



## DETERMINING THE MINIMUM COST OF PRODUCTS DISTRIBUTION, USING NORTH-WEST CORNER AND LEAST COST MODELS

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### Abstract

This study is carried out to determine the schedule with the minimum cost for distributing Guinness Malt from two production plants in Ghana to Nine major dealers. The data used for this study is a secondary data. The data were modeled using two Linear Programming models namely; the North-West corner and the Least cost methods. These methods were chosen because of its simplicity and ability to generate good feasible solution without tedious calculations. The findings of the study revealed that the least cost method gave the transportation schedule with the minimum cost. Based on the findings of this study, it is recommended that the transportation scheduled obtained using the least cost method should be followed when distributing products from the two plants in order to minimize cost.

**Keywords:** Determining, minimum cost, distributing, product, production plants, linear programming

### Introduction

Industries, governments and businesses are often faced with the problem of minimizing cost and maximizing profit. Maximizing profit is dependent on the kind of decision taken in any profit oriented organization. Consequently, decision making has become a very complex task for policy makers, managers and government; this may be due to high cost of raw materials, and labour and other economic variables in the production process in industries and business (Abdallah & Mohammad, 2012). Operations Research consist of mathematical procedures that deals with the problem of allocating scarce resources, minimizing cost of production and transportation of products in an organization. It involves the optimization of the value of some objectives subject to some constraints (Kanuet al. 2014). The objective may be to maximize profit or to minimize costs.

Effective management in a production firm requires high level of professionalism and realistic abilities to analyze the manufacturing process in order to make appropriate decision. This implies that, managerial activities in organisation are tied to masking decision. Decisions are taken to maintain the activities of all businesses for the effective functioning of an organization (Marek & Krzysztof, 2018). Thus, the inability to take appropriate decisions may cause a lot of drawback in the achievements of the organizational goals. The decision making process encompasses checks and balance. According to Zuhaimy, (2011), the dynamism of decision making lies in the fact that the solution of one problem may lead to another problem. Mathematical modeling could be used as a guide in taking appropriate decisions. Particularly, linear programming models are some of such mathematical models that could be utilized to enhance the profitability of making decision.

In this paper the problem of distributing products from two production plants to nine major dealers given the maximum demand of each dealer is considered. The aim is to find out the distribution plan that will reduce cost.

Problems of this sort are referred to as transportation problem and could be solved using the Transportation Models (TM). Transportation models are among the most significant models in optimization problem. It is in this light that Asase (2011) opined that transportation problem is a unique form of linear programming models in operation research. The goal of the transportation model is to determine the method of distribution that will minimize the cost. Transportation problem involves several transportation routes from a number of production plants or supply origin to a received points. The aim is to establish unit point of products to be distributed from one production point to a

receiver in order to meet the demand at the receiving end and minimized the total cost of distributing the products while distributing all the products.

In transportation problems, factories or production plants where goods are manufactured are referred to as origins. The location where a required quantity of the products is to be supplied is called the receiving point. Thus, we can also call point of production as the starting and the point of supply as the receiving ends.

Transportation theory is the name given to the study of optimal transportation and allocation of resources. Transportation models are valuable for making important decisions involving the selection of best movement path, in order to efficiently distribute products from several production points to many dealers. Thus, for organisation to have the best schedule for distribution of goods, they can apply this model. The model aids in allocating a fresh property when two or more number of points is under review.

Several real life problems have been solved using the transportation models. Equi (1996) as cited in Asase (2011) applied the transportation problem in modeling transporting and scheduling of timber, sugar cane, from different production points (origins) to several demand destinations. The optimal solution of the formulated problem was determined using Lagrangean Decomposition method. Similarly, Zhang et al., (2009) carried out a study on determining least cost transportation plan. The mathematical model for cost minimization was developed, the model was converted to a relaxation total distance minimization model, and further changed to network problems. The tour construction and improvement procedures were adopted to solve the problem. When linear programming techniques was used, the transportation problem was almost certainly one of the most significant problems earlier studied. The problem can be articulated by the formulation of a linear model, which could be solved using the simplex algorithm. On the other hand, because of the unique nature of the linear model in the issue, more efficient ways now exist for tackling the issue, which is the purpose of this research. The issue is concern with the transportation of any product from an origin to a receiver, with the aim of minimizing the total distribution cost (Adlakha et al., 2006). According to Suganthi and Tamilarasi (2012), in a transportation problem, the main focus is on the original point which may represent factories that produce some goods, and a destination which may represent a supply point where the products are demanded. Thus a certain amount of the goods is to be transported to a particular receiving point.

### Statement of Problem

Most of the approaches used in tackling transportation issues are geared towards getting an optimal solution. The shortcoming of some of these methods is that they are seen as difficult and very extensive in term of the execution time and computation. In this study the Least Cost method and the North-West corner method are applied to a real life problem. The aim is to determine the method that will give the minimum cost schedule for distributing drinks from two production plants to nine dealers.

### Aim and Objectives of the Study

The aim of this study is to determine the most cost effective schedule in distributing finished products from two production plants using linear programming models, specifically transportation models. The objectives of the study are as follows:

- I determine the cost of distributing product from two production plants using North-West Corner method
- ii determine the cost of distributing product from two production plants using Least Cost method

### Materials and Methods

Suppose the goods produced by a company with  $m$  production plants are to be distributed to  $n$  dealers. A single product is to be distributed from the production plants to the dealers. Each plant has a specific production capacity, and each dealer has a specific demand. Given that the cost of transporting the products to each of the dealers is known, and these costs are considered to be linear. Simply put, this is a linear programming issue whose goal is to reduce the cost of distributing the products.

The problem is formulated as follows.

Let  $m$  be the production plants  $P_1, P_2, \dots, P_m$  with  $a_j$  ( $j = 1, 2, \dots, m$ ) production capacity to be distributed to  $n$  dealers  $D_1, D_2, D_3, \dots, D_n$  with  $b_i$  ( $i = 1, 2, 3, 4, \dots, n$ ) demands respectively.

Let the cost of moving goods from production plant  $j$  to dealer  $i$  be  $c_{ji}$ .

If the number of product distributed from production plant  $j$  to dealer  $i$  is  $x_{ji}$  the problem is to determine the distribution schedule that will result in a minimum cost of transportation while satisfying the total demand by the dealers. The problem could be stated mathematically as follows:

Minimize the total cost ( $Z$ )

$$Z = \sum_{j=1}^m \sum_{i=1}^n c_{ji} x_{ji} \tag{1}$$

Subject to the constraints

$$\sum_{j=1}^m x_{ji} \leq a_j, \quad j = 1, 2, \dots, m \quad (\text{supply constraint}) \tag{2}$$

$$\sum_{j=1}^m x_{ji} \leq b_i, \quad j = 1, 2, \dots, n \quad (\text{demand constraint}) \tag{3}$$

$$x_{ji} \geq 0 \text{ for all } i \text{ and } j$$

The objective function encloses the total costs associated with each of the variables. This problem is a minimization problem.

When the total supply = the total demand then the transportation problem is said to be balanced. That is when

$$\sum_{j=1}^m a_j = \sum_{i=1}^n b_i$$

There will be  $(m + n - 1)$  basic independent variables out of  $(m \times n)$  variables.

**Transportation Tableau:** The transportation problem can be illustrated by means of linear programming mathematical model as shown in equation (1) above. For the sake of simplicity and easy solution, a table called transportation tableau is preferred. Transportation tableau is a tabular representation of the transportation problem.

To Destination →	D <sub>1</sub>	D <sub>2</sub>	... D <sub>j</sub> ...	D <sub>n</sub>	Source Supply
From Source ↓					
S <sub>1</sub>	c <sub>11</sub> x <sub>11</sub>	c <sub>12</sub> x <sub>12</sub>		c <sub>1n</sub> x <sub>1n</sub>	a <sub>1</sub>
S <sub>2</sub>	c <sub>21</sub> x <sub>21</sub>	c <sub>22</sub> x <sub>22</sub>		c <sub>2n</sub> x <sub>2n</sub>	a <sub>2</sub>
... S <sub>i</sub> ...			c <sub>ij</sub> x <sub>ij</sub>		... a <sub>i</sub> ...
S <sub>m</sub>	c <sub>m1</sub> x <sub>m1</sub>	c <sub>m2</sub> x <sub>m2</sub>		c <sub>mn</sub> x <sub>mn</sub>	a <sub>m</sub>
Destination Requirements	b <sub>1</sub>	b <sub>2</sub>	... b <sub>j</sub> ...	b <sub>m</sub>	∑ a <sub>i</sub> ∑ b <sub>j</sub>

Figure 1: Network Representation of the Transportation Problem. The transportation problem model by Asase, (2011)  
Where,

$x_{ji}$  is the number of product distributed from production plant  $j$  to dealer  $i$   
 $c_{ji}$  is the cost of moving goods from production plant  $j$  to dealer  $i$   
 $a_j$  is the production capacity of the  $j$  plant  
 $S_j$  is the  $j$ th production plant or Supply

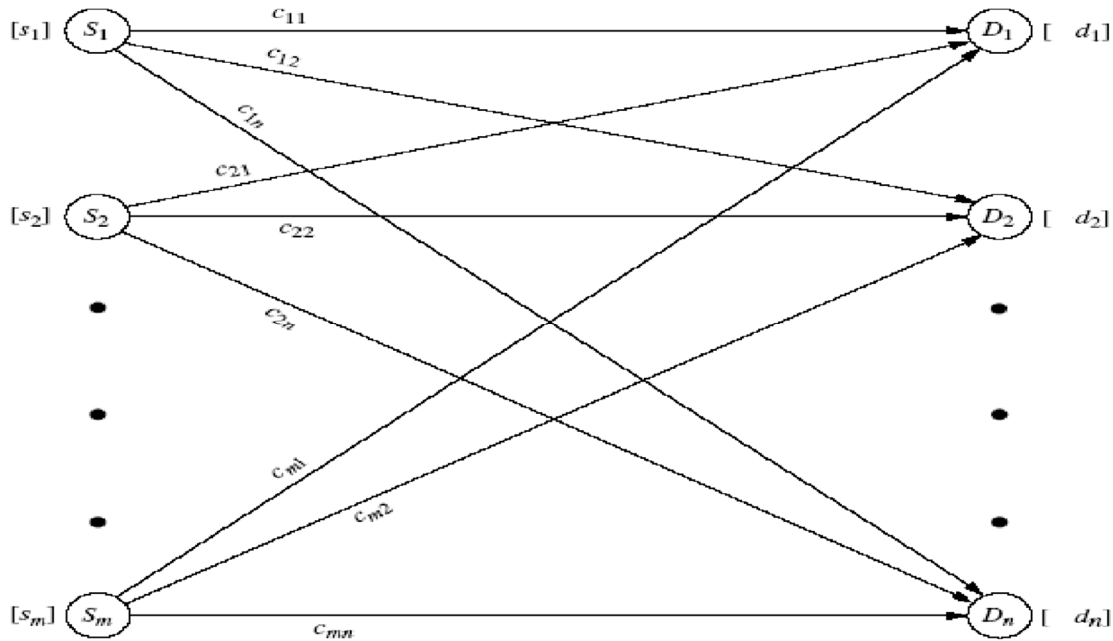


Figure 2: Network Representation of the Transportation Problem. The transportation problem model by Asase (2011)

From Figure 2, there are  $S_1 \dots S_n$  production plants and  $D_1 \dots D_n$  dealers. The arrows show the distribution of products from production plant to the dealers. Each dealer is linked to each plant by an arrow. The number  $C_1, C_2 \dots C_n$  above each arrow represents the cost of distributing on that route.

**Assumption of the models**

1. Only a single type of commodity is being shipped from an origin to a destination.
2. Total supply is equal to the total demand.

$$\sum_{j=1}^m a_j = \sum_{i=1}^n b_i$$

$a_j$  (supply) and  $b_i$  (demand) comprise of only positive integers.

3. The unit transportation cost of the product from all the production plants to the dealers is known.
4. The objective is to minimize the total cost.

When the second assumptions above are satisfied, then there exists a feasible solution to the problems in equations (1) to (3)

**Data Collection**

The data set used in this study was reported by Asase (2011). The data was collected from Guinness Ghana Ltd. The data consist of the production capacity of two production plants of Malta Guinness, the demand by nine major dealers, the cost of distributing the drinks from the two production plants located at Achimota and Kaasi to the dealers in nine different locations in Ghana. The study covered data collected within July 2007 to June 2008. The

production capacity and demand are given in crates and the cost of transportation is given in Ghana currency. The data is presented in the transportation tableau below:

**Results**

**Table 1: Transportation Tableau for July 2007 to June 2008 (10<sup>2</sup>)**

Production PLANT	MAJOR DEALERS									CAPACITY
	FTA	RICKY	OBIBJK	KADOM	NATO	LESK	DCEE	JOEMA	KBOA	
ACHESE	399.9	1262.7	1027.0	816.8	388.1	719.9	312.1	222.8	3210.4	<b>12980</b>
KAS	1453.6	338.2	1540.5	641.9	879.0	1079.8	654.5	390.8	1673.8	<b>19480</b>
<b>DEMAND</b>	<b>4650</b>	<b>6050</b>	<b>4510</b>	<b>3380</b>	<b>2600</b>	<b>1830</b>	<b>2820</b>	<b>1270</b>	<b>5350</b>	

Table 1 shows the transportation matrix indicating the supply (capacity), demand and the unit cost per full truck.

Various methods exist for determining the basic feasible solution of the transportation problem. In this study two of such methods namely, the north-west corner method and the least cost method have been adopted. They are summarized below.

**The North-West Corner Method**

In the North-West corner method, the basic variables are selected beginning from the top left (North – West corner) of the transportation tableau. This method can be summarized in the steps below.

1. Begin with the cell at the top left (North-West corner) of the transportation tableau. Apportion as much as possible to the cell, considering the supply and demand conditions.
2. Assign as much as possible to the next adjacent feasible cell.
3. Repeat step 2 until all rim requirements are met (the demand and supply are satisfied).

**The Least Cost Method**

The least cost method is one of the methods for tackling simple transportation problem where the fundamental parameters are pick with respect to the unit cost of transportation.

This method finds a starting solution by distributing through the cheapest routes. It starts by allocating as much as possible to the cell with the least unit cost in the tableau. Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted accordingly. This method is summarized in the steps below.

1. Identify the box having smallest unit transportation cost ( $C_{ij}$ ).
2. If demand is met, remove the column.
3. If supply is all taken, remove the row.
4. Go through steps 1-3 until all the demands are met and the products from each of the production plant (supply) are distributed.

**Data Analysis**

From Table1, the problem under study is formulated as follows.

Let  $x_{ji}$  be the number of product to be distributed from production plant  $j$  ( $j=1,2$ ) to dealer  $i$  ( $i=1,2,3,4,5,6,7,8,9$ )

The linear programming model for the transportation problem is

$$\text{Minimize the total distribution cost (Z) = } 399.9x_{11} + 1262.7x_{12} + 1027.0x_{13} + 816.8x_{14} + 388.1x_{15} + 719.9x_{16} + 312.1x_{17} + 222.8x_{18} + 3210.44x_{19} + 1453.6x_{21} + 338.2x_{22} + 1540.5x_{23} + 641.9x_{24} + 879.0x_{25} + 1079.8x_{26} + 644.5x_{27} + 390.8x_{28} + 1673.8x_{29}$$

**Subject to:**

**(i) Production capacity constraints**

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \leq 12980 \quad (5)$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} \leq 19480 \quad (6)$$

**(ii) Demand Constraint**

$$\begin{aligned} x_{11} + x_{21} &\leq 4650 \\ x_{12} + x_{22} &\leq 6050 \\ x_{13} + x_{23} &\leq 4510 \\ x_{14} + x_{24} &\leq 3380 \\ x_{15} + x_{25} &\leq 2600 \\ x_{16} + x_{26} &\leq 1830 \\ x_{17} + x_{27} &\leq 2820 \\ x_{18} + x_{28} &\leq 1270 \\ x_{19} + x_{29} &\leq 5350 \end{aligned}$$

$$x_{ji} \geq 0$$

To solve the problem formulated, the North-West corner and Least Cost methods has been adopted. The results gotten from these methods are summarized in the Table 2, 3, 4 and 5 below.

Table 2: Feasible Solution for the Problem in Table1 Using the North-West Corner Method

Production PLANT	MAJOR DEALERS										
	FTA	RICKY	OBIJK	KADOM	NATO	LESK	DCEE	JOEMA	KBOA	CAPACITY	
<b>Achimota</b>	399.9 (465)	1262.7 (605)	1027.0 (228)	816.8 *	388.1	719.9 *	312.1 *	222.8 *	3210.4 *	<b>12980</b>	<b>a<sub>1</sub></b>
<b>Kaasi</b>	1453.6 *	338.2 *	1540.5 (223)	641.9 (338)	879.0 (260)	1079.8 (183)	654.5 (282)	390.8 (127)	1673.8 (535)	<b>19480</b>	<b>a<sub>2</sub></b>
<b>DEMAND</b>	<b>4650</b>	<b>6050</b>	<b>4510</b>	<b>3380</b>	<b>2600</b>	<b>1830</b>	<b>2820</b>	<b>1270</b>	<b>5350</b>		<b>32460</b>

Table 3: Summary of the Results Using the North-West Corner Method.

Production Plant	Dealers (Destination)	FULLTRUCK PER CASE (000)	COST PER FULL TRUCK LOAD	TOTAL COST
Achimota	FTA	465	399.9	185953.5
Achimota	RICK	605	1262.7	234156.0
Achimota	OBIB JK	228	1027.0	763933.5
Achimota	KADOM	0	816.8	0
Achimota	NATO	0	388.1	0
Achimota	LEK	0	719.9	0
Achimota	DCEE	0	312.1	0
Achimota	JOEMA	0	22.28	0
Achimota	KBOA	0	3210.4	0
Kaasi	FTA	0	145.36	0
Kaasi	RICK	0	338.2	0
Kaasi	OBIB JK	223	1540.5	343531.5
Kaasi	KADOM	338	641.9	216962.2
Kaasi	NATO	260	879.0	228540
Kaasi	LEK	183	1079.8	197603.4
Kaasi	DCEE	282	654.5	18456.9.0
Kaasi	JOEMA	127	390.8	49631.6
Kaasi	KBOA	535	167.38	895483.0

The total transportation cost is:3115794.7

Table 4: Feasible Solution for the Problem in Table1 Using the Least Cost Method.

Production PLANT	MAJOR DEALERS									CAPACITY	
	FTA	RICKY	OBIBJ K	KADOM	NAT O	LESK	DCEE	JOEMA	KBOA		
Achimota	399.9 (465)	1262.7	1027.0	816.8	388.1 (260)	719.9 (164)	312.1 (282)	222.8 (127)	3210.4	<b>1298</b>	<b>a<sub>1</sub></b>
Kaasi	1453.6	338.2 (605)	1540.5 (451)	641.9 (338)	879.0	1079.8 (19)	654.5	390.8	1673.8 (535)	<b>1948</b>	<b>a<sub>2</sub></b>
<b>DEMAND</b>	<b>4650</b>	<b>6050</b>	<b>4510</b>	<b>3380</b>	<b>2600</b>	<b>1830</b>	<b>2820</b>	<b>1270</b>	<b>5350</b>		<b>32460</b>
	<b>b<sub>1</sub></b>	<b>b<sub>2</sub></b>	<b>b<sub>3</sub></b>	<b>b<sub>4</sub></b>	<b>b<sub>5</sub></b>	<b>b<sub>6</sub></b>	<b>b<sub>7</sub></b>	<b>b<sub>8</sub></b>	<b>b<sub>9</sub></b>		

Table5: Summary of Result Using the Least Cost Method.

Production Plant	Dealers (Destination)	FULLTRUCK PER CASE (000)	COST PER FULL TRUCK LOAD	TOTAL COST
ACH	FTA	465	399.9	185953.5
ACH	RICKY	0	1262.7	0
ACH	OBIB JK	0	1027.0	0
ACH	KADOM	0	816.8	0
ACH	NAATO	260	388.1	100906.0
ACH	LESK	164	719.9	118063.6
ACH	DCEE	282	312.1	88012.2
ACH	JOEMA	127	22.28	28295.6
ACH	KBOA	0	3210.4	0
Kaasi	FTA	0	145.36	0
Kaasi	RICK	605	338.2	204611.0
Kaasi	OBIB JK	451	1540.5	694765.5
Kaasi	KADOM	338	641.9	216962.2
Kaasi	NATO	0	879.0	0
Kaasi	LESK	19	1079.8	20516.2
Kaasi	DCEE	0	654.5	0
Kaasi	JOEMA	0	390.8	0
Kaasi	KBOA	535	167.38	895483.0
<b>The total transportation cost is:</b>				<b>255356.88</b>

Total cost=  $39.99 \times 465 + 260 \times 38.8 + 71.99 \times 164 + 31.21 \times 282 + 22.28 \times 127 + 33.82 \times 605 + 154.05 \times 451 + 64.19 \times 338 + 107.98 \times 19 + 167.38 \times 535$   
 = 255356.88.

### Discussion of Findings

In this study, two transportation models have been adopted to establish the transportation path that will minimize the cost of distributing Guinness Malt from two production plant to nine dealers. Table 2 reveals that the North-West model gave a total transportation cost of 3115794.7 Ghana cedis. While the least cost method gave a total transportation cost of 255356.88 Ghana cedis. Thus, the distribution schedule obtain from the least cost method in Table 4 is recommended. From Table 4, the following transportation schedule will minimize the cost of distributing the products from the two production plants to the nine dealers.

- 465000 crates of malt Guinness should be distributed from Plant Achimota to dealer FTA.
- 260000crates of malt Guinness should be distributed from Achimota to OBIB JK
- 164000cratesmalt Guinness should be distributed from Achimota NATO
- 282000 crates should be distributed from Achimotato LESK
- 127000 crates should be shipped from Achimota to Jeoma
- 605000 crates could be transported from the production plant at KaasiRisk
- 451000 crates of malt Guinness should be transported from Kaasito distributor OBj
- Ship 338000 crates of malt Guinness from Plant Kaasi to KADO
- Transport 19000 crates from Kaasito distributor LESK
- 535000 crates should be transported from Kaasito dealer KBOA



The finding of this study reveals that the least cost method gives the minimum distribution cost therefore, better than North-West corner method. This result is in consonance with the findings of Asase (2011) that the least-cost method always gives improved or a better initial solution when compared with the North-West corner method. This method is considered the best, compare to the one discussed above because it generally gives an optimum, or close to optimum, initial solutions.

It is in this light that Zuhaimy et al., (2011) noted that transportation models are efficient linear programming models that could be used to improve or optimize the cost of transporting goods in a production system. It could further be used to improve the service quality of the public transport systems. Asase (2011) added that, the initial solution gotten by the least cost method could be used to determine the best optimum solution. Similarly, Norozi et al., (2010) opined that the least cost solution method could be efficiently applied in the distribution of electronic in a competitive market in order to reduce cost and maximize profit. Eisakhani et al., (2012) suggested the use of linear programming models, specifically in scientific fields such as the transportation of chemicals and oxygen from different supply points to demands centers, the least cost method could be used in designing a transportation plan that will minimize cost. From the foregoing, it is obvious that the complex algebraic procedures involved in the use of the simplex method could be drastically reduce when the North-west corner method and least cost method is applied. It has also been observed that the least cost method gives the optimum distribution cost for the data set used in this study when compared with the North-West corner method.

### Conclusion

The cost of distributing products is part of the total cost in the production process. The transportation problem was formulated in this study, the north-west corner and least cost methods were used. The findings of the study revealed that the distribution schedule provided by the least cost method gave the minimum cost of distributing the product from the plants to the major dealer. Thus, the distribution pattern shown in Table four is suggested.

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