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Time-Varying Covariance in Major Energy Portfolios

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Abstract
Utilizing the primary crude oil markets of Average, Brent, Dubai and West Texas Intermediate from 1982 to 2023, this research aim to estimate the Multivariate VECH between returns on Average, Brent, Dubai, and WTI crude oil price to see how the conditional covariance matrix of the crude oil market variables have a flexible dynamic structure, report time-varying covariance and impact of lagged shocks on conditional volatility and select the appropriate model to model the energy markets. After preliminary investigation have been conducted, like the time graph to determine trend in the evolution of the series, Augmented Dickey-Fuller test to ascertain unit root and logarithmic return for stability, the Multivariate Vector Error Conditional Heteroskedasticity (MVECH) and Diagonal Conditional Correlation (DCC) models were applied to the study variables. The findings demonstrate that the "positive semidefinite" property is satisfied by the diagonal multivariate VECH model as the estimates on the leading diagonal of the variance-covariance matrix are positive, it may be inferred that the variables are traveling in the same direction. Also, each asset in the portfolio exhibits time-varying volatility as captured by the significant ARCH and GARCH coefficients. Market returns and crude oil prices exhibit time-dependent oscillations, according to the Diagonal Conditional Correlation (DCC). By applying the DCC-GARCH's constant conditional correlation to the relationship between Dubai raw prices and Brent, Average and West Texas Intermediate, the impact of lagged shocks on the conditional variance was statistically significant. According to the model selection approach that use the Akaike information criterion (AIC), the Diagonal Conditional Correlation (DCC) model performs better than the Diagonal Multivariate VECH model. All sufficiency tests also show that the model is adequate. Since crude oil market factors such as time-varying covariance, and volatility imbalance are interdependent, it is necessary to use a multivariate GARCH Model to assess the advantages of this dependency. Some recommendations were proffered.

Keywords: Covariance, Time-Varying, Portfolios, Volatility

Introduction

Econometric modeling of financial markets has long recognized the dynamic nature of asset returns, particularly their time-varying volatility and correlations. In the context of crude oil markets, understanding these dynamics is crucial for risk management, portfolio diversification, and pricing of energy derivatives. The inherent complexities of oil prices, influenced by geopolitical events, supply-demand imbalances, and speculative activities, necessitate sophisticated statistical tools to capture their evolving interdependencies. This introduction sets the stage for an in-depth analysis of time varying covariance and volatility dynamics in major crude oil markets, employing the multivariate Vector Error Conditional Heteroskedasticity (MVECH) and the Diagonal Conditional Correlation (DCC) model. According to Zhang (2013), key crude oil benchmarks, such as Crude Oil Average (a composite index representing global average or specific blend), West Texas Intermediate (US crude), Crude Oil Brent (North Sea), Dubai crude oil benchmark (middle East), exhibit distinct characteristics and are influenced by regional and global factors leading to varying level of volatility and interconnectedness. Bollerslev (1990) used Vector Error Conditional Heteroskedasticity (VECH) model to allow the conditional covariance matrix of the dependent variable to follow an elastic dynamic structure. These methodologies offer robust frameworks for uncovering the intricate relationships among different crude oil bench marks, providing valuable insight into market behavior and potential opportunities for hedging and arbitrage. Ejukwa and Tuaneh (2025), examine volatility contagion in the crude oil market to assess how the association between benchmark crude oil prices evolve over time. The BEKK model along with the constant conditional correlation model was deployed. Findings show that the historical conditional volatility and squared error had substantial impact on the conditional variance. Bollerslev (1986) in his work Generalized Autoregressive Conditional Heteroskedasticity introduced the GARCH model, extending ARCH to allow for more flexible and persistent volatility dynamics. Nomikos and Voukelatos (2014) examined dynamic volatility and correlation of crude oil spot and futures market offering insight into the relationship between spot and futures prices. Engle (2002) in his study, dynamic conditional correlation: A simple way to estimate varying correlations, presents the DCC model, a widely used and computationally efficient approach for estimating time-varying conditional correlations. Existing literature utilize GARCH-type models to capture time-varying volatility in financial assets, including crude oil. Univariate GARCH models provide insights into individual volatility of each crude oil series, while multivariate GARCH models such as the Diagonal BEKK and Constant Conditional Correlation models (CCC) have been employed to analyze volatility transmission and co-movement among crude oil prices. However, while these models offer valuable insight into static and time-varying volatilities and correlation, a comprehensive examination of the dynamic and evolving covariance structures among a diverse set of globally significant crude oil benchmarks –Crude Oil Average, Crude Oil Brent, Crude Oil Dubai and West Texas Intermediate – using advanced multivariate GARCH specifications remains an area with scope for deeper investigation. While some studies have used DCC-GARCH to investigate relationships between crude oil and other markets and others have examined volatility spillovers between a subset of these crude oil benchmarks, there is a gap in a detailed and comprehensive analysis of time-varying covariance and volatility dynamics across all four major crude oil benchmarks simultaneously using both MVECH and DCC models. This research seeks to bridge this gap by providing a nuanced understanding of how the interdependencies and volatility characteristics of these crude oil market evolves over time.

Statement of the Problem

This study examines the main crude oil markets through the lens of the multivariate GARCH model in order to ascertain averages, evolving correlations, and volatility transmission. Contradictory results have been produced by several GARCH model examinations of the crude oil market. Many hypotheses seek to clarify the discrepancy between the ability of random volatility to explain returns and the ability of trends or moving averages to do so. Two variables show the unequal effects when two or more potentially incorrect financial time series change at the same time. To stabilize volatility, the GARCH model employs conditional variables and historical returns. Optimal models for multivariate GARCH calculations and future fluctuation predictions do not yet exist, according to Serletis and Elder (2011). Many essential goods are influenced by oil prices. As important as risk assessment is for investors to make informed decisions, price forecasting is as crucial. An abrupt drop in output happens if oil prices abruptly rise or fall, say Serletis and Elder (2011). Maximizing earnings while minimizing risk should be an investor's purpose. Estimating and predicting Value at Risk (VaR) is a good fit for GARCH models. Few studies have made use of the multivariate GARCH model and variance prediction using the same variables. This research found that the world's biggest crude oil markets had notoriously difficult-to-model mean volatility, time-varying covariance and conditional correlation. However, multivariate GARCH models like the VECH, the DCC and other risk model prediction methods have been introduced that account for different distributions. The results of this research could be useful for students, entrepreneurs, professors, and others in positions of power. As they consider new laws and regulations, lawmakers may use this study as a resource for a deeper grasp of the market's inner workings.

Aim of the Study

This study utilizes the multivariate GARCH Model to model four most significant crude oil markets' time-varying covariance, and conditional volatility.

Objectives of the Study

The objectives include to;

i. Find the dynamic structure of the variables' conditional covariance matrices in the crude oil market by estimating the multivariate VECH between the returns on Average, Brent, Dubai, and WTI crude oil prices.

ii. Investigate the impact of lagged shocks on volatility in the four crude oil markets and how long they last.

iii. Investigate the conditional correlation between the Crude Oil Average, Crude Oil Brent, Crude Oil Dubai and West Texas Intermediate crude oil market.
iv. Select the MGARCH model that best matches the simulations of the crude oil market under review.

Scope of the Study

In order to determine time-varying covariance, and persistence of shocks on volatility of crude oil benchmarks using multivariate GARCH Model, the scope covers the use of Vector Error Conditional Heteroskedasticity (VECH-GARCH) and Diagonal Conditional Correlation (DCC) models. The study mainly takes into account four metrics: Crude Oil Average, Crude Oil Brent, Crude Oil Dubai and West Texas Intermediate ranging from January,1982 to April, 2023. One standard used in the crude oil market is the benchmark crude, often called a marking crude.

Materials and Method

Source of Data

Our data set includes monthly Brent, Average, Dubai and West Texas Intermediate crude oil prices sourced from the Central Bank of Nigeria (CBN) website (www.cbn.gov.ng). The data covers the period from January 1982 to April 2023, encompassing 1984 observations.

Software use for Data Analysis

To analyze this data, EViews 13 was utilized. EViews is a versatile statistical software package that caters to researchers across disciplines. It is well suited for organizing, visualizing, and analyzing data trends and patterns.

Preliminary Analysis

This verifies the accuracy of the measurements, the validity of the analysis and the appropriateness of the assumptions underlying the analysis. This include checking for normality of the variable distribution, the absence of outliers and the suitability of the data to the proposed model. In addition, correlation analysis, descriptive statistics, the ARCH Effect, and logarithmic return and volatility estimations is dealt with. For the logarithmic return the data is fitted with a conditionally compound monthly return that is determined by the price of crude oil benchmark. A conditionally compound monthly return, calculated as;

$$RCOA = Log\left(\frac{COA_t}{COA_{t-1}}\right)X\,\frac{100}{1} \tag{1.0}$$

$$RCOB = Log\left(\frac{COB_t}{COB_{t-1}}\right)X\,\frac{100}{1} \tag{2.0}$$

$$RCOD = Log\left(\frac{COD_t}{COD_{t-1}}\right)X\,\frac{100}{1} \tag{3.0}$$

$$RCOWTI = Log\left(\frac{COWTI_t}{COWTI_{t-1}}\right)X\,\frac{100}{1} \tag{4.0}$$

is suited to the crude oil price, where t-1 represents one period lag on the price of crude oil, each of the four types of returns on crude oil prices at time t are represented by equation 1, 2, 3 and 4. The returns is done to achieve stationarity and make the series comparable across different time periods and assets with varying price levels.

Time Plot

A time plot provides a visual snapshot of how variables change over time making it an essential tool for understanding and analyzing time series data. It detects anomalies like outliers or unusual data point, reveal patterns like seasonality or cyclical behavior.

Descriptive Statistics

The Jarque-Bera test as popularize by Ejukwa and Nanaka (2024) to test for normality and other features in the data set focuses on describing the main features and characteristics of the dataset without making any generalizations or inference to a larger population. The test statistics is presented as;

$$X^2 \approx \frac{N}{6}\left[S^2 + \frac{(K-3)^2}{4}\right] \tag{5.0}$$

The variable's magnitude is denoted by N, kurtosis by K, and skewness by S.

Unit Root Test for Stationarity

This statistical test is used to determine whether a time series variable is non-stationary and possesses a unit root. This test involves comparing the estimated coefficient of the lagged dependent variable in an autoregressive model to a critical value. The Augmented Dickey-Fuller (ADF) and the Phillip Perron Test (PPT) was used in the analysis. Assuming a series moves randomly is the assumption of the unit root test.

$$Y_t = b_1 y_{t-1} + \varepsilon_t \quad \text{Random walk} \tag{6.0}$$

$$Y_t = b_0 + b_1 y_{t-1} + \varepsilon_t \quad \text{Random walk with drift} \tag{7.0}$$

$$Y_t = b_0 + b_1 y_{t-1} + b_2 t + \varepsilon_t \quad \text{Random walk with drift and trend} \tag{8.0}$$

ARCH Effect

This test is a statistical phenomenon that is used to evaluate whether the variance of a variable is not constant over time but rather depends on the size of its past shocks. This is a requirement for the use of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models The regression is then reported as follows;

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t+}^2 +,...,+ \alpha\, p\, \varepsilon_{t-p}^2 + \mu_t \tag{9.0}$$

where; $\alpha_1,...,\alpha_p$ are the regression coefficients and α_0 the intercept.

H₀: $\alpha_1 = \alpha_2 =,...,= \alpha_p = o,$

The null hypothesis that there is no ARCH impact in the residuals.

H₀: $\alpha_1 \neq \alpha_2 \neq,...,\neq \alpha_p \neq o$

The alternative hypothesis that there is ARCH impact in the residuals.

Optimal Lag Length Order

The order of the VAR Lag Length is determined by the following criteria, as stated by Tuaneh (2018): (a) Final Predator Error (FPE), (b) Akaike Information Criteria (AIC), and (c) Hannan-Quinn Information Criteria (HQ). Due to its importance in determining the length, it was preserved for inclusion in the study despite its potential to diminish the model selection criteria. To fit a VAR model, the order of L must be between 0 and L_{max}. While more degrees of freedom are provided by models with more delays, residue volatility may be enhanced in these models. You may use Information Criteria (IC) to find the "correct" amount of delays in a VAR(p) model, much as in a univariate AR(p) model.

$$\text{AIC} = \text{In}\,|\Omega| + \frac{2(n^2 p + n)}{T} \tag{10.0}$$

$$\text{SBC} = \text{In}\,|\Omega| + \frac{(n^2 p + n)\,\text{In}\,(\text{T})}{T} \tag{11.0}$$

Co-integration Test

It is necessary to validate the existence of co-integrating interactions between the components in question. Two likelihood ratio tests yielded the following findings, which was used for this purpose:

$$\lambda_{\text{Trace}} = -\text{T} \sum_{t=r+1}^{n} In\,(1 - \lambda_i) \tag{12.0}$$

For i = 0, 1,..,n -1

$$\lambda_{Max} = -T\text{In}\,(1 - \lambda\text{r} + 1) \tag{13.0}$$

It compares and contrasts two hypotheses: one says there is no co-integrating relationship and the other says otherwise. If two series are co-integrated and show a long-run relationship, then they are likely connected and can be combined linearly. While it is feasible that several data sets will eventually converge, unexpected events might change their present trajectories at any time.

The Multivariate GARCH Model Specification

Expanding univariate models into multivariate models can improve decision-making tools for value-at-risk (VaR) forecasting, hedging, portfolio selection, and asset pricing. This study emphasizes the diagonal VECH-GARCH model, and the Constant Conditional Correlation (GARCH) model.

Vector Error Conditional Heteroskedasticity (VECH)-GARCH

Bollerslev et al., (1988), used the Vector Error Conditional Heteroskedasticity (VECH) GARCH models to allow the conditional covariance matrix of the dependent variables to follow an elastic dynamic structure. All diagonal multivariate VECH models are "positive semi-definite," as declared by Brooks (2001). According to the results of variance-covariance or correlation matrix models, the series returns of a positive definite matrix must be equal and have zero variance, according to Deebom et al., (2020). In the case of the VECH, the conditional variance and covariance would each depend upon lagged values of all the variances and covariances and on lags of the squares of both error terms and their cross products. Suppose that there are four variables used in the model. The conditional covariance matrix denoted H_t, would be 4×4. H_t and VECH (H_t) are written in matrix form as thus:

$$\sigma_{i,t}^2 = M(i) + A1(i) * \varepsilon_{1,t-1}^2 * \left(\varepsilon_{1,t-1}^2\right)^1 + B1(i) * \sigma_{1,t-1}^2 , \quad \Xi_t / \Psi_{t-1} \sim N(0, H_t) \quad (14.0)$$

Where M(i) , A1(i), and B1(i) are parameters of an indefinite matrix

$$M(i) = \begin{bmatrix} M(1) \\ M(2) \\ M(3) \\ M(4) \end{bmatrix} = \begin{bmatrix} M(1,1) \\ M(2,2) \\ M(3,3) \\ M(4,4) \end{bmatrix}, A1(i) = \begin{bmatrix} A1(1) \\ A1(2) \\ A1(3) \\ A1(4) \end{bmatrix} = \begin{bmatrix} A1(1,1) \\ A1(2,2) \\ A1(3,3) \\ A1(4,4) \end{bmatrix}, \quad B1(i) = \begin{bmatrix} B1(1) \\ B1(2) \\ B1(3) \\ B1(4) \end{bmatrix} = \begin{bmatrix} B1(1,1) \\ B1(2,2) \\ B1(3,3) \\ B1(4,4) \end{bmatrix}$$

$$\sigma_{i,t}^2 = \begin{bmatrix} M(1) \\ M(2) \\ M(3) \\ M(4) \end{bmatrix} + \begin{bmatrix} A1(1) \\ A1(2) \\ A1(3) \\ A1(4) \end{bmatrix} * \varepsilon_{i,t-1}^2 + \begin{bmatrix} B1(1) \\ B1(2) \\ B1(3) \\ B1(4) \end{bmatrix} * \sigma_{i,t-1}^2 ,$$

The variance-covariance estimates of the Diagonal VECH Multivariate GARCH model in Equation form:

$$\sigma_{1,t}^2 = M(1,1) + A1(1,1)\varepsilon_{i,t-1}^2 + B1(1,1)\sigma_{i,t-1}^2 \quad (14.1)$$
$$\sigma_{2,t}^2 = M(2,2) + A1(2,2)\varepsilon_{i,t-1}^2 + B1(2,2)\sigma_{i,t-1}^2 \quad (14.2)$$
$$\sigma_{3,t}^2 = M(3,3) + A1(3,3)\varepsilon_{i,t-1}^2 + B1(3,3)\sigma_{i,t-1}^2 \quad (14.3)$$
$$\sigma_{4,t}^2 = M(4,4) + A1(4,4)\varepsilon_{i,t-1}^2 + B1(4,4)\sigma_{i,t-1}^2 \quad (14.4)$$

The covariance model in equation form is thus

$$\rho_{1,2,t} = M(1,2) + A1(1,1)\varepsilon_{1,t-1} * \varepsilon_{2,t-1} + B1(1,1)\rho_{1,2,t-1} \quad (14.5)$$
$$\rho_{1,3,t} = M(1,3) + A1(1,3)\varepsilon_{1,t-1} * \varepsilon_{3,t-1} + B1(1,3)\rho_{1,3,t-1} \quad (14.6)$$
$$\rho_{1,4,t} = M(1,4) + A1(1,4)\varepsilon_{1,t-1} * \varepsilon_{4,t-1} + B1(1,4)\rho_{1,3,t-1} \quad (14.7)$$
$$\rho_{2,3,t} = M(2,3) + A1(2,3)\varepsilon_{2,t-1} * \varepsilon_{3,t-1} + B1(2,3)\rho_{1,3,t-1} \quad (14.8)$$
$$\rho_{2,4,t} = M(2,4) + A1(2,4)\varepsilon_{2,t-1} * \varepsilon_{4,t-1} + B1(2,4)\rho_{2,4,t-1} \quad (14.9)$$
$$\rho_{3,4,t} = M(3,4) + A1(3,4)\varepsilon_{3,t-1} * \varepsilon_{4,t-1} + B1(3,4)\rho_{3,4,t-1} \quad (14.10)$$

Therefore, the specification of VECH model is given as

$$VECH(H_t) = C + AVECH(E_{t-1}E'_{t-1}) + BVECH(H_{t-1}) \quad (14.11)$$

Where H_t = NxN conditional variance-covariance matrix, E_t= Nx1 disturbance vector, $VECH(\cdot)$ = the column stacking operator applied to the upper portion of the symmetric matrix, C = N(N+1)/N parameter vector, A and B = N(N+1)/N parameter matrices.

Constant Conditional Correlation (CCC) Model

According to Hansen et al., (2012), the constant conditional correlation (CCC) model was developed by Bollerslev in 1990 to model the correlation coefficient matrix but the coefficients are constant, describing univariate fluctuation characteristics and negatively capturing the dynamic correlation between sequences.

Let (η_t) be a sequence of iid variables with distribution η. A process (ϵ_t) is called CCC-GARCH(p,q) if it satisfies

$\epsilon_t = H_t^{1/2}\eta_t$

$H_t = D_t R D_t$

$h_t = \omega + \sum_{i=1}^{q} A\, i\epsilon_{t-1} + \sum_{j=1}^{p} B_j h_{i-j}$

where R is a correlation matrix, ω is an mx_1 vector with positive coefficients, A_i and B_j are mxm matrices with nonnegative coefficients, D_t is a diagonal matrix of conditional variance, $H_t^{1/2}$ is the cholesky factor of the time-varying conditional covariance matrix H_t, Dt is a diagonal matrix of conditional variance and η_t is an mx1 vector of normal, independent and identically distributed innovations.

The advantage of this specification is that a simple condition ensuring the positive definiteness of H_t is obtained through the positive coefficients for the matrices A_i and B_j and the choice for a positive definite matrix for R. However, this model is limited by its non-stability by aggregation and arbitrary nature of the assumption of constant conditional correlations.

Method of Estimating Parameters of Multivariate GARCH Model

The quasi maximum likelihood (QML) method is often used for estimating the conditional covariance matrix of an MGARCH model. That is, if it is stated in the statement that θ is a parameter for a residual vector t with dimensions Nx1, and that the conditional covariance matrix of t, H_t (θ), is positive definite and NxN. By applying the log probability of a normal distribution to θ, the estimate can be optimize using the QML approach.

$$\log L_T(\theta) = \frac{-N.T}{2} Log(2\Pi) - \frac{1}{2}\sum_{t=1}^{T} Log / H_t / - \frac{1}{2}\sum_{t=1}^{T} \Xi_t^{1} H_t^{-1} \Xi_t \qquad (15.0)$$

Post Estimation Technique

Conducting conditional heteroscedasticity tests is essential for ensuring that the chosen model provides the most accurate estimates. In this work, two tests were used to measure conditional heteroscedasticity. They are portmanteau analysis and the Q-Q plot

The Portmanteau test statistics is given as: $Q_k(m) = T^2 \sum_{i=1}^{m} \frac{1}{T-i} b_i (\hat{p}_o^{\,(a)-1} \otimes \hat{p}_o^{\,(a)-1}) b_i \qquad (16.0)$

Results

Table 1: Descriptive Statistics on Raw and Return of Crude Oil Price Benchmarks

	COA	COB	COD	COWTI	RCOA	RCOB	RCOD	RCOWTI
Mean	44.9	46.0	43.9	44.7	0.2	0.2	0.2	0.2
Median	30.7	31.0	28.9	31.7	0.9	0.6	0.9	0.8
Maximum	132.9	133.9	131.2	133.9	43.0	43.3	49.1	54.7
Minimum	9.6	9.5	7.9	11.3	-50.5	-51.1	-54.0	-59.3
Std. Dev.	30.4	31.7	31.2	28.5	9.2	9.4	9.5	9.4
Skewness	0.9	0.9	0.9	0.9	-0.7	-0.5	-0.7	-0.7
Kurtosis	2.6	2.7	2.6	2.6	8.3	6.9	9.3	11.0
Jarque-Bera	68.7	71.6	70.5	66.0	620.1	331.8	853.1	1367.0
Probability	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Sum	22263.1	22823.4	21774.1	22191.6	85.6	85.3	90.2	81.1
Sum Sq. Dev.	457293.6	498712.3	480674.9	401026.6	41482.2	43372.7	44335.8	43753.3
Observations	496	496	496	496	495	495	495	495

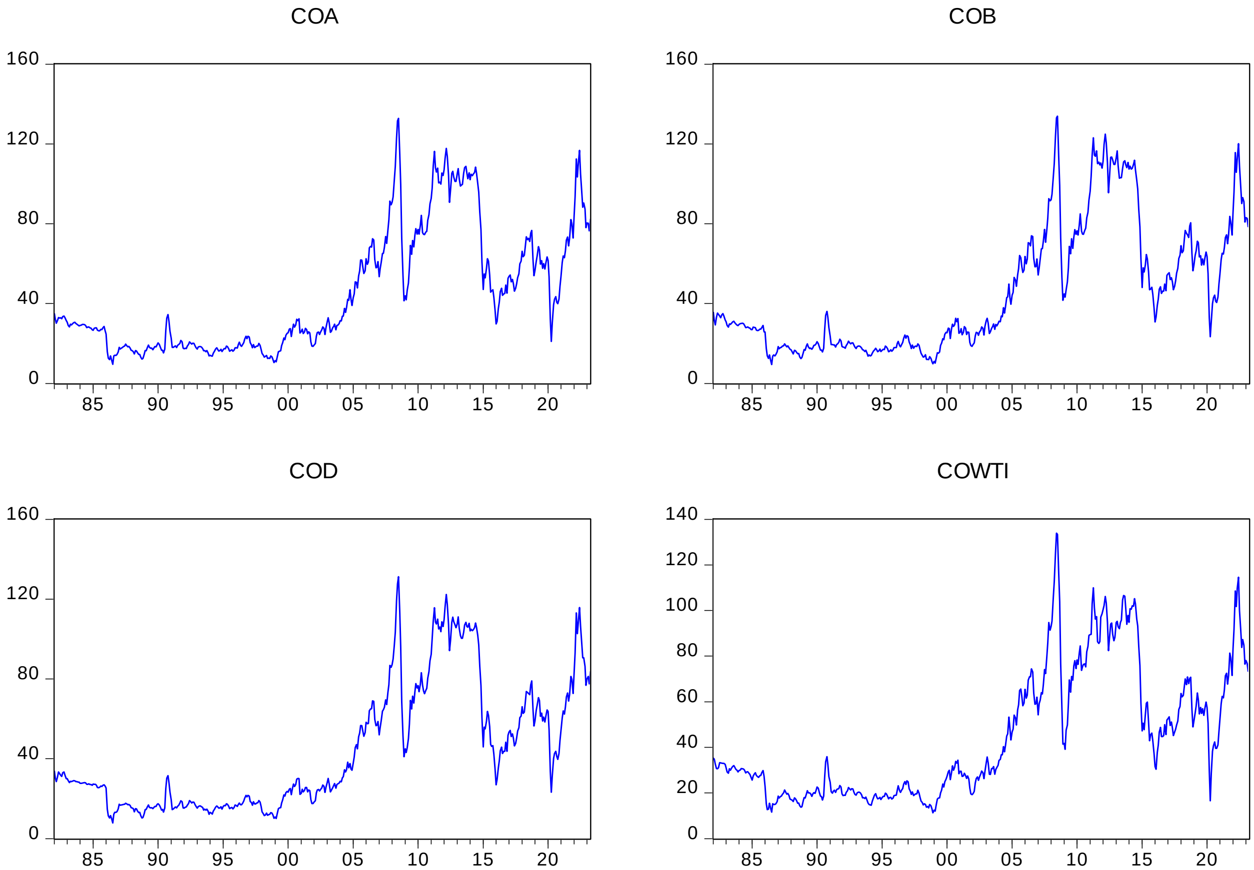

Figure 1: Time Plot on Crude Oil Price Average, Crude Oil Price Brent, Crude Oil Price Dubai and Crude Oil Price West Texas Intermediate from 1982, January to May, 2023.

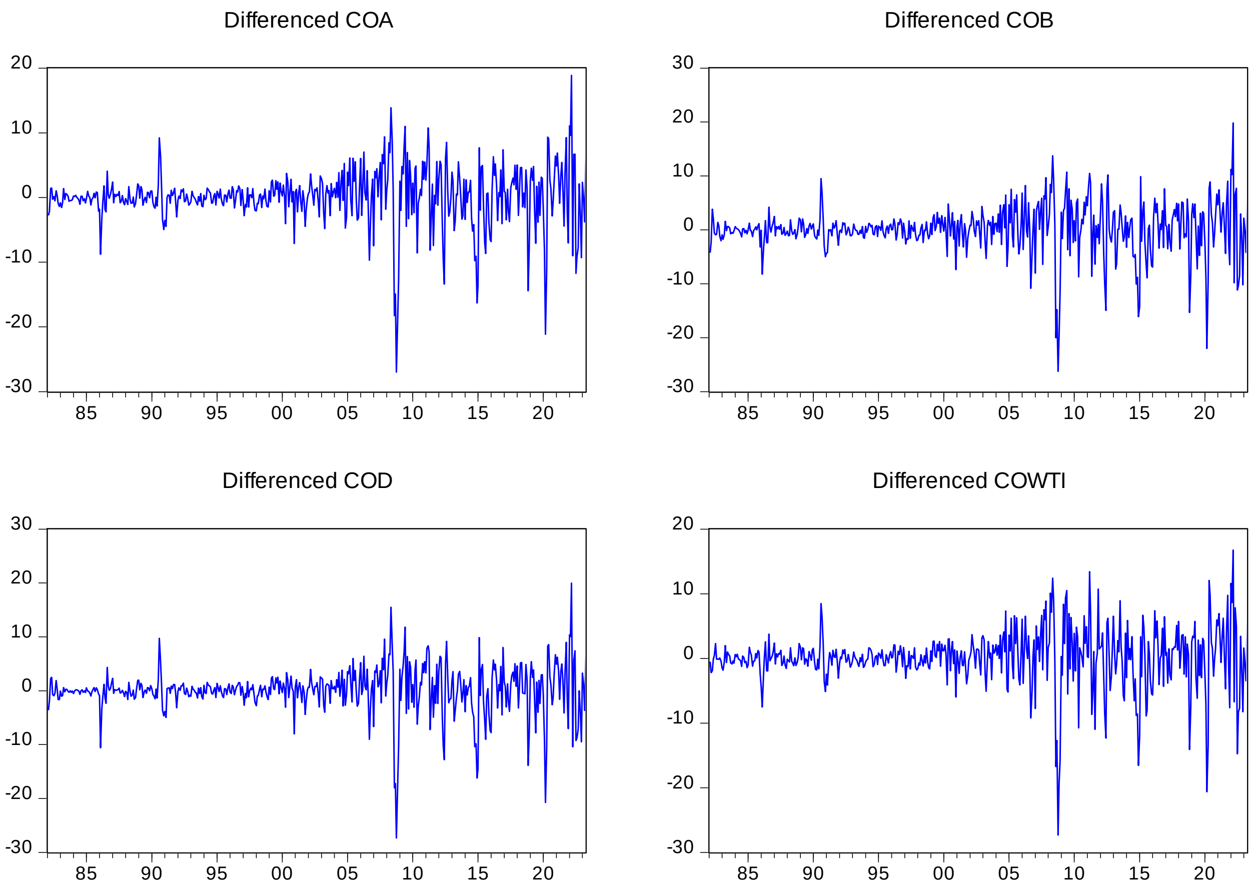

Figure 2: Time Plot on the Difference Crude Oil Price Average, Crude Oil Price Brent, Crude Oil Price Dubai and Crude Oil Price West Texas Intermediate from 1982, January to May, 2023.

Table 2: Results of Unit Root Test on Raw and Return on Crude Oil Price Benchmarks

Variable	t-Statistic	P-Value	Remarks
COA	-2.220	0.199	
D(COA)	-14.976	0.000	**1(1)**
COB	-2.172	0.217	
D(COB)	-15.415	0.000	**1(1)**
COD	-2.171	0.217	
D(COD)	-14.637	0.000	**1(1)**
COWTI	-2.367	0.152	
D(COWTI)	-15.396	0.000	**1(1)**

Table 3: Result of Test for Co-integration

Unrestricted Co-integration Rank Test (Trace)					**Unrestricted Co-integration Rank Test (Maximum Eigenvalue)**			
Hypothesized No. of CE(s)	**Eigenvalue**	**Trace Statistic**	**0.05 Critical Value**	**Prob.****	**Eigenvalue**	**Max-Eigen Statistic**	**0.05 Critical Value**	**Prob.****
r= 0	0.350	599.027	47.856	0.000	0.291	168.822	21.132	0.0001
r≤ 1	0.291	388.084	29.797	0.000	0.222	122.785	14.265	0.0001
r≤ 2	0.222	219.263	15.495	0.000	0.179	96.478	3.841	0.0000
r≤ 3	0.179	96.478	3.841	0.000	0.350	210.942	27.584	0.0001

Table 4: Results of VAR lag Order Selection Criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-3784.515	NA	67.160	15.559	15.593	15.572
1	-3726.289	115.26	56.467	15.385	15.557*	15.453
2	-3680.701	89.491	50.007	15.264	15.573	15.385*
3	-3655.770	48.532	48.208	15.227	15.674	15.403
4	-3636.499	37.197	47.568	15.214	15.798	15.443
5	-3613.905	43.239	46.301	15.186	15.909	15.470
6	-3585.311	54.251*	43.974*	15.135*	15.995	15.473
7	-3574.231	20.842	44.881	15.155	16.153	15.547
8	-3564.590	17.976	46.081	15.181	16.316	15.627

* Indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Table 5: Result of Error Correction Model and ARCH Effect

Crude oil Price Benchmark series	**Cointegration Rank**	**ECT**	**Residuals Heteroscedasticity Test (statistics)**	**P-Value**
Crude oil Price Average	4	29.29453(4.920)	10701.210	(0.0000)
Crude oil Price in Brent Blend	4	-15.445(1.979)	22431.320	(0.0000)
Crude oil Price in Dubai	4	-7.031(1.426)	8643.737	(0.0000)
Crude oil Price in West Texas Intermediate (WTI),	4	-8.320(1.67884)	7906.609	(0.0000)

Note: in parenthesis p-value is attached

Model Estimation

Results of Vector Error Conditional Heteroskedasticity (VECH) GARCH

The matrix representation of results of the Vector Error Conditional Heteroskedasticity (VECH) GARCH is presented as thus:

$$M = \begin{bmatrix} 20.973 & 12.072 & 30.695 & 19.154 \\ 12.072 & 7.118 & 17.661 & 10.715 \\ 30.695 & 17.661 & 46.087 & 26.488 \\ 19.154 & 10.715 & 26.488 & 18.709 \end{bmatrix}, \quad A = \begin{bmatrix} 0.555 & 0.528 & 0.561 & 0.485 \\ 0.528 & 0.502 & 0.537 & 0.460 \\ 0.561 & 0.537 & 0.565 & 0.490 \\ 0.485 & 0.460 & 0.490 & 0.430 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.268 & 0.377 & 0.120 & 0.316 \\ 0.377 & 0.524 & 0.177 & 0.439 \\ 0.120 & 0.177 & 0.047 & 0.138 \\ 0.316 & 0.439 & 0.138 & 0.376 \end{bmatrix}$$

Alternatively, the model is represented in equation form as:

$$\sigma^2_{\text{i,t}} = \begin{bmatrix} 20.973 \\ 7.118 \\ 46.087 \\ 18.709 \end{bmatrix} + \begin{bmatrix} 0.555 \\ 0.502 \\ 0.565 \\ 0.430 \end{bmatrix} * \varepsilon^2_{i,t-1} + \begin{bmatrix} 0.268 \\ 0.524 \\ 0.047 \\ 0.376 \end{bmatrix} * \sigma^2_{\text{i,t-1}} \qquad (16.0)$$

Variance Equation:

$$\sigma^2_{1,t} = 20.974 + 0.555\varepsilon^2_{i,t-1} + 0.268\sigma^2_{i,t-1} \qquad (16.1)$$
$$\sigma^2_{2,t} = 7.118 + 0.502\varepsilon^2_{i,t-1} + 0.524\sigma^2_{i,t-1} \qquad (16.2)$$
$$\sigma^2_{3,t} = 46.087 + 0.565\varepsilon^2_{i,t-1} + 0.047\sigma^2_{i,t-1} \qquad (16.3)$$
$$\sigma^2_{4,t} = 18.708 + 0.430\varepsilon^2_{i,t-1} + 0.376\sigma^2_{i,t-1} \qquad (16.4)$$

Covariance Equation

$$\rho_{1,2,t} = \underset{(0.000)}{12.072} + \underset{(0.000)}{0.528}\varepsilon_{1,t-1} * \varepsilon_{2,t-1} + \underset{(0.000)}{0.377}\rho_{1,2,t-1} \qquad (16.5)$$
$$\rho_{1,3,t} = \underset{(0.000)}{30.695} + \underset{(0.000)}{0.562}\varepsilon_{1,t-1} * \varepsilon_{3,t-1} + \underset{(0.000)}{0.120}\rho_{1,3,t-1} \qquad (16.6)$$
$$\rho_{1,4,t} = \underset{(0.000)}{19.154} + \underset{(0.000)}{0.485}\varepsilon_{1,t-1} * \varepsilon_{4,t-1} + \underset{(0.000)}{0.316}\rho_{1,2,t-1} \qquad (16.7)$$
$$\rho_{2,3,t} = \underset{(0.000)}{17.661} + \underset{(0.000)}{0.537}\varepsilon_{2,t-1} * \varepsilon_{3,t-1} + \underset{(0.000)}{0.177}\rho_{2,3,t-1} \qquad (16.8)$$
$$\rho_{2,4,t} = \underset{(0.000)}{10.715} + \underset{(0.000)}{0.460}\varepsilon_{2,t-1} * \varepsilon_{4,t-1} + \underset{(0.000)}{0.439}\rho_{2,4,t-1} \qquad (16.9)$$
$$\rho_{3,4,t} = \underset{(0.000)}{26.488} + \underset{(0.000)}{0.490}\varepsilon_{3,t-1} * \varepsilon_{4,t-1} + \underset{(0.000)}{0.134}\rho_{3,4,t-1} \qquad (16.10)$$

Results of Variance-Covariance Estimates of the Constant Conditional Correlation Diagonal Multivariate GARCH Model

The matrix representation of the variance-covariance estimates of the diagonal constant conditional correlation multivariate GARCH model are thus;

$$\sigma^2_{i,t} = M(i) + A1(i)\varepsilon^2_{i,t-1} + B1(i)\sigma^2_{i,t-1}$$
$$\rho(i,j) = R(i,j) * \sqrt{A1(i)\sigma^2_{i,t-1} * B1(i)\sigma^2_{i,t-1}}$$

$$M(i) = \begin{bmatrix} 58.095 \\ 76.976 \\ 54.338 \\ 56.325 \end{bmatrix}, A1(i) = \begin{bmatrix} 0.085 \\ 0.096 \\ 0.092 \\ 0.083 \end{bmatrix}, \quad B1(i) = \begin{bmatrix} 0.152 \\ 0.030 \\ 0.248 \\ 0.159 \end{bmatrix} \qquad (17.0)$$

$$R(i,j)=\begin{bmatrix} 0 & \underset{(0.000)}{0.989}*\sqrt{\sigma^2_{1,t-1}*(i)\sigma^2_{2,t-1}} & \underset{(0.000)}{0.979}*\sqrt{\sigma^2_{1,t-1}*(i)\sigma^2_{3,t-1}} & \underset{(0.000)}{0.970}*\sqrt{\sigma^2_{1,t-1}*(i)\sigma^2_{4,t-1}} \\ 0 & 0 & \underset{(0.000)}{0.935}*\sqrt{\sigma^2_{2,t-1}*(i)\sigma^2_{3,t-1}} & \underset{(0.000)}{0.936}*\sqrt{\sigma^2_{2,t-1}*(i)\sigma^2_{4,t-1}} \\ 0 & 0 & 0 & \underset{(0.000)}{0.915}*\sqrt{\sigma^2_{3,t-1}*(i)\sigma^2_{4,t-1}} \end{bmatrix} \quad (18.0)$$

Variance-Covariance Estimates of the Constant Conditional Correlation Diagonal Multivariate GARCH Model in equation form

$$\sigma^2_{1,t}=\underset{(0.000)}{58.095}+\underset{(0.000)}{0.085\varepsilon^2_{1,t-1}}+\underset{(0.000)}{0.152\sigma^2_{1,t-1}} \quad (19.1)$$

$$\sigma^2_{2,t}=\underset{(0.000)}{76.976}+\underset{(0.000)}{0.096\varepsilon^2_{2,t-1}}+\underset{(0.000)}{0.030\sigma^2_{2,t-1}} \quad (19.2)$$

$$\sigma^2_{3,t}=\underset{(0.000)}{54.338}+\underset{(0.000)}{0.092\varepsilon^2_{3,t-1}}+\underset{(0.000)}{0.0248\sigma^2_{3,t-1}} \quad (19.3)$$

$$\sigma^2_{4,t}=\underset{(0.000)}{56.325}+\underset{(0.000)}{0.083\varepsilon^2_{4,t-1}}+\underset{(0.000)}{0.159\sigma^2_{4,t-1}} \quad (19.4)$$

$$\rho(1,2)_{,t}=\underset{(0.000)}{0.989}*\sqrt{\sigma^2_{1,t-1}*(i)\sigma^2_{2,t-1}} \quad (19.5)$$

$$\rho(1,3)_{,t}=\underset{(0.000)}{0.979}*\sqrt{\sigma^2_{1,t-1}*(i)\sigma^2_{3,t-1}} \quad (19.6)$$

$$\rho(1,4)_{,t}=\underset{(0.000)}{0.970}*\sqrt{\sigma^2_{1,t-1}*(i)\sigma^2_{4,t-1}} \quad (19.7)$$

$$\rho(2,3)_{,t}=\underset{(0.000)}{0.935}*\sqrt{\sigma^2_{2,t-1}*(i)\sigma^2_{3,t-1}} \quad (19.8)$$

$$\rho(2,4)_{,t}=\underset{(0.000)}{0.936}*\sqrt{\sigma^2_{2,t-1}*(i)\sigma^2_{4,t-1}} \quad (19.9)$$

$$\rho(3,4)_{,t}=\underset{(0.000)}{0.915}*\sqrt{\sigma^2_{3,t-1}*(i)\sigma^2_{4,t-1}} \quad (19.10)$$

Table 6: Result of Model Selection and Heteroskedasticity Test

Parameters	**DVECH Model**	**DCC**	**MINIMUM AIC**	**Decision**	**Orthogonalization: Cholesky (Lutkepohl)**
Log likelihood	-4696.795	-3647.117			
Avg. log likelihood	-2.372119	-1.841978			248.523(0.4194)
Akaike info criterion	19.11432	14.82471	14.82471	DCC	3650.747(0.654)
Schwarz criterion	19.40312	15.01158			3899.270(0.819)
Hannan-Quinn criter.	19.22770	14.89807			

Table 7: Estimation Results for Portmanteau Tests

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	Df
1	22.93649	0.1154	23.05721	0.1122	16
2	46.25668	0.0494	46.62417	0.0458	32
3	59.27973	0.1274	59.85504	0.1171	48
4	91.70694	0.0132	92.97588	0.0105	64
5	100.8715	0.0574	102.3868	0.0466	80
6	113.1738	0.1113	115.0881	0.0896	96
7	122.4939	0.2343	124.7627	0.1931	112
8	155.4524	0.0497	159.1621	0.0322	128
9	172.4273	0.0532	176.9764	0.0321	144
10	192.1867	0.0420	197.8274	0.0225	160
11	204.9934	0.0664	211.4168	0.0352	176
12	223.8200	0.0576	231.5055	0.0271	192

Source: Researcher's Calculation using Eviews Version 10

Discussion of Results

Vector Error Correction Model

The results of vector error correction model as presented in table 5 shows that the coefficient of error correction term 29.29453 in Crude oil Price Average market was unable to fulfill the condition of negativity and significant but the ECT is negative and significant for Crude oil Price in Brent Blend (-15.445), Crude oil Price in Dubai (-7.031) and Crude oil Price in West Texas Intermediate (-8.320), which indicates that when Crude oil Price in Brent Blend, Crude oil Price in Dubai and Crude oil Price in West Texas Intermediate deviate from equilibrium level, they tend to correct at -15.445, -7.031 and -8.320 respectively back towards long run equilibrium level in the next period.

Vector Error Conditional Heteroskedasticity (VECH) GARCH

The leading diagonal estimates (0.555, 0.502, 0.565, 0.430) of ARCH coefficient matrix A measures the sensitivity of the current conditional variance of COA, COB, COD and COWTI to the squared lagged error. A larger ARCH coefficient means that past shocks have a stronger impact on current volatility. From equations 16.1, 16.2, 16.3 and 16.4 past shocks has the strongest impact on current volatility in COD, followed by COA, COB then COWTI. The GARCH estimates COA 0.268, COB 0.524, COD 0.047 and COWTI 0.376 in matrix B measures the persistence of volatility. COB has the highest volatility persistence with COD representing the lowest volatility shocks to die out. In the covariance equation, the coefficient of the ARCH terms in equations 16.1, 16.2, 16.3 and 16.4 measures the sensitivity of the current conditional covariance to the product of the lagged errors of the two assets. Equation 16.5 shows a positive long-run average covariance (12.072) of COA and COB. Past co-shocks have a small positive impact (0.528) and a moderate positive persistence in the covariance (0.377). Equation 16.6 shows a positive long-run average covariance (30.695) of COA and COD. Past co-shocks have a small positive impact (0.562) and a small positive persistence in the covariance (0.120). Equation 16.7 shows a positive long-run average covariance (19.154) of COA and COWTI, past co-shocks have a moderate impact (0.485) and a moderate persistence in covariance (0.316). Equation 16.8 shows a positive long-run average covariance (17.661) of COB and COD. Past co-shocks have a small positive impact (0.537) and a small positive persistence in the covariance (0.177). Equation 16.9 shows a positive long-run average covariance (10.715) of COB and COWTI. Past co-shocks have a small positive impact (0.460) and a moderate positive persistence in the covariance (0.439). Equation 16.10 shows a positive long-run average covariance (26.488) of COD and COWTI. Past co-shocks have a small positive impact (0.490) and a small positive persistence in the covariance (0.134). The variance-covariance matrices of the four primary crude oil benchmarks are positive integers and symmetric around the leading diagonal, these features are mathematically significant and attractive. This feature ensures a positive anticipated value at risk, independent of the weight of the asset series in the portfolio. Our findings are consistent with those of Deebom et al. (2020), who evaluated diagonal MGARCH models with conditional variance-covariance using interest and currency rates from Nigerian commercial banks. Interest rates on time deposits held by commercial banks are affected by shocks to exchange rates; both variables have a positive conditional variance; yet, the diagonal multivariate VECH GARCH model that attempts to explain the relationship between them fails to show any temporal clustering. When the estimates on the leading diagonal are positive, the elements in the variance-covariance matrix become symmetrical, then diagonal multivariate VECH is characterised as "positive semi-definite" according to Deebom et al. (2020).

Variance-Covariance Estimates of the Constant Conditional Correlation Diagonal Multivariate GARCH Model

All the values of the variance-covariance estimates are non-zero, suggesting that both markets exhibit ARCH and GARCH effects. By the 5% level of significance, all ARCH parameters estimated for the Diagonal Dynamic Conditional Correlation (DCC) (α) are robust, positive, and statistically significant. When the total of α and β is less than one, the model exhibits volatility due to the fact that conditional correlations evolve with time. Market returns and crude oil prices exhibit time-dependent oscillations, according to the Diagonal Conditional Correlation (DCC) test. From equations 19.5, 19.6, 19.7, 19.8, 19.9 and 19.10, the positive and highly significant conditional correlation (0.989), between crude oil price Average and Brent, crude oil price Average and Dubai (0.979), crude oil price Average and West Texas Intermediate (0.970), Brent and Dubai (0.935), Brent and West Texas Intermediate (0.936), and Dubai and West Texas Intermediate (0.915) reflect the presence of strong direct interconnections between the returns on crude oil price markets. The coefficient of correlation ranges from 0.915 to 0.989. Positive association between these variables show evidence of increase in volatility with high degree of correlation during the period under investigation. These findings agree with Kanchan et al., (2017), on volatility spillover using multivariate GARCH Model and the application in futures and spot market price of black pepper. In Kanchan et al., (2017), it was found that there is a dynamic and time varying conditional correlation between spot and futures market reflecting significant volatility spillover during the period of the year 2012. Estimates presented in table 6 reveal that the Diagonal Conditional Correlation (DCC) model had the smallest Akaike Information Criteria of 14.82471 as against 19.11432 for MVECH model. This confirms that the DCC model better captures volatility of crude oil benchmark than the DVECH model. According to the model diagnostics, the DCC-GARCH model fared better than the DVECH model. This criterion was satisfied by the portmanteau test in table 7 when out of the twelve lags tested eight of the lags were statistically significant.

Conclusion

Since the coefficients of error correction term in the model using Crude Oil Price Average market as the dependent variable, was positive and insignificant at the 5% level of significance essentially means that changes in the average crude oil price benchmark do not have significant association with the other variables in question over a long term. This could suggest that other variables are having a greater influence on COB, COD and COWTI. Furthermore, the tendency for crude oil prices to revert to back to their equilibrium over time is not feasible because of its explosiveness. Also, crude oil Brent price benchmark (15.445) has the highest speed of reverting to stability in the long term after a shock, followed by crude oil West Texas Intermediate price benchmark (8.320) and then crude oil Dubai price benchmark (7.031). Diagonal multivariate VECH model exhibit "positive semidefinite" property as the leading diagonal of the variance-covariance matrix has positive estimates, indicating that the variables move together and irrespective of the weight of all the crude oil price return the Value at Risk (VaR) will be positive. Also, there is evidence of positive and strong correlation between these crude oil markets, this simply means that regardless of what the markets might experience within the period under investigations volatility persist overtime. Each asset in the portfolio exhibits time-varying volatility as captured by the significant ARCH and GARCH coefficients. The constant conditional correlation between COA and COB, COD, COWTI are very high (around 0.97 to 0.99), this suggests a strong tendency for these assets to move in the same direction. The diagonal conditional correlation model is identified as the best model for modeling volatility of crude oil price benchmark when compared to multivariate VECH. This conclusion is based on its lower Akaike Information Criterion (AIC). The findings provide credence to the idea that residual autocorrelations do not exist in the chosen model, indicating a good fit and valid statistical properties.

Recommendations

i. Hedging and sharing of information are essential in the crude oil market because of its inherent volatility.

ii. To better manage risk and rebalance their portfolios, marketers and investors should benefit from a better understanding of the link between crude oil prices.

iii. The most effective MGARCH model for crude oil markets within the scope of the COA, COB, COD and WTI is the DCC-GARCH.

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