

Impact of 150% Depletion Rate Coefficient of the Forest Resources Biomass due to population

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Abstract

Anthropogenic activities considerably impact forest biodiversity. Disturbances such as deforestation (clearing forests for agriculture, urbanization, and logging), habitat fragmentation (breaking up of forests into smaller, isolated areas) and climate change (altering temperature and precipitation patterns), significantly impact forest biodiversity. This study examines the effect of 150% depletion rate coefficient of the forest resources biomass due to population. Our analysis reveals that anthropogenic activities lead to loss of habitat (reducing available space for species), reduced species richness/ decline (decreasing population size which drives species into extinction), and altered ecosystem functioning by altering nutrient cycling, pollination and seed dispersal. The consequences of these include: Loss of ecosystem services: which impacts on water regulation and soil conservation; Decreased resilience (making forests more vulnerable to disturbances). We discuss the implications of these findings for conservation efforts and sustainable forest management, highlighting the need for integrated approaches to mitigate the impact of human activities on forest biodiversity.

Keywords: Anthropogenic Activities, Forest Biodiversity, Deforestation, Habitat Fragmentation, Climate Change, Conservation.

Introduction

Forests are one of the most bio diverse ecosystems on the planet, providing habitat for a vast array of plant and animal species (Eguiguren et al., 2019). However, these ecosystems are facing unprecedented threats from anthropogenic activities, which are significantly impacting forest biodiversity (Mbakwe, 1996). This introduction will provide an overview of the impact of human-induced disturbances on forest ecosystems, highlighting the importance of preserving biodiversity and the need for sustainable forest management. Forests are crucial ecosystems that support a wide range of plant and animal species. They provide essential ecosystem services including: carbon sequestration (the process of capturing and storing atmospheric carbon dioxide; either naturally or through human intervention. This helps mitigate climate change by reducing the amount of carbon dioxide in the atmosphere) Water regulation (forests play a critical role in maintaining water cycles, ensuring the availability of fresh water resources). Other areas include soil conservation and biodiversity (Mohammed, 2014).

Some of the most significant anthropogenic activities affecting forests include: deforestation, habitat fragmentation, climate change, and overexploitation (Ekaka-a, 2009). Some of the consequences of these anthropogenic activities on forest biodiversity include: loss of ecosystem services such as carbon sequestration, water regulation, species decline and decreased resilience (Sale & Agbidye 2011). It is also important to adopt sustainable forest management practices; which includes: protected areas, such as parks and wild life reserves to safeguard biodiversity, sustainable harvesting, reforestation and afforestation (creating a forest where it never existed before), and community engagement (Shukla et al. 2014). It is a natural phenomenon that as the population in an area grows, this growth will

trigger the population pressure, and these will in turn impact on the forest resources biomass of the area. But a scenario where the forest resources biomass is increased by one half (150%), what will be the effect on the forest?

Materials and Methods

Ramdhani and Nugrahani (2015) stated the following equation:

$$\frac{dB}{dt} = S \left(1 - \frac{B}{L}\right) B - S_0 B - \beta_2 NB - S_1 IB - \beta_3 B^2 I \quad (1)$$

$$\frac{dN}{dt} = r \left(1 - \frac{N}{K}\right) N - r_0 N + \beta_1 NB \quad (2)$$

$$\frac{dP}{dt} = \lambda N - \lambda_0 P - \theta I \quad (3)$$

$$\frac{dI}{dt} = \pi \theta P + \pi_1 S_1 IB - \theta_0 I \quad (4)$$

With the following constraints conditions:

$$B(0) > 0, N(0) > 0, P(0) > 0, I(0) > 0 \text{ and } 0 < \pi \leq 1, 0 < \pi_1 \leq 1$$

Where the notations:

$B(t)$ = the density of forestry resource biomass at time t

$N(t)$ = the density of population dependent on the resource at time t

$P(t)$ = the density of population pressure at time t

$I(t)$ = the density of industrialization at time t

S = the intrinsic growth rate coefficient of the forest resources biomass

S_0 = the coefficient of natural depletion rate of resource biomass

S_1 = the coefficient of the depletion rate of biomass density caused by industrialization

r = the intrinsic growth rate of the population density

r_0 = the coefficient of natural depletion rate of population

L = the carrying capacity of the forestry resources biomass

K = the carrying capacity of the population density

β_1 = the growth rate of cumulative density of human population effect of resources

β_2 = corresponding depletion rate coefficient of the resource biomass density due to population

β_3

= the depletion rate coefficient of forestry resources biomass due to crowding by industrialisation

λ = the growth coefficient of population pressure

λ_0 = the natural depletion rate coefficient of population pressure

θ = depletion rate coefficient of population pressure due to industrialisation

θ_0 = coefficient of control rate of industrialisation which is applied by government

π = growth rate of industrialisation effect of population pressure

$\pi_1 S_1$ = growth rate of industrialisation due to forestry resource.

Existence of Solutions for Initial Value Problems

This subsection deals with qualitative analysis related to the solution of the initial-value problems for ordinary differential equations. Referring to the above class of mathematical formulations, the following definitions can be given:

Well-Posedness:

A problem is well formulated if the evolution equation is associated with the correct number of initial (or boundary) conditions for its solution, while a problem is well posed if the solution exists, it is unique and depends continuously on the initial data (Akpodee & Ekaka-a 2019). The main purpose of a model related to a certain physical system is to predict, for a certain time interval, the behavior of the system starting from the knowledge of the state at the initial time t_0 . The predictions of the model are then obtained by solving the initial-value problem. To do that, there are some basic requirements that a problem should satisfy:

- i) The solution should exist at least for the period of time desired.
- ii) The solution should be unique.
- iii) The solution should depend continuously on the initial data and on the parameters of the model so that if a small error is made in describing the present state, one should expect the effect of this error to be small in the

future. As already stated, if these requirements are satisfied, then the initial-value problem is said to be well posed.

Numerical Iterations Mathematical Preliminaries

Following Akpodee (2019), when numerical solutions to initial value problems (IVPs) are required that cannot be obtained by analytical means, it is necessary to use numerical methods. From the numerical methods that exist in solving initial value problems, we have only considered the popular fourth-order Runge-Kutta method in this study as part of the mathematical preliminaries. The mathematical structure and the theoretical definitions of this method are presented as follows:

The fourth-order Runge-Kutta (R-K) method is an accurate and flexible method based on a Taylor series approximation to the function $f(x, y)$ in the initial value problem

$$\frac{dy}{dx} = f(x, y)$$

Subject to the initial condition $y(x_0) = y_0$

The increment h in x may be changed at each step, but is usually kept constant so that after n steps, we have

$$x_n = x_0 + nh$$

The Runge-Kutta algorithm for the determination of the approximation y_{n+1} to $y(x_{n+1})$ is

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \\ k_3 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \\ k_4 &= hf(x_{n+1}, y_n + k_3) \end{aligned}$$

The local error involved in the determination of y_{n+1} from y_n is $O(h^5)$

The above method can be extended to find solution to a system of differential equations such as

$$\frac{dy}{dx} = f(x, y, z)$$

$$\frac{dz}{dx} = g(x, y, z)$$

Subject to the initial condition $y(x_0) = y_0$ and $z(x_0) = z_0$

These are the types of equations considered by this study which consists of a system of two first order nonlinear differential equations.

At the n th integration step, using a step of length h , the Runge-Kutta Algorithm for the system takes the form

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$z_{n+1} = z_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

Where,

$$\begin{aligned} k_1 &= hf(x_n, y_n, z_n) \\ k_2 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1) \\ k_3 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2) \\ k_4 &= hf(x_n + h, y_n + k_3, z_n + K_3) \end{aligned}$$

and

$$\begin{aligned} K_1 &= hg(x_n, y_n, z_n) \\ K_2 &= hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1) \\ K_3 &= hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2) \\ K_4 &= hg(x_n + h, y_n + k_3, z_n + K_3) \end{aligned}$$

1. Results

The results and discussion will be given in the next section:

Table 1: Impact of Experimental Time for the Interaction Between Forest Resource Biomass, Human Population Density, Population Pressure and Industrialization, When all the Parameter Values are Fixed for the Time Interval of $t \in 0(1)25$ Months.

Time, t (month)	N1	N2	N3	N4
0	1.0000	1.0000	1.0000	1.0000
1.0000	9.5736	2.1157	1.2095	0.4366
2.0000	7.0728	3.4008	3.3726	0.2041
3.0000	3.7447	4.1844	4.7828	0.1059
4.0000	1.9659	4.4796	5.3807	0.0679
5.0000	1.2419	4.5639	5.5731	0.0540
6.0000	0.9567	4.5826	5.6248	0.0489
7.0000	0.8453	4.5845	5.6357	0.0470
8.0000	0.8049	4.5833	5.6365	0.0463
9.0000	0.7928	4.5823	5.6358	0.0460
10.0000	0.7907	4.5817	5.6346	0.0459
11.0000	0.7912	4.5815	5.6306	0.0458
12.0000	0.7919	4.5815	5.6347	0.0458
13.0000	0.7924	4.5815	5.6358	0.0458
14.0000	0.7926	4.5815	5.6363	0.0458
15.0000	0.7926	4.5815	5.6366	0.0458
16.0000	0.7927	4.5815	5.6315	0.0458
17.0000	0.7927	4.5815	5.6355	0.0458
18.0000	0.7927	4.5815	5.6365	0.0458
19.0000	0.7926	4.5815	5.6391	0.0458
20.0000	0.7926	4.5815	5.6375	0.0458
21.0000	0.7927	4.5815	5.6331	0.0458
22.0000	0.7927	4.5815	5.6360	0.0458
23.0000	0.7926	4.5815	5.6381	0.0458
24.0000	0.7927	4.5815	5.6359	0.0458
25.0000	0.7927	4.5815	5.6346	0.0458

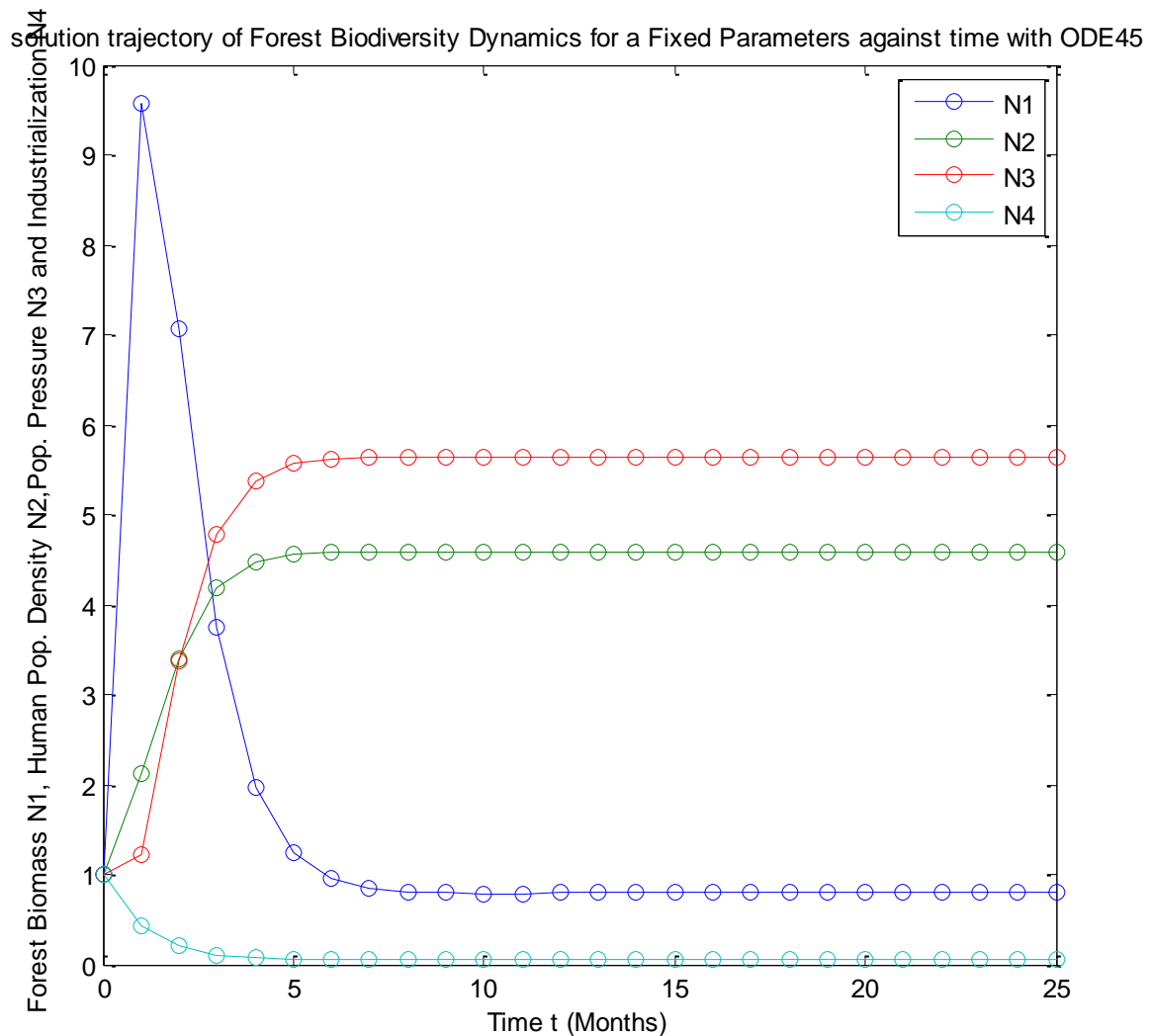


Figure 1: Solution Trajectory of the Impact of Experimental Time for the Interaction Between Forest Resource Biomass, Human Population Density, Population Pressure and Industrialization, When all the Parameter Values are Fixed for the Time Interval of $t \in 0(1)25$ Months.

Table 2: Impact of 150% Variation of the Depletion Rate Coefficient of Forest Resource Biomass Density due to Population. 1.0e+003 *

Time,t(month)	N1	N11	N2	N21	N3	N31	N4	N41
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	9.5736	6.0071	2.1157	2.0762	1.2095	1.2073	0.4366	0.4181
2.0000	7.0728	1.1877	3.4008	3.2422	3.3726	3.2654	0.2041	0.1780
3.0000	3.7447	0.0028	4.1844	3.9657	4.7828	4.5588	0.1059	0.0865
4.0000	1.9659	0.0000	4.4796	4.3135	5.3807	5.1794	0.0679	0.0569
5.0000	1.2419	0.0000	4.5639	4.4573	5.5731	5.4370	0.0540	0.0480
6.0000	0.9567	0.0000	4.5826	4.5126	5.6248	5.5354	0.0489	0.0455
7.0000	0.8453	-0.0000	4.5845	4.5333	5.6357	5.5717	0.0470	0.0448
8.0000	0.8049	0.0000	4.5833	4.5410	5.6365	5.5849	0.0463	0.0447
9.0000	0.7928	-0.0000	4.5823	4.5438	5.6358	5.5897	0.0460	0.0447
10.0000	0.7907	0.0000	4.5817	4.5448	5.6346	5.5914	0.0459	0.0447
11.0000	0.7912	-0.0000	4.5815	4.5452	5.6306	5.5920	0.0458	0.0447
12.0000	0.7919	0.0000	4.5815	4.5454	5.6347	5.5922	0.0458	0.0447
13.0000	0.7924	-0.0000	4.5815	4.5454	5.6358	5.5923	0.0458	0.0447
14.0000	0.7926	-0.0000	4.5815	4.5454	5.6363	5.5923	0.0458	0.0447
15.0000	0.7926	-0.0000	4.5815	4.5455	5.6366	5.5923	0.0458	0.0447
16.0000	0.7927	0.0000	4.5815	4.5455	5.6315	5.5923	0.0458	0.0447
17.0000	0.7927	-0.0000	4.5815	4.5455	5.6355	5.5923	0.0458	0.0447
18.0000	0.7927	0.0000	4.5815	4.5455	5.6365	5.5923	0.0458	0.0447
19.0000	0.7926	-0.0000	4.5815	4.5455	5.6391	5.5923	0.0458	0.0447
20.0000	0.7926	0.0000	4.5815	4.5455	5.6375	5.5923	0.0458	0.0447
21.0000	0.7927	-0.0000	4.5815	4.5455	5.6331	5.5923	0.0458	0.0447
22.0000	0.7927	0.0000	4.5815	4.5455	5.6360	5.5923	0.0458	0.0447
23.0000	0.7926	-0.0000	4.5815	4.5455	5.6381	5.5923	0.0458	0.0447
24.0000	0.7927	0.0000	4.5815	4.5455	5.6359	5.5923	0.0458	0.0447
25.0000	0.7927	0.0000	4.5815	4.5455	5.6346	5.5923	0.0458	0.0447

Table 2a: Impact of 150% Variation of the Depletion Rate Coefficient of Forest Resource Biomass Density due to Population showing expected biodiversity EBD.

Time,t(month)	N1	N11	EBD%
0	1.0000	1.0000	0
1.0000	9.5736	6.0071	37.2528
2.0000	7.0728	1.1877	83.2081
3.0000	3.7447	0.0028	99.9261
4.0000	1.9659	0.0000	100.0000
5.0000	1.2419	0.0000	100.0000
6.0000	0.9567	0.0000	100.0000
7.0000	0.8453	*0.0000	100.0001
8.0000	0.8049	0.0000	99.9999
9.0000	0.7928	*0.0000	100.0000
10.0000	0.7907	0.0000	99.9999
11.0000	0.7912	*0.0000	100.0001
12.0000	0.7919	0.0000	100.0000
13.0000	0.7924	*0.0000	100.0001
14.0000	0.7926	*0.0000	100.0000
15.0000	0.7926	*0.0000	100.0000
16.0000	0.7927	0.0000	100.0000
17.0000	0.7927	*0.0000	100.0000
18.0000	0.7927	0.0000	99.9999
19.0000	0.7926	*0.0000	100.0000
20.0000	0.7926	0.0000	99.9999
21.0000	0.7927	*0.0000	100.0001
22.0000	0.7927	0.0000	100.0000
23.0000	0.7926	*0.0000	100.0001
24.0000	0.7927	0.0000	100.0000
25.0000	0.7927	0.0000	100.0000

*indicates areas of biodiversity loss

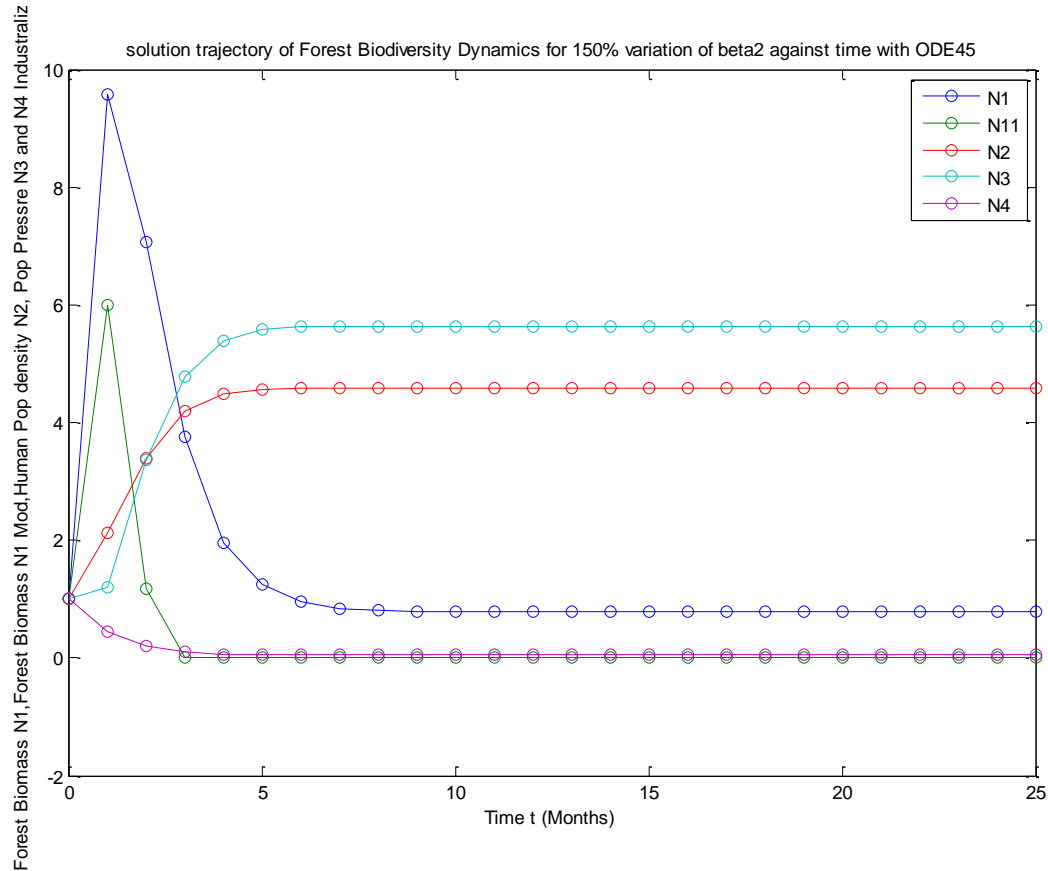


Figure 2: Solution Trajectory of the Impact of 150% Variation of the Depletion Rate Coefficient of Forest Resource Biomass Density due to Population.

Discussion

The Impact of experimental time for the interaction between forest resource biomass, human population density, population pressure and industrialization, when all the parameter values are fixed for the time interval of $t \in 0(1)25$ months, as shown in Figure 1. Four (4) coordinates were examined namely N1 being the forest resource biomass for fixed values, N2 being the human population density for fixed parameter values, N3 being the population pressure for fixed parameter values and N4 being the industrialization for fixed parameter values. From the numerical result obtained, we observed that on the base day of our experimental time, here called the initial condition; all the parameter values were fixed for the time interval of $t \in 0(1)25$ months, the initial values of the interacting variables; forest resource biomass N1, human population density, N2, population pressure, N3 and industrialization, N4, here called the initial conditions on the base day were recorded as 1.0000, 1.0000, 1.0000 and 1.0000. It was observed that the forest resource biomass, N1, decreased steadily from 9.5736 to 0.7907 for the first ten (10) months, after which it slight increased from 0.7912 in the eleventh month to 0.7926 in the fifteenth month. The value fluctuated between 0.7926 and 0.7927 till the twenty fifth month; indicating a convergence. On the other hand, the human population density, N2, increased steadily for the first seven (7) months, from 2.1157 to 4.5845 then for the next three months, it dropped slightly from 4.5833 on the eighth month to 4.5815 on the eleventh month where it stagnated till the twenty fifth month. This indicates that there were no new human arrivals into the area under study, hence, the stability of the population density. The population pressure, N3, increased steadily for the first eight (8) months; from 1.2095 to 5.6365. It declined for another three (3) months; from 5.6258 to 5.6306, it rose again for another three (3) months; from 5.6347 to 5.6363 in the sixteenth month. This trend of increasing for three months and dropping for one month continued; an indication that the forest resources tried to manage the rising population at intervals. Finally on this table, the impact of industrialization steadily decreased for ten (10) months; from 0.4366 to 0.0459 before stabilizing for the rest of the remaining fifteen (15) months at 0.0458. This gives a picture of the fact that at the time of constructing the industry, the forest resource was greatly impacted upon; and once the construction was over, the impact was minimized.

The impact of 150% Variation of Depletion rate Coefficient of Forest Biomass Density due to Population, eight (8) coordinates were examined namely N1 being the forest resource biomass for fixed values, N11 being the modified forest resource biomass, N2 being the human population density for fixed parameter values, N21 being the modified population density, N3 being the population pressure for fixed parameter values, N31 being the modified population pressure and N4 being the industrialization for fixed parameter values, and N41 being the modified industrialization. We observed that the initial values of the interacting variables; forest resource biomass N1, human population density, N2, population pressure, N3 and industrialization, N4, here called the initial conditions on the base day were recorded as 1.0000, 1.0000, 1.0000 and 1.0000. From the result obtained in Table 4.2c as well as figure 4.2b, on the Impact of 150% Variation of Depletion rate Coefficient of Forest Biomass Density due to Population, there was a biodiversity loss. When the depletion rate coefficient on the forest biomass was increased by half (150%), there was a significant decrease on the modified forest resource biomass, N11; which decreased from 6.0071 for the first month to 0.0000 in the twenty fifth (25th) month. this is a clear case of the total loss of the forest biomass, when the human population density rises astronomically, this translates to a massive increase in the human population pressure, which in turn impacts on the forest resource biomass. In this case, the forest was totally erased. In the same way, the modified human population density N21 increased steadily from the first to the eleventh month; with a value of 2.0762 to 4.5452, after which it converged to 4.5454. The modified population pressure N31 increased steadily from 1.2073 in the first month to 5.5922 in the twelfth month where it converged to 5.5923. The modified industrialization recorded a steady decline from 0.4181 for the first month to 0.0447 in the seventh month; where it also converged to 0.0447 till the twenty-fifth month. This means that, when the impact on the forest resource biomass is increased, the impact of industrialization will diminish.

Conclusion

The use of Mat lab ODE45 numerical scheme has proven to be robust in predicting biomass loss and projecting future trends. Our projections indicate that without effective intervention, the trend of biomass loss is likely to continue, intensifying environmental and climatic challenges. When the depletion rate coefficient of the forest resource biomass

was increased by half (150%), the forest resources biomass declined greatly, the human population density rose significantly; this also triggered the population pressure.

Recommendations

1. Effective policy measures should focus on mitigating deforestation, promoting reforestation, and enhancing land-use planning to minimize adverse impacts on forest biomass. Additionally, the integration of socio-economic data into biomass assessments provides valuable insights into the driving forces behind these changes, facilitating more informed decision-making.
2. The computational approach demonstrated in this study offers a scalable and detailed framework for monitoring forest resources and assessing the impact of human activities. It provides valuable tools for policymakers, conservationists, and researchers, enabling them to make data-driven decisions and implement effective strategies for forest conservation and sustainable development.
3. Finally, addressing the challenges posed by anthropogenic activities on forest biomass requires a collaborative effort that combines advanced computational techniques with actionable policy measures. By leveraging the insights gained from this study, stakeholders can work towards preserving forest ecosystems, enhancing their resilience, and ensuring the continued provision of essential ecosystem services.

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