6 months of the year. The following information is also available:

1. The company has a regular time production capacity of 115 units per month and 25 units per month for overtime production capacity.
2. The cost of regular time (RT) production is #15,000 per unit and #17,000.00 per unit for overtime (OT).
3. The company can carry inventory to the next month and the holding cost is #50 per unit per month.

Formulation of LPP for the production planning Decision Variables:

Let $x_j = Units\ produced\ using\ regular\ time$

Let $y_j = Units\ produced\ using\ over\ time$

Let $I_j = Units\ carried\ at\ the\ end\ of\ month\ to\ the\ next\ month$

Objective function

$$Min(Z) = 15000\sum_{j=1}^{4} x_j + 17000\sum_{j=1}^{4} y_j + 50\sum_{j=1}^{3} I_j$$

S t.

Constraints: Demand

$x_1 + y_1 = 130 + I_1$ (1st Month requirement)

$I_1 + x_2 + y_2 = 140 + I_2$ (2nd Month requirement)

$I_2 + x_3 + y_3 = 135 + I_3$ (3rd Month requirement)

$I_3 + x_4 + y_4 = 110 + I_4$ (4th Month requirement)

$I_4 + x_5 + y_5 = 100 + I_5$ (5th Month requirement)

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$I_5 + x_6 + y_6 = 100$ (6th Month requirement)

Production Capacity

$x_j \leq 115$, for all $j = 1, 2, 3, 4$

$y_j \leq 25$, for all $j = 1, 2, 3, 4$

x_j and $y_j \geq 0$ (non-negativity constraints)

The canonical form of the model

This model is transformed to its Canonical form by substituting for variable I_j to give the following: Objective function

$$Min\ (Z) = 15000\sum_{j=1}^{4} x_j + 17000\sum_{j=1}^{4} y_j + 50(x_1 + y_1 - 130 + x_1 + y_1 + x_2 + y_2 - 270 + x_1 + y_1 + x_2 + y_2 + x_3 + y_3 - 405 + x_1 + y_1 + x_2 + y_2 + x_3 + y_3 + x_4 + y_4 - 515 + x_1 + y_1 + x_2 + y_2 + x_3 + y_3 + x_4 + y_4 + x_5 + y_5 - 615 + x_1 + y_1 + x_2 + y_2 + x_3 + y_3 + x_4 + y_4 + x_5 + y_5 + x_6 + y_6 - 715)$$

Subject to:

Constraints: Demand constraint

$x_1 + y_1 \geq 130$

$x_1 + y_1 + x_2 + 2 \geq 270$

$x_1 + y_1 + x_2 + {}_2 + x_3 + {}_3 \geq 405$

$x_1 + y_1 + x_2 + {}_2 + x_3 + {}_3 + x_4 + y_4 \geq 515$

$x_1 + y_1 + x_2 + y_2 + x_3 + y_3 + x_4 + y_4 + x_5 + y_5 \geq 615$

$x_1 + y_1 + x_2 + y_2 + x_3 + y_3 + x_4 + y_4 + x_5 + y_5 + x_6 + y_6 \geq 715$

Product Capacity constraint

$x_j \leq 115$, for all $j = 1, 2, 3, 4$

$y_j \leq 25$, for all $j = 1, 2, 3, 4$

x_j and $y_j \geq 0$ (non-negativity constraints)

Discussion

The answer report is divided into three parts: Objective Cells, Variable Cells, and Constraints. The result in the Objective Cell in Table 3 of the appendix shows the type of optimization done (i.e., cost minimization) the original value and the Final Value (Objective function) of the production planning. The result shows that the Final Value (Objective function) is #7,842,500. This value represents the minimum cost the company must incur to produce the required estimated units of Palm Oil for the six months concerning available resources. This value is obtained by substituting the optimal solution (Final Value) for each variable into the Objective function. The statistics in the Variable Cells are the Original Value and Final Value (optimal solution). The Original Value in the variables in the Objective function was dormant before production commenced, hence the value zeros associated with each variable. The interesting statistic is the Final Value of the variables. In this work, the values show the units of products that must be produced at regular times and during overtime in other to meet the estimated demand for individual months. The result shows that 115 units of regular-time production and 15 units of overtime production are required to meet the estimated requirement for January. In February, 115 units must be produced at regular time while 25 units must be produced during overtime to meet the estimated demand. 115 units must be produced during regular time while 20 units must be produced during overtime to meet up estimated demand for March. For April and May, the company must produce 110 units and 100 units respectively during regular time and no production must be done during overtime. Finally, for June, the company should produce 50 units of Palm Oil at regular and overtime production.

The value of the slack variables is given in Table 4 and they represent the amount of resources that were not used up during the production process. According to the result in Table 4, for the Company to achieve the optimum solution and minimize its cost of production, all the 115 units of regular-time production capacity available should be used for production in January leaving zero units for slack. To compensate for the 130 units

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estimated requirement for January, 15 units must be produced during overtime leaving 10 units of the available overtime production capacity as slack value. In February, all the 115 units of regular-time production capacity available should be used leaving 0 units of slack. All the 25 units of overtime production capacity available should be used to complement the 140 units estimated requirement for the month, leaving a slack of 10 units of the overtime production capacity. A similar regular time procedure for February should be replicated in March leaving 0 slack value of regular time production capacity. However, 20 units of the available 25 units of overtime production capacity are required to supplement the 135 units estimated demand, leaving the remaining 5 units of overtime production capacity as slack. In April, 110 units of the available regular-time production capacity should be used leaving 5 units of regular-time production capacity as slack, and since the 110 units produced during regular time is equal to the estimated units' requirement for the month, no overtime production will be used leaving 25 units of overtime production capacity as slack. A similar process has been replicated for the last two months. For May, 110 units of available regular-time production capacity should be used, leaving the remaining 15 units of regular-time production capacity as slack. 0 units of overtime production capacity are required since the units produced during regular time equals the units' requirement for the month, leaving all 25 units of overtime production capacity available as slack. Finally, 50 units of the available units of regular time production capacity should be used leaving the remaining 50 units of the available regular time production capacity as slack for June. However, no unit is to be produced during overtime, so all 25 units available of overtime production capacity remain as slack. This implies that for June, only 50 units of Palm Oil should be produced concerning the available resources if the cost of production must be minimized. This simply means that the constraints x_1, x_2, x_3, and y_2 are binding while the constraints x_4, x_5, x_6, y_1, y_3, y_4, y_5, and y_6 are not binding.

The Sensitivity Report is divided into two parts namely: Variable Cells and Constraints. The Variable Cells given in Table 5 of the Appendix show: Variable name, Final Value, Reduced Cost, Objective Coefficient, Allowable Increase, and Allowable Decrease.

The Final Value is similar to that reported in the variable cell above. The allowable increase in the production mix for January is #2000 and the allowable decrease is 1E+30 (which is an infinitely large number). This means that if the cost of producing one unit of Palm Oil increases to a maximum of #2000.00 in January, which is within the allowable increasable range, the optimal solution of the production mix, will not change (the value of x_1 in the Final Value). Similarly, if the cost of producing a unit of Palm Oil decreases to 0, it still will not change the optimal solution of x_1. A similar interpretation applies to other variables.

The constraints table of the Sensitivity Report in Table 6 of the Appendix, shows results for Final Value, Shadow Price, Constraint R.H. Side, Allowable Increase, and Allowable Decrease. The Final Value is similar to the one discussed in the Answer Report.

From the Sensitivity report in Table 4, the Shadow Prices of -2000, -2500, and -2000 for x_1, x_2 and x_3 respectively, are the interesting values of the result. It suggests that with the Allowable Increase of 15 units, 0 units and 20 units respectively if the company wishes to further minimize their cost of production, the best solution would be to try and increase the units produced during regular time for January and March to the maximum allowable increase.

The Limits Report is presented in Table 7 of the Appendix. The values for the Lower Limit and the Upper Limit of the Objective function given in this report indicate that the minimum cost of #7842500 will be incurred on regular-time production in January. And since the Upper Limit and Lower Limit are the same, this value will be the same for both Upper Limit and Lower Limit regular time production. If 15 units of overtime production are in the production planning, a minimum cost of #7842500 will be incurred and should the units of overtime production increase to 25 units, then the minimum cost of production will be #8015500 for January. Producing 115 units of Palm Oil during the regular time in February will incur a minimum cost of #7842500, since the Lower Limit and Upper Limit are the same the cost of production is the same. Since the Lower Limit and the Upper Limit for regular time and overtime production are 25 units, the inclusion of overtime production in the objective function will give #7842500. This interpretation is similar for other months.

Conclusion

Based on the analysis carried out, the optimal solution of the production planning shows that since resources are limited and the units of product demanded in some months exceeds the monthly regular time production capacity of the Company, the Company must consider producing during overtime in other to meet the expected

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monthly demand. The Optimal solution (Final Value) of the production (Table 5) indicates that to meet the units' requirement for the first three months (January, February, and March), the Company must engage in overtime production. Conversely, the regular time production is just enough for the Company to meet its estimated demands for the last three months (April, May, and June).

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APPENDIX

Table 3: Answer Report for Objective Cell & Variable Cells

Objective Cell (Min)			
Name	**Original Value**	**Final Value**	
Minimize	-132500	7842500	
Variable Cells			
Name	**Original Value**	**Final Value**	**Integer**
x1	0	115	Contin
y1	0	15	Contin
x2	0	115	Contin
y2	0	25	Contin
x3	0	115	Contin
y3	0	20	Contin
x4	0	110	Contin
y4	0	0	Contin

x5	0	100	Contin
y5	0	0	Contin
x6	0	50	Contin
y6	0	0	Contin

Table 4: Constraints with their respective slack variables

Slack type	Cell Value	Status	Slack
s_1	130	Binding	0
s_2	270	Binding	0
s_3	405	Binding	0
s_4	515	Binding	0
s_5	615	Binding	0
s_6	715	Binding	0
s_7	115	Binding	0
s_8	115	Binding	0
s_9	115	Binding	0
s_{10}	110	Not Binding	5
s_{11}	100	Not Binding	15
s_{12}	50	Not Binding	65
s_{13}	15	Not Binding	10
s_{14}	25	Binding	0
s_{15}	20	Not Binding	5
s_{16}	0	Not Binding	25
s_{17}	0	Not Binding	25
s_{18}	0	Not Binding	25

Table 5: Sensitivity Report for Variable Cells

		Final	Reduced	Objective	Allowable	Allowable
Cell	Name	Value	Cost	Coefficient	Increase	Decrease
B2	x1	115	0	15300	2000	1E+30
B3	y1	15	0	17300	1E+30	50
B4	x2	115	0	15250	2050	1E+30
B5	y2	25	0	17250	50	1E+30
B6	x3	115	0	15200	2000	1E+30
B7	y3	20	0	17200	100	2000
B8	x4	110	0	15150	2000	15050
B9	y4	0	0	17150	1E+30	2000
B10	x5	100	0	100	15050	75
B11	y5	0	0	100	1E+30	75
B12	x6	50	0	50	150	50
B13	y6	0	0	50	1E+30	50

Table 6: Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
LHS	130	0	130	0	1E+30
LHS	270	100	270	10	0
LHS	405	2050	405	5	5
LHS	515	15050	515	5	15
LHS	615	75	615	15	100
LHS	715	25	715	130	100
LHS	115	-2000	115	15	10
LHS	115	-2050	115	0	10
LHS	115	-2000	115	20	5
LHS	110	0	115	1E+30	5
LHS	100	0	115	1E+30	15
LHS	50	0	115	1E+30	65
LHS	15	0	25	1E+30	10
LHS	25	-50	25	0	10
LHS	20	0	25	1E+30	5
LHS	0	0	25	1E+30	25
LHS	0	0	25	1E+30	25
LHS	0	0	25	1E+30	25
LHS	115	0	0	115	1E+30
LHS	115	0	0	115	1E+30
LHS	115	0	0	115	1E+30
LHS	110	0	0	110	1E+30
LHS	100	0	0	100	1E+30
LHS	50	0	0	50	1E+30
LHS	15	0	0	15	1E+30
LHS	25	0	0	25	1E+30
LHS	20	0	0	20	1E+30
LHS	0	2000	0	25	0
LHS	0	75	0	25	0
LHS	0	50	0	25	0

Table 7: Limits Report

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
B2	x1	115	115	7842500	115	7842500
B3	y1	15	15	7842500	25	8015500
B4	x2	115	115	7842500	115	7842500
B5	y2	25	25	7842500	25	7842500
B6	x3	115	115	7842500	115	7842500
B7	y3	20	20	7842500	25	7928500
B8	x4	110	110	7842500	115	7918250

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B9	y4	0	0	7842500	25	8271250
B10	x5	100	100	7842500	115	7844000
B11	y5	0	0	7842500	25	7845000
B12	x6	50	50	7842500	115	7845750
B13	y6	0	0	7842500	25	7843750