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MODELING THE IMPACT OF COMPETITIVE COEFFICIENT OF TWO INTERACTING HERBIVORES IN A MILD ENVIRONMENT ON RESOURCE BIOMASS

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Abstract
The paper formulates and studies a non-linear first-order mathematical model to understand the dynamics of the impact of resource biomass on two competing resource-dependent interacting biological inter-specific species. Analysis of the model was performed by finding the equilibrium points and their dynamic behaviours with the conditions for stability and instability of the system established. As a consequence, the analytic solution obtained was tested using parameter values as given by George, (2019) for the simulation of the system (3.1)-(3.3) and MATLAB ODE45 was used for simulation as depicted in figures 1,2,3 and 4 and the results indicate that a biodiversity gain could be achieved where the two interacting species co-exist as shown in figure 5. This is possible where there is a steady nutrient supply for the two species (Rabbit and Goat) and the resource biomass (Grass) and this will require appropriate farm management practices and policies to sustain the ecosystem of discussion.

Keywords: Mathematical model, resource biomass, differential equations, competing resource-dependent biological species, intra-specific competition.

Introduction
Observably, our environment is replete with several interacting relationships, and the need to understand such real-life phenomena has attracted the attention of many scholars. In his quest to apprehend the nature of prey-predator interactions, Solomon (1949) came up with the concept of functional and numerical responses and describes functional response in the light of the prey consumption rate by a single predator as a function of prey abundance, while the numerical response describes the impact of prey consumption on the predator enlistment. Generally, predator-prey models such as the Lotka–Volterra model assume that the production of new predators is directly proportional to the food consumption and thus numerical response relates directly to the functional response with the constant of proportionality in the model reflecting the efficient rate of conversion of the prey to newborns predators.
Several biological interactions have been identified such as commensalism, mutualism, parasitism, neutralism, and competition. These relationships could be long-term or short-term depending on the interacting biological species. If the relationship involves the same species living in the same ecological area then such is known as intra-specific interaction whereas the interaction between different species inhabiting the same ecological area is known as inter-specific. Interactions in an ecosystem may be positive, negative, or neutral. Positive where the interacting specie benefits from such a relationship, negative where one of the interacting species suffers harm, and neutral where there is no actual benefit derived from the relationship yet the interacting species doesn't suffer any form of harm.

Mathematical models are used to illustrate many real-world phenomena. Hence, a mathematical model is the interpretation of a dynamical system using a set of parameters, variables, and equations to create a relationship between the components of the system being considered. Biwas et.al. (2016), posited that to efficiently and sustainably manage natural resources and develop adaptable control measures in such an environment then such systems mainly depend essentially on the understanding of the mechanisms of their evolution over a while and mathematical modeling as applied to non-linear differential equations (ODEs) can be pivotal in unraveling the system under consideration. Thus, mathematical modeling is used extensively in scientific research undertakings to discuss various types of real-life phenomena that lead to the design of better prediction, prevention, management and control strategies in the environment of interest. For example, Dike and George (2020) studied the effect of competition on the resource biomass of a resource-dependent interacting biological species with emphasis on two species (Goat and sheep).

However, the Lotka -Volterra equations only examine how inter-specific competition and species coexistence affect the population size but ignore the mechanism through which the competition occurs. This model limitation was however addressed by Tilman's model in van Opheusden et al.(2015) work, which explored competition between two species over scarce resources. For Tillman (1982), resources refer to any factor consumed by the organism and its abundance in the environment characterizes the growth of the population. Ultimately, an abundance of a resource will lead to population growth. Moreso, when two populations of separate species compete for the same resource, the consumption rate varies between the two species.

The paper deals with modeling the impact of two interacting biological species (Rabbit & Goat) in a mild environment where the competing species are affected by the resource biomass using the Lotka-Volterra model with emphasis on the analysis of the equilibria points that can sustain the existence of the interacting species and the resource biomass in the habitat of interest. Animals that have inter-specific competition amongst themselves specifically rabbits and goats will be considered in this paper. Inter-specific competition occurs when two or more interacting species compete for limited resources in the same area. Thus when the resources available are in limited supply for the population, in this instance Rabbit and Goat, then the interacting species will experience lowered productivity (poor meat or dairy production), growth rate, and in extreme cases, one of the species may go into extinction. Hence the need for innovative approaches in the management of these resources sustainably is of great concern to various stakeholders including governments worldwide.

Mathematical Formulation

We will develop a deterministic model in which each of the competing species (Rabbit& Goat) and the resource biomass (Grass) are represented in a system of non-linear differential equations using the modified Lotka-Volterra equations (Dike & George, 2020)

Assumption of the Model

The model is based on the following assumptions:

1. Intrinsic rate growth (r_1, s_1, σ_1) Rabbit (species 1), Goat (Specie 2) and the (Grass) resource biomass, competition coefficient (α, β) species 1, and 2, and carrying capacity (*K,L,M*) of species 1,2 and resource biomass are all constants.
2. Every individual within each population is similar.
3. Diversity of the populations are not allowed.
4. The environment (habitat) is homogenous.

Table 1. Definition of parameters and variables

Variables	Description
$x(t)$	at time t, represents the population size of Rabbit (specie 1)
y(t)	at time t, represents the population size of Goat (specie 2)
$R(t)$	at time t, represents the amount of resource available

Parameters	Description
r_1	is the intrinsic growth rate of specie 1
s_1	is the intrinsic growth rate of species 2
r_2	is the intra-competition coefficient of specie 1
s_2	is the intra-competition coefficient of specie 2

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α_1	is the growth rate coefficient of specie 1
β_1	is the growth rate coefficient of specie 2
α	is the inter-competitive coefficient of specie 1
β	is the inter-competitive coefficient of specie 2
σ_1	is the intrinsic growth rate of the Resource Biomass
σ_2	is the intra-competitive coefficient of the Resource Biomass
K	is the carrying capacity for specie 1
L	is the carrying capacity for specie 2
M	is the carrying capacity for the resource biomass

$$\frac{dx}{dt} = r_1 x - r_2 x^2 - \alpha xy + \alpha_1 xR$$
$$\frac{dy}{dt} = s_1 y - s_2 y^2 - \beta xy + \beta_1 yR \qquad (2.1)$$
$$\frac{dR}{dt} = \sigma_1 R - \sigma_2 R^2 - \alpha_1 xR - \beta_1 yR$$

subject to

$$x(0) = x_0 \geq 0$$
$$y(0) = y_0 \geq 0 \qquad (2.2)$$
$$R(0) = R_0 \geq 0$$

Model Analysis

If the interacting biological species in the ecosystem have adequate food supply then they are expected to grow exponentially otherwise the less powerful specie will shrink in weight and go into extinction. Again, if there are no interactions between the three biological species, that is if

$\alpha = \alpha_1 = r_2 = s_2 = \sigma_2 = \beta = \beta_1 = 0$ thus equation (2.1) becomes

$$\frac{dx}{dt} = r_1 x$$
$$\frac{dy}{dt} = s_1 y \qquad (2.3)$$
$$\frac{dR}{dt} = \sigma_1 R$$

subject to

$$x(0) = x_0 \geq 0$$
$$y(0) = y_0 \geq 0 \qquad (2.4)$$
$$R(0) = R_0 \geq 0$$

Solving the above systems of equation will yield solutions :

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$$x(t) = x_0 e^{r_1 t}$$
$$y(t) = y_0 e^{s_1 t}$$
$$R(t) = R_0 e^{\sigma_1 t} \qquad (2.5)$$

The above equations (2.5) clearly indicate that as $t \to \infty, x(t), y(t)$ and $R(t)$ will grow exponentially. This is mathematically true but scientifically inaccurate and premised on the fact that no population grows exponentially considering that space and resource constraints can affect such growth.
According to Ekaka-a, (2009), the carrying capacity of a biological species in an environment is the maximum sustainable population size in that environment and this can be expressed mathematically as the ratio of the quotient of the intrinsic growth rate to the intra-competition coefficient. Let K, L, M be the carrying capacities of species 1 , 2, and the resource biomass populations respectively. Thus the various subpopulation carrying capacities become;

$$K = \frac{r_1}{r_2}, L = \frac{s_1}{s_2}, M = \frac{\sigma_1}{\sigma_2},$$

The non-linear equations (2.1) are analyzed qualitatively in this section to determine the equilibrium points and the dynamical behaviour of the points.

3.1 ***Positivity of analysis***

Lemma 1 Supposed that $x(t) \geq 0, y(t) \geq 0, R(t) \geq 0$ then the solutions $x(t), y(t)$ and $R(t)$ of the model in equation (2.1) are positive.

Proof: To prove this lemma, we obtain from equation (2.1) hereunder restated,

$$\frac{dx}{dt} = r_1 x - r_2 x^2 - \alpha xy + \alpha_1 xR$$
$$\frac{dy}{dt} = s_1 y - s_2 y^2 - \beta xy + \beta_1 yR$$
$$\frac{dR}{dt} = \sigma_1 R - \sigma_2 R - \alpha_1 xR - \beta_1 yR$$

Now $\frac{dx}{dt} - \alpha_1 xR \geq 0 \Rightarrow \frac{dx}{dt} + (-\alpha_1 R)x \geq 0$ and then integrating factor, $I(t) = e^{\int(-\alpha_1 R)dt} = e^{-(\alpha_1 R)t}$. Multiplying by the integrating factor on both sides we have

$\frac{dx}{dt} e^{-(\alpha R)t} + e^{-(\alpha R)t}((\alpha_1 R)x \geq 0 \Rightarrow e^{-(\alpha R)t} \geq 0 \geq c_1 \Rightarrow x \geq c_1 e^{(\alpha R)t}$ where c_1 is an integer constant. Applying the initial conditions for the value of the integer constant, when $t = 0$ then $x(t) = x(0),$ hence $x(0) \geq c_1$. Putting the value of c_1 we get $x(t) \geq x(0)e^{(\alpha R)t}$. Because $x(t) \geq 0$ and $\alpha R \geq 0$ hence $x(t) \geq 0 \forall t$. Similarly,

$\frac{dy}{dt} = s_1 y - s_2 y^2 - \beta xy + \beta_1 yR \Rightarrow \frac{dy}{dt} + (-\beta_1 yR) \geq 0$ and the integrating factor $I(t) = e^{(-\beta_1 R)t}$. Now multiplying $e^{(-\beta_1 R)t}$ on both sides we have

$$e^{(-\beta_1 R)t}\frac{dy}{dt} + e^{(-\beta_1 R)t}(-\beta_1 R)y \geq 0 \Rightarrow \frac{d}{dt}e^{(-\beta_1 R)t} y \geq 0 \Rightarrow e^{(-\beta_1 R)t} y \geq c_2 \Rightarrow y(t) = c_2 e^{(\beta_1 R)t}$$

where c_2 is an integer constant. For the value of c_2, applying the initial conditions, when t=0, then $y(t) = y(0)$ and hence $y(0) \geq c_2$. When the value of c_2 is put into the equation we have,

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$y(t) = y(0)e^{(\beta_1 R)t}$. Since $y(t) \geq 0$ and $-\beta_1 R \geq 0$ hence $y(t) \geq 0 \forall t$. Repeating the process for the remaining equations we have, for

$\frac{dR}{dt} = \sigma_1 R - \sigma_2 R^2 - \alpha_1 xR - \beta_1 yR \Rightarrow \frac{dR}{dt} + \alpha_1 xR + \beta_1 yR \geq 0 \Rightarrow I(t) = e^{\int(\alpha_1+\beta_1)Rdt} = e^{(\alpha_1+\beta_1)Rt}$ which also yield $R(t) \geq R(0)e^{-(\alpha_1+\beta_1)Rt}$. Hence it is proven that $x(t), y(t)$ and $R(t)$ is positive for all $t \geq 0$

3.2 Existence of equilibrium

Next we obtain the steady states of the system by equating the derivatives on the left hand sides and solving the resulting algebraic equations. That is $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dR}{dt} = 0$, this implies

$$r_1 x - r_2 x^2 - \alpha xy + \alpha_1 xR = 0 \qquad (3.21)$$

$$s_1 y - s_2 y^2 - \beta xy + \beta_1 yR = 0 \qquad (3.22)$$

$$\sigma_1 R - \sigma_2 R^2 - \alpha_1 xR - \beta_1 yR = 0 \qquad (3.23)$$

From equations (3.1)- (3.3)

$x(r_1 - r_2 x - \alpha x + \alpha_1 R) = 0 \Rightarrow x = 0$ or $r_1 - r_2 x - \alpha y - \alpha_1 R = 0 \Rightarrow r_2 x = r_1 - \alpha y + \alpha_1 R$

$$x = \frac{1}{r_2}(r_1 - \alpha y - \alpha_1 R) \qquad (3.24)$$

$y(s_1 - s_2 y - \beta x + \beta_1 R) = 0 \Rightarrow y = 0$ or $s_1 - s_2 y - \beta x + \beta_1 R = 0 \Rightarrow s_2 y = s_1 - s_2 y - \beta x + \beta_1 R$

$y = \frac{1}{s_2}(s_1 - \beta x + \beta_1 R)$ and $\sigma_1 R - \sigma_2 R^2 - \alpha_1 xR - \beta_1 yR = 0 \Rightarrow R(\sigma_1 - \sigma_2 R - \alpha_1 x - \beta_1 y)$

$$R = 0 \text{ or } R = \frac{1}{\sigma_2}(\sigma_1 - \alpha_1 x - \beta_1 y) \qquad (3.25)$$

$$y = \frac{(r_2\sigma_2 + \alpha\alpha_1)(r_2 s_1 - \alpha\beta_1) + (r_2\beta_1 - r_1\beta)(r_2\sigma_1 - r_2\alpha)}{(r_2\sigma_2 + \alpha\alpha_1)(r_2 s_2 - \alpha\beta) + (r_2\beta_1 - r_2\beta)(r_2\beta_1 - \alpha^2)} \qquad (3.26)$$

$$R = \frac{(r_2\sigma_2 + \alpha\alpha_1)(r_2 s_2 - \alpha\beta)x - (r_2\sigma_2 + \alpha\alpha_1)(r_2 s_1 - r_1\beta)}{(r_2\sigma_2 + \alpha\alpha_1)(r_2\beta_1 - \alpha_1\beta)} \qquad (3.27)$$

Putting (3.26) and (3.27) into (3.25)

$$x = \frac{(r_2\beta_1 - \alpha_1\beta)[(r_2 s_1 - r_1\beta)(r_2\beta_1 - r_2\beta)(r_2\beta_1 - \alpha^2) + 2\sigma_1(r_1\beta_1 - \alpha_1\beta)(r_2\beta_1 - r_2\beta)(r_2\beta_1 - \alpha^2)]}{\alpha_1(r_2\beta_1 - \alpha_1\beta) + \sigma_2(r_2 s_2 - \alpha\beta)(r_1\beta_1 - \alpha_1\beta)(r_2\beta_1 - r_2\beta)(r_2\beta_1 - \alpha^2)} \qquad (3.28)$$

Stability of steady states

Now, the mathematical structure under consideration from the above has the following form:

$$\frac{dx}{dt} = f_1(x, y, R) \qquad x(0) = x_0 \geq 0 \qquad (3.31)$$

$$\frac{dy}{dt} = f_2(x, y, R) \qquad y(0) = y_0 \geq 0 \qquad (3.32)$$

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$$\frac{dR}{dt} = f_3(x, y, R) \qquad R(0) = R_0 \geq 0 \qquad (3.33)$$

Where f_1, f_2 and f_3 are continuous functions of the variables x, y, z and R. Thus the steady -state solution satisfies (x_e, y_e, R_e), at arbitrary point.

$$f_1(x_e, y_e, R_e) = 0 \qquad (3.34)$$

$$f_2(x_e, y_e, R_e) = 0 \qquad (3.35)$$

$$f_3(x_e, y_e, R_e) = 0 \qquad (3.36)$$

Having constructed the necessary condition for the existence of a steady-state solution at an arbitrary point (x_e, y_e, R_e), the next step will be to establish the condition sufficient for the existence of steady-state solutions. The interacting functions in equations (3.34)-(3.36) are differentiated partially for each of the state variables to obtain the Jacobian elements as analyzed. The Jacobian elements are defined thus:

$$J_{11} = \frac{\partial f_1}{\partial x}, \qquad J_{12} = \frac{\partial f_1}{\partial y} \qquad J_{13} = \frac{\partial f_1}{\partial z}$$

$$J_{21} = \frac{\partial f_2}{\partial x} \qquad J_{22} = \frac{\partial f_2}{\partial y} \qquad J_{23} = \frac{\partial f_2}{\partial z}$$

$$J_{31} = \frac{\partial f_3}{\partial x} \qquad J_{32} = \frac{\partial f_3}{\partial y} \qquad J_{33} = \frac{\partial f_3}{\partial z}$$

The Jacobian matrix is set up as follow:

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

The eigenvalues of the system are found in the Jacobian matrix.
Based on linear stability analysis, a steady-state solution is stable if all the eigenvalues of the Jacobian, evaluated at that particular steady-state solution are negative. If at least one system is positive then the system is unstable.

Let the functions $\lambda = F_1(x, y, R) = \frac{dx}{dt}, \tau = F_2(x, y, R) = \frac{dy}{dt}$, and $\eta = F_3(x, y, R) = \frac{dR}{dt}$ from a steady-state ($(x^*, y^*, R^*) = (0,0,0)$ of the equation (2.1)

$$J_{(\bar{x},\bar{y},\bar{R})} = \begin{bmatrix} r_1 - r_2 x - \alpha y - \alpha_1 R & -\alpha x & \alpha_1 x \\ -\beta y & s_1 - 2s_2 x - \beta x + \beta_1 R & \beta_1 y \\ -\alpha_1 R & -\beta_1 R & \sigma_1 - 2\sigma_2 R - \alpha_1 x - \beta_1 y \end{bmatrix}$$

For steady state $E_1 \; \overline{(x,y,R)} = (0,0,0)$ we get

$$J_{(0,0,0)} = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & s_1 & 0 \\ 0 & 0 & \sigma_1 \end{bmatrix}$$

The characteristic equation of the matrix with eigenvalue ω is,

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$$|J-\omega I| = \begin{bmatrix} r_1-\omega & 0 & 0 \\ 0 & \sigma_2-\omega & 0 \\ 0 & 0 & \sigma_1-\omega \end{bmatrix}$$

Now, $r_1-\omega=0$ or $s_2-\omega=0$ or $\sigma_1-\omega=0$

$\therefore \omega_1=r_1, \omega_2=s_1, \omega_3=\sigma_1$. Here the eigenvalue ω of the matrix "J' determines the stability of the states.
Depending on ω the following stability conditions are established:
i) If the eigenvalue $\omega<0$, then the steady-state is stable
ii) If the eigenvalue $\omega>0$, then the system is unstable

$$E_2(x^*,y^*,R^*)=(x^*,y^*,0)$$

Numerical Simulation and Discussion
The mathematical simulation was carried out using parameter values given by George,(2019) for the system (3.1) - (3.3) thus:

$$r_1=5, r_2=0.22, \alpha=0.007, \alpha_1=0.02, \sigma_1=10,$$
$$s_1=3, s_2=0.26, \beta=0.008, \beta_1=0.04, \sigma_2=0.3$$

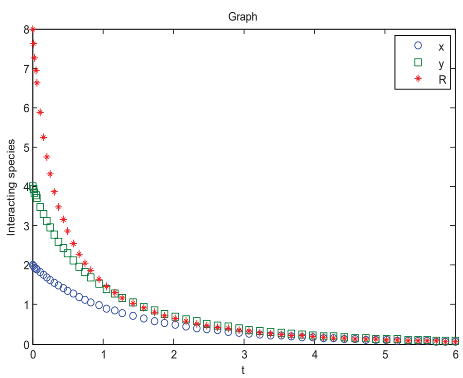

Figure 1

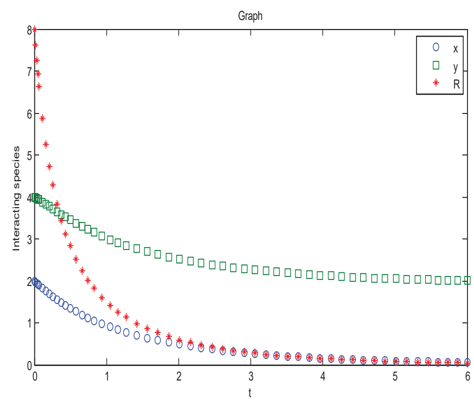

Figure 3

Figure 1-2 illustrate the interaction between species 1, 2 and the resource biomass where the two species and the biomass witness negative growth in Figure 1, while both species 2 and the resource biomass witness negative in figure 2, specie 1 experience positive growth rate with the parameter values :

$$r_1=-0.5; r_2=0.22; s_1=-0.5; s_2=0.26; \sigma_1=-0.5;$$
$$\sigma_2=0.3; \alpha=0.007; \alpha_1=0.02; \beta=0.008; \beta_1=0.04;$$
$$r_1=0.5; r_2=0.22; s_1=-0.5; s_2=0.26; \sigma_1=-0.5; \sigma_2=0.3;$$
$$\alpha=0.007; \alpha_1=0.02; \beta=0.008; \beta_1=0.04;$$

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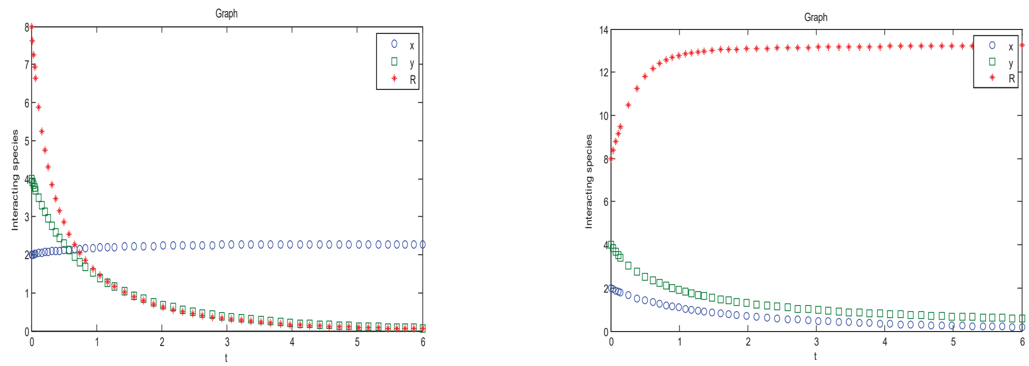

Figure 3-4 illustrates the interaction between species 1, 2 and the resource biomass where species 1 and the biomass witness a negative growth rate while specie 2 witness a positive growth with the parameter values:

$$r_1 = -0.5; r_2 = 0.22; s_1 = 5; s_2 = 0.26; \sigma_1 = -0.5; \sigma_2 = 0.3;$$

$$\alpha = 0.007; \alpha_1 = 0.02; \beta = 0.008; \beta_1 = 0.04;$$

$$r_1 = -0.5; r_2 = 0.22; s_1 = 5; s_2 = 0.26; \sigma_1 = -0.5; \sigma_2 = 0.3;$$

$$\alpha = 0.007; \alpha_1 = 0.02; \beta = 0.008; \beta_1 = 0.04;$$

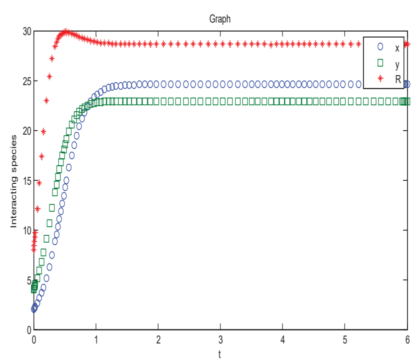

Figure 5 illustrates the interaction between species 1, 2 and the resource biomass where the two species and the biomass witness a stable relationship with the parameter values below

$$r_1 = 5; r_2 = 0.22; s_1 = 5; s_2 = 0.26; \sigma_1 = 10; \sigma_2 = 0.3; \alpha = 0.007; \alpha_1 = 0.02; \beta = 0.008; \beta_1 = 0.04;$$

Figures 1-5 illustrate the simulations results of the study. While figure 1 illustrates the scenario where the growth rates of the interacting species are negative which indicates a stable steady-state solution and this though mathematically correct, is unphysical in a real biological sense since any population species that witness a negative growth will gradually diminish and eventually go into extinction. Again, figure 2 illustrates the steady-state solution that is unstable, and thus specie (1) witnesses a positive growth rate while specie (2) and the resource biomass have negative growth rates. This situation may not be sustainable for a long period since a negative growth rate of the resource biomass portends greater danger for both species (1&2) and ultimately the two species and the resource biomass will go into extinction. Also, Figure 3 represents another unstable steady-state solution and is similar to the case earlier described but in this instance, specie (1) and the resource biomass exhibit growth rate reduction whereas specie (2) experienced a positive growth rate which is unsustainable in the long run. Finally, figure 4 demonstrates a steady-state solution that is unstable and indicates that the two interacting biological species' populations will gradually reduce and go into extinction with time, while the resource biomass will grow without bound.

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Conclusion

The impact of competition of two biological resource dependent interacting species were considered in this paper using a mathematical model of a system of three non-linear ordinary differential equations with appropriate initial conditions. Positivity analysis was performed by proving **lemma 1** which indicated that $x(t), y(t)$ and $R(t)$ are all positive for all t. Analytic solutions to the model equations were obtained at steady-state and the equilibria of the steady states were analyzed to examine the behaviour of the dynamical system. The Jacobian matrix was constructed to further the study of the two interacting species and the resource biomass and the conditions for stability and instability of the system. While a negative eigenvalue signifies a stable steady-state solution, a positive eigenvalue indicates an unstable steady-state solution. The four simulations scenarios indicate a mathematically accurate deduction for stable, unstable steady states respectively. In Figure 1, the two species and the resource biomass experienced a negative growth rate. In Figure 2, specie (1) experience a positive growth rate while specie (2) and resource biomass had negative growth rates. The same was observed for Figure 3 where specie (2) had a positive growth rate while specie (1) and resource biomass had a negative growth rate finally Figure 4 was seen to have species (1&2) with a negative growth rate while the resource biomass had positive growth rate. From our analysis of the study, the question that arose bothers on the possibilities of the two interacting competing species coexisting sustainably given the same resource availability. There appear to be only two possibilities for this to occur: If the constant nutrient supply is insufficient to sustain either species (Rabbit and Goat), then both species will go into extinction and the resource biomass (Grass) reaches its stable level. In the alternative, the specie with lower food requirements reaches a stable level while the other becomes extinct. Furthermore, equation (3.26) indicates the steady-state solution where the three populations can survive together (x^*, y^*, R^*) but this begs for more biological questions premised on the fact of availability of resource biomass for the two interacting species. If $x = y = R = 0$ corresponding to the steady-state solution of the dynamical system at (0,0,0) then the three populations will go into extinction which again raises biological teasers. However, if appropriate farm practice is employed where there is a constant replenishment of the resource biomass then the feasibility of coexistence of the interacting biological species could be attained.

It is recommended based on figure 5 which best fits our co-existence assumptions (scenarios) that to maintain a balance in the ecosystem of discussion, adopting appropriate policies and management practices that will ensure rapid replenishment of the resource biomass for the two interacting biological species to coexist. Thus the availability of the resource biomass in abundance will make life easier for species (1 &2) and a little investment in finding the resource is expressed in an increased growth rate of species (1& 2) or a decreased possibility of starvation for them. It is seen from the model that the net effect is the same hence the subpopulation approaching the equilibrium point is true but a generalization of the analysis could not be considered since no actual information about the dynamics of the system was obtained other than the steady states.

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