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Modelling the Interconnectedness and Lag Effects in Crude Oil Price Benchmarks

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Abstract
This study focuses on analyzing the dynamic interaction of crude oil price benchmarks with emphasis on investigating the short and long-term impact of changes in the crude oil price benchmark and select an appropriate model for modeling the interaction of crude oil Brent, crude oil Dubai and crude oil West Texas Intermediate oil price benchmarks. To achieve this, the Autoregressive Distributed Lag (ARDL) model with monthly data from May, 1994 to March 2024 was introduced. Preliminary investigation such as unit root test, ARDL bound test together with ARDL model estimation were conducted on the study variables. The result confirms strong evidence of time-varying interdependence between COB, COD and COWTI. COWTI is negatively related to its second and third lags. This means that past innovations in COB and COD have great influence on present changes in COWTI. The first lag of COD is also negatively correlated with COWTI. However, COB and COD influence COWTI in a positive way. In the long run there is no interconnection between COB, COD and COWTI. More so, the combine lags of COWTI (-1), COWTI (-2) and COWTI (-3) significantly caused own shocks in the short-run. Since own shocks have been identified as determinants of impulse. Some recommendations were made.

Keywords: Modeling, Interconnectedness, Benchmark, Lag

Introduction

Crude oil has become a major source of energy for all economic activity as it provides for jet oil, kerosene, premium motor spirit (PMS), asphalt etc. It is arguably the most important driver of the global economy over the past decade and changes in the price of crude oil will have significant impact on the economy of most countries like Nigeria. According to Musa et al. (2019), crude oil is a major source of foreign exchange earnings and the dominant source of revenue for the Nigerian economy. The Nigerian economy has been completely reliant on oil and the basis upon which government budgeting, revenue distribution and capital allocations are determined. However, Musa et al. (2019) emphasize that the recent shock in crude oil price has adversely affected the country's foreign exchange earnings, foreign reserves, decline in government revenue and inability to meet financial obligation as at when due. The crude oil benchmark is a standard pricing or ceiling for the sale and purchase of crude oil. There are several crude oil benchmarks but for the purpose of this research, emphasis will be on crude oil Brent (COB), crude oil Dubai (COD), crude oil West Texas Intermediate (COWTI). These benchmarks serve as a reference point for buyers and sellers, influencing the prices of various crude oil types across the globe. Brent crude, sourced primarily from the North Sea is a global benchmark heavily influencing oil prices in Europe and Asia. Whereas, West Texas Intermediate (WTI) extracted from the Permian Basin in Texas, is the primary benchmark for crude oil in the United States and a significant global indicator. The Dubai crude is a medium sour crude oil extracted from Dubai and it is used as an oil marker in the Persian Gulf. Most of the literature has argued that the Brent, Dubai and WTI oil benchmarks have historically moved in tandem. But according to Caporin et al. (2019) this equilibrium relationship has been broken in the early 2011. Since then the price trend has reversed and the WTI has started to be exchanged for a larger discount at Brent and Dubai. This price decoupling could be an indication that the two markets are not fully integrated at all times. Therefore, the proximity of prices and their convergence to their long term pattern is always associated with globalized market. Since the dynamic interplay or interaction between these variables can have significant impact on each other it is important to understand pattern, direction and magnitude of adjustment in the case of disequilibrium. The autoregressive distributed lag (ARDL) is a popular econometric technique introduced by Pesaran and Shin (1999), that allows for the examination of both short-term and long-term relationships between variables (Dinh, 2022). By applying the autoregressive distributed lag (ARDL) model to the crude oil price benchmark data, countries like Nigeria that trade crude oil with Broom, Rannoch, Etive, Ness and Tarbert (BRENT) benchmark can better understand the causation and interaction and make informed predictions about the correlation or effect of the crude oil benchmarks on each other in the short and long run. Several studies have tried to model and analyze the impact of these variables on each other, among them are;

Alabi and Ojo (2024) studied a comparative analysis of extreme gradient boosting and support vector regression for modeling benchmark crude oil price. This article compares two machine learning models, Support Vector Regression (SVR) and Extreme Gradient Boosting (XGBoost) for predicting crude oil price across different benchmarks, including OPEC reference basket (ORB), NYMEX WTI, and ICE Brent. Finding reveal that while SVR is suited for stable pricing environment, XGBoost is better equipped to handle the unpredictability of more volatile markets. Ejukwa and Tuaneh (2025) examine volatility contagion in the crude oil market by adopting four primary crude oil benchmarks. The diagonal BEKK model along with the constant conditional correlation model was deployed to analyze the effect of returns and volatility spillover between benchmark crude. Findings show that historical conditional volatility and squared errors had impact on the conditional variance of the four main crude oil benchmarks. Ejukwa et al. (2023)1 investigated the relationship between gross domestic product, motor vehicle assembly and oil refining in Nigeria. The autoregressive distributed lag error correction model was introduced to evaluate the relationship that exist among the study variables. Annual data from 1981 to 2022 was used. The estimates from the error correction model analysis infer that oil refining had a strong effect on gross domestic product and the combined lags of oil refining and motor vehicle assembly cause its own shocks.

Ejukwa et al. (2023) examine trend and causal relationship between gross domestic product, agriculture and industry interaction in Nigeria over the period 1981 to 2022. The data for the study is from National Bureau of Statistics. The autoregressive distributed lag(ARDL) model was used to analyze the data and from the result obtained agriculture and industry contributed significantly to gross domestic product growth in Nigeria in the short term. While in the long run, gross domestic product has significant influence on agriculture. However, most of these studies reviewed could not address the dynamic interaction of COA, COB, COD and COWTI benchmarks. This study therefore focusses on the relationship between crude oil Brent (COB), crude oil Dubai (COD), and crude oil West Texas Intermediate (COWTI) and examines the impact of changes in one variable on the other. This research has the potential to provide valuable insights for policymakers, government, researchers, economists and investors in and out of Nigeria. The study will also serve as a useful guide to the marketers and the agricultural institutes in regulating the prices of agricultural commodities in Nigeria. This research work primarily covers the causality between crude oil Brent, crude oil Dubai and crude oil West Texas Intermediate using Autoregressive Distributed Lag (ARDL) Model. The time frame is from May, 1994 to March 2024, which gives a total data point of 359.

Statement of the Problem

Crude oil price volatility has significant economic and political implications, affecting global markets, energy security, and geographical stability. Understanding the factors driving these fluctuations is crucial for policy makers, investors and businesses. While numerous studies have examined crude oil price determinants, many rely on traditional econometric models with limitations in capturing complex dynamics. Autoregressive Distributed Lag (ARDL) model offers a powerful tool to analyze the long run relationship between crude oil price benchmarks. However, the choice of benchmark crude oil price significantly impacts the model's results and interpretations. Different benchmarks, such as Brent, Average, West Texas Intermediate and Dubai exhibit varying price sensitivities to different factors due to regional differences in supply, demand and geographical risks. The problem lies in determining the most appropriate benchmark crude oil price for ARDL modeling in a specific context. This requires careful consideration of the research objectives, the geographical scope of the analysis and the availability and reliability of data for different benchmarks. An incorrect choice of benchmark can lead to biased estimates, misleading conclusions, and ineffective policy recommendations. Furthermore, the dynamics of crude oil price benchmarks are subject to continuous shift due to evolving market conditions, geographical events, and technological advancements. Therefore, the choice of benchmark and the ARDL specification need to be regularly reviewed and updated to ensure their relevance and accuracy. Existing studies on modeling crude oil price benchmark in Nigeria have limitations, including lack of consideration for long-term relationship between the various crude oil price benchmarks and its determinants. This study aims to fill this gap by using the Autoregressive Distributed Lag (ARDL) model to investigate the interaction of the various crude oil price benchmarks considered in this study.

Aim and Objectives

The aim of the study is to analyze the dynamic interaction of crude oil price benchmark using autoregressive distributed lag (ARDL) model. The objectives include to;

(i) determine trend in the evolution of crude oil Brent, crude oil Dubai and crude oil West Texas Intermediate benchmarks.
(ii) model and estimate the interaction among crude oil Brent, crude oil Dubai and crude oil West Texas Intermediate and determine the direction and magnitude of the speed of adjustment between study variables.
(iii) analyze the short and long-term impact of changes in the crude oil price benchmark.
(iv) select an appropriate model for modeling the interaction of crude oil Brent, crude oil Dubai and crude oil West Texas Intermediate oil price benchmarks.

Materials and Methods

The emphasis of this section is focused mainly on the techniques and approaches employed in arriving at the results and they include the following: Research design, ARDL model specification, model identification, model estimation and diagnostic check.

Research Design

According to Kerlinger (1986), research design is the plan, structure and strategy of investigation conceived so as to obtain answers to research questions and to control variance. The ex-post factor research design was utilized in this study as the researcher cannot influence the outcome of the data analyze in the study. He further describes an ex-post factor research design, otherwise referred to as causal comparative research design, as a type of research design used when the researcher intends to determine cause and effect relationship between a dependent variable and an independent variable in a way to establish a link in their relationship.

Source of Data/Software Used for Data Analysis

The data used for this study was extracted from the National Bureau of statistics data base. The variables used was monthly data on crude oil Price benchmarks within the period May, 1994 to March 2024. The software used for the data analysis is Eview version 10 released by IHS Markit (2020).

Method of Data Analysis

The study relied on Autoregressive Distributed Lag (ARDL) and Error Correction Model (ECM) techniques to achieve its goals. Co-integrated vector autoregressive models of first difference, and mixed order of integration are the criteria to which these terms pertain. We first checked for consistency in the series' means and variances changing over time using a unit root test and an autoregressive distributed lag (ARDL) bounds test. Also, we checked to see whether the variables under study, both internal and external, are in an equilibrium condition over the long run.

Error Correction Model

To understand how a time series variable gets closer to equilibrium is by using the error correction model to account for times when it was out of whack. Stock (1987), showed that the error correcting system consistently and efficiently estimates disequilibrium in the system as shown in equation one. If the vector autoregressive variables show integration of order one or higher, predicting unconstrained parameters in a regression model with nonstationary variables might be challenging. Ankargren and Lyhagen (2018) found that when it comes to economic time series modelling, the Error Correction Model (ECM) is the most successful strategy.

$$\Delta COB_t = \sigma_1 + \sum_{t-1}^{k} \varphi_1 \, \Delta COB_{t-1} + \sum_{t-1}^{k} \theta_1 \, \Delta COD_{t-1} + \sum_{t-1}^{k} \vartheta_1 \, \Delta COWTI_{t-1} + \lambda_1 ECT_{t-1} + \pi_{1,t} \quad (1)$$

Where ECT_{t-1} is the error correction mechanism, $\lambda_1$is the coefficient of the error correction term called speed of adjustment to stability.

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Pre-EstimationTest

Ejukwa et al. (2023)1 emphasize that it is more reasonable to conduct a pre-estimation test by asking questions about the traits and actions of the study variables. The time plot, summary statistics, unit root and ARDL bounds tests were the first steps in the investigation portfolio. Pre-estimation tests are essential in time series analysis as they enable researchers to assess model adequacy, identify data issues, guide model selection, improve forecast accuracy and enhance the robustness and reliability of the analysis results. By incorporating pre-estimation tests into the modeling process, analyst can make informed decision and produce more accurate and reliable forecast base on the time series data in question.

Graphical Evolution of the Variables

A time plot is a graphical representation of data collected at various point in time. The x-axis of the plot represents the time period, while the y-axis represents data points collected. This type of plot helps to visualize trends and patterns in data over time, making it easier to identify any seasonal variation, trends or outliers.

Unit Root Test

One of the challenges of time series data is the problem of unit root. However, the study adopted the Augmented Dickey-Fuller and Phillip-Perron unit root test. The null hypothesis is that there is presence of unit root in the variables. Whereas, the alternative hypothesis is that there is no unit root. The null hypothesis is rejected if the ADF test statistic is greater than the critical value at five percent level of significance. Otherwise, the null hypothesis is accepted against the alternative hypothesis. If the null hypothesis is rejected, it implies that there is no unit root and the series has zero mean and constant variance over time. Assuming a series moves randomly is the premise of the unit root test.

$$Y_t = b_1 y_{t-1} + \varepsilon_t \qquad \text{Random walk} \qquad (2)$$

$$Y_t = b_0 + b_1 y_{t-1} + \varepsilon_t \qquad \text{Random walk with drift} \qquad (3)$$

$$Y_t = b_0 + b_1 y_{t-1} + b_2 t + \varepsilon_t \qquad \text{Random walk with drift and trend} \qquad (4)$$

The ARDL Bounds Test

In order to empirically analyze the short run relationship and long run dynamics among crude oil Brent (COB), crude oil Dubai (COD) and crude oil West Texas Intermediate (COWTI) benchmark, the autoregressive Distributed Lag (ARDL) co-integration technique of Pesaran and Shin (1999) will be utilized. Comparatively, the ARDL is relatively more efficient because it does not necessarily need all the variables under investigation to be integrated of the same order. More so, it is more efficient for small and finite sample data size (Harris & Sollis, 2003). The ARDL bounds test is based on the assumption that the variables are I(0) and I(1) or I(1) only using. The hypothesis for ARDL bounds test for the joint significance of the coefficient of the lagged levels of the variable are;

H_o: $\lambda_1 = \lambda_2 = \lambda_3 = 0$ (No long run relationship exist)

H_1: $\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq 0$ (Long run relationship exist)

There are several selection criterions like the Hannan Quinn, Schwarz information criteria, Akaike information criteria etc. For the purpose of this study, the Akaike information criteria (AIC) is used to select the order of the ARDL (p, q_1, q_2) model once co-integration is established

Lag Length Specification

The lag length order describes the number of previous time periods used to forecast future values or correlation of a time series. The lag length order is determine by the auto-correlation function (ACF) and partial auto-correlation function (PACF). The ACF measures the correlation between a data point and its lagged values at different time lags, while the PACF measures the correlation between a data point and its lagged value after controlling for the effect of intermediate lags. Selecting the appropriate lag length order is crucial as using too few or too many lags can lead to inaccurate forecasts and unreliable results. Statistical criteria such as the Akaike information criteria (AIC) or Bayesian information criteria (BIC) is employed to determine the optimal lag order. Given the time variable, $Y_1, Y_2, \ldots, Y_n$ at time $X_1, X_2, \ldots, X_n$, the k lag autocorrelation function is defined as

$$\rho_k = \frac{\sum_{t=k+1}^{n}(Y_t - \bar{Y})\,(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2} \qquad (5)$$

where,

ρ_k – Autocorrelation at lag k
Y_t - Value of the series at time t
$\bar{Y}$ - Mean of the series
n - Number of observations
For the partial autocorrelation function (PACF), let X_t be a stationary time series with $EX_t = 0$. The PACF is defined as $\pi_1 = Corr(X_1, X_2) = \rho_1$ (6)

ARDL Models Specification

In specifying the parameters of the autoregressive distributed Lag Model (ARDL), two dimensions will be examined, the short and long run relationship between the vectors. When the I(1) variables have one or more co-integrating vectors, it is safe to say that the appropriate approach for estimation is the autoregressive distributed lag model. This will provide a basis for evaluating the short run properties of the co-integrating series as well as the long run dynamics, more so, the short run changes and deviations from equilibrium is adjusted in the error correction model (ECM).
The basic form of the ARDL model for the three variables in its first lag with error correction mechanism is prescribed thus:

$$\Delta COB_t = \sigma_1 + \sum_{t-1}^{k} \varphi_1 \, \Delta COB_{t-1} + \sum_{t-1}^{k} \theta_1 \, \Delta COD_{t-1} + \sum_{t-1}^{k} \vartheta_1 \, \Delta COWTI_{t-1} + \lambda_1 ECT_{t-1} + \pi_{1,t} \quad (7)$$

$$\Delta COD_t = \sigma_2 + \sum_{t-1}^{k} \varphi_2 \, \Delta COD_{t-1} + \sum_{t-1}^{k} \theta_2 \, \Delta COB_{t-1} + \sum_{t-1}^{k} \vartheta_2 \, \Delta COWTI_{t-1} + \lambda_1 ECT_{t-1} + \pi_{2,t} \quad (8)$$

$$\Delta COWTI_t = \sigma_3 + \sum_{t-1}^{k} \varphi_3 \, \Delta COWTI_{t-1} + \sum_{t-1}^{k} \theta_3 \, \Delta COD_{t-1} + \sum_{t-1}^{k} \vartheta_3 \, \Delta COB_{t-1} + \lambda_1 ECT_{t-1} + \pi_{3,t} \quad (9)$$

where:

COB - Crude oil Brent
COD - Crude oil Dubai
COWTI - Crude oil West Texas Intermediate
λ - Speed of Adjustment
ECT - Error Correction Term
π - Impulse
P - Maximum Lag Length
σ - Intercept
Σ - Summation
Δ - Difference Operator
$\varphi, \theta, \vartheta$ - Short run coefficients

Post Estimation Test

Post estimation diagnosis tells us about the robustness of the estimated coefficients. The type of diagnostic test depends on the modelling technique employed. However, the commonly used diagnostic test are residual diagnostic, Stability diagnostic and the lag structure. This study will utilize the Breusch-Godfrey LM test and CUSUM stability test since the regression models try to minimize residuals which are independently and identically distributed (i.id). Shrestha (2017), emphasized that the stability diagnostic examines the stability of the estimated parameters across various sub-samples of the data.

Results

The various preliminary test conducted in this section are the descriptive statistics, time plot, unit root test, bounds test for co-integration using the F-statistic and the t-statistic, lag order selection. This test is a necessary pre-requisite for parameter estimation to be conducted.

Descriptive Statistics

Table 1: Summary Statistics for the Raw Data on COB, COD and COWTI

	COB	COD	COWTI
Mean	55.00482	52.81838	52.95797
Median	51.97000	50.43000	49.83000
Maximum	133.8700	131.2200	133.9300
Minimum	9.800000	10.05000	11.31000
Std. Dev.	32.91104	32.19072	29.33917
Skewness	0.466342	0.462097	0.429726
Kurtosis	2.070809	2.035781	2.122131
Jarque-Bera	25.92721	26.68344	22.57678
Probability	0.000002	0.000002	0.000013
Sum	19746.73	18961.80	19011.91
Sum Sq. Dev.	387762.9	370974.7	308161.8
Observations	359	359	359

Table 1 is the summary statistics for crude oil Brent, crude oil Dubai and crude oil West Texas Intermediate. It outlines measures that can describe COB, COD and COWTI by utilizing the mean, median, mode, skewness, kurtosis and Jaque-Bera statistic for normality.

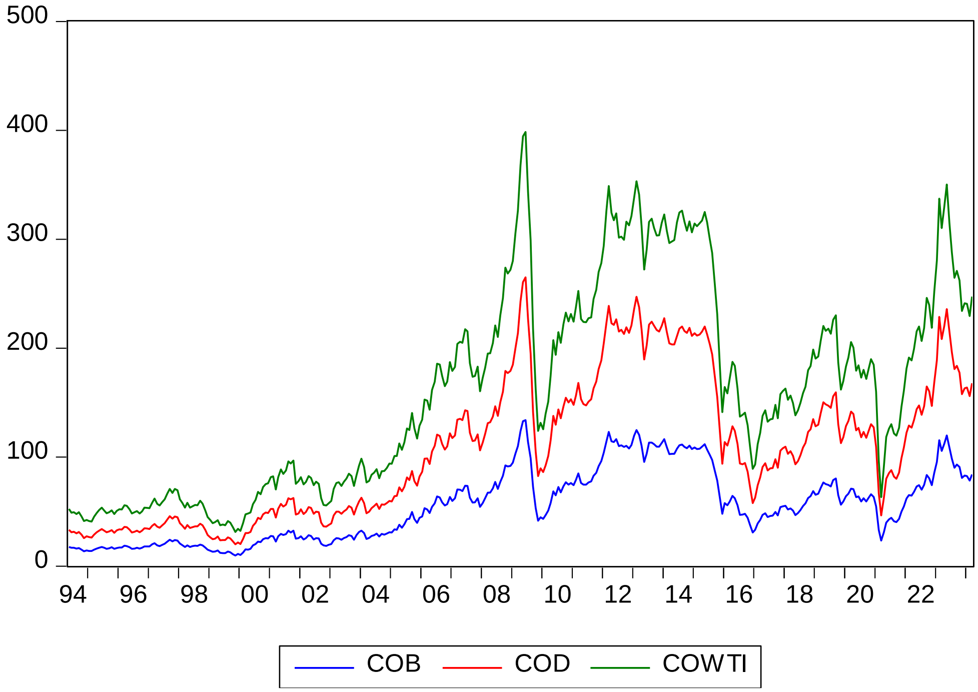

Figure 1: Graph of the raw Crude Oil Brent, Crude Oil Dubai and Crude Oil WTI

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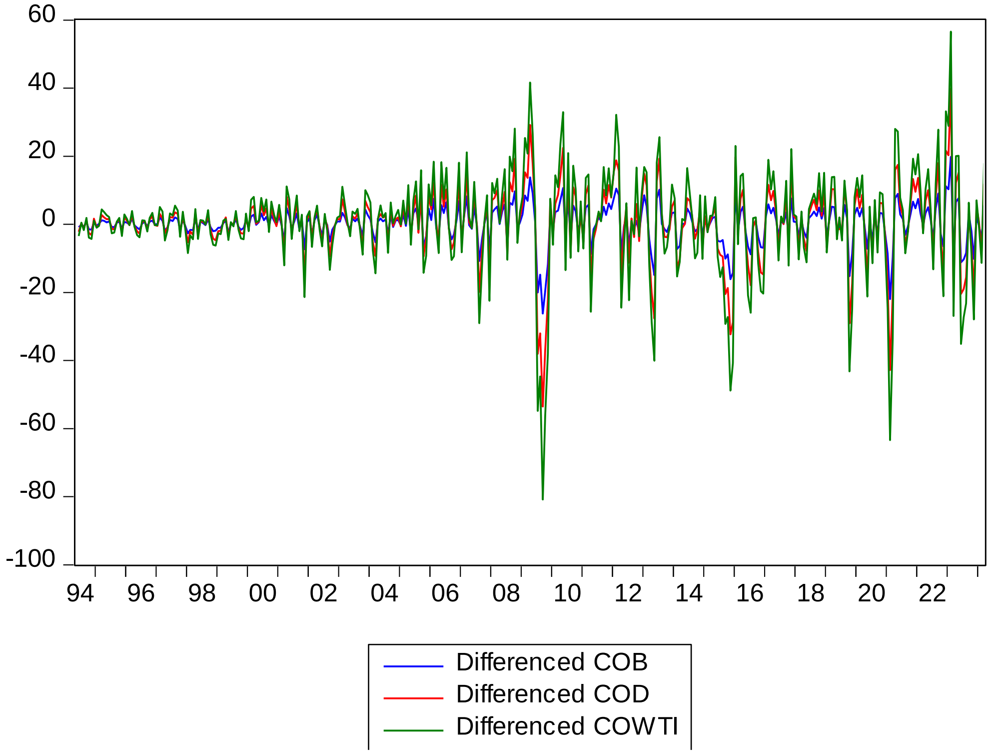

Figure 2: Graph of the Differenced Crude Oil Brent, Crude Oil Dubai and Crude Oil WTI

Figures 1 and 2 are the plots for the raw and differenced series on COB, COD and COWTI respectively. The crude oil price benchmark is on the vertical axis and the time (months) is on the horizontal axis. This helps to visually identify trend in the variable's characteristics.

Unit Root Test

Table 2: Results of Unit Root Test

	ADFT			PPT		
Variable	Level	1st Diff	C.V	Level	1st Diff	C.V
COB	-2.23633 (0.1939)	-13.13699 (0.0000)	-2.869419	-1.933466 (0.3167)	-13.01697 (0.0000)	-2.869396
COD	-2.263152 (0.1847)	-8.790247 (0.0000)	-2.869396	-1.785089 (0.3877)	-11.93282 (0.0000)	-2.869419
COWTI	-2.442208 (0.1309)	-8.998032 (0.0000)	-2.869534	-1.992970 (0.2900)	-13.08094 (0.0000)	-2.869419

Source: Researcher's computation. P-value in bracket ( )

Table 2 show the result of unit root test for the raw price series on COB, COD and COWTI. The result show that the COB, COD and COWTI are all integrated at order one, I(1). This fulfils the condition for the use of autoregressive distributed lag (ARDL) model.

AKaike Information Criteria Graphs

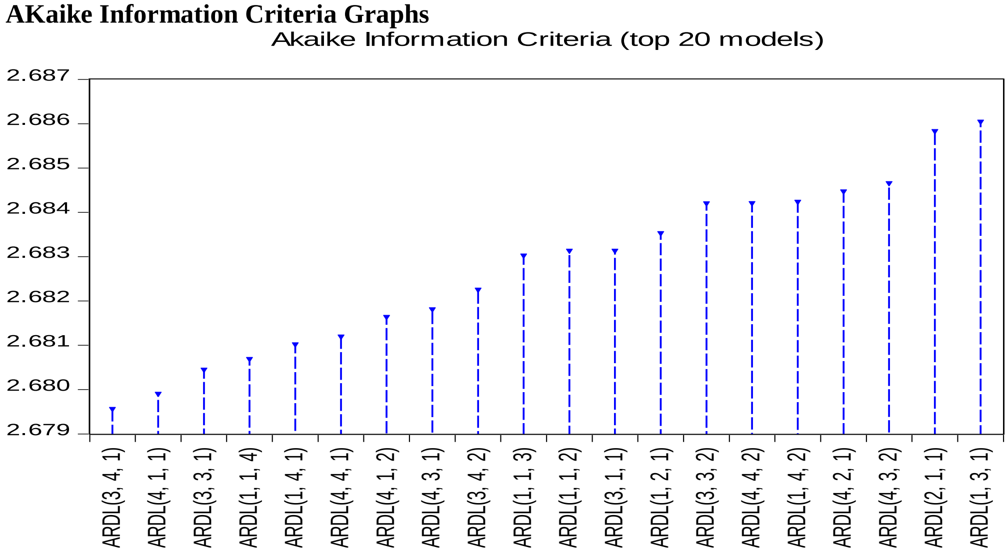

Figure 3: Akaike Information Criteria Graph for COB

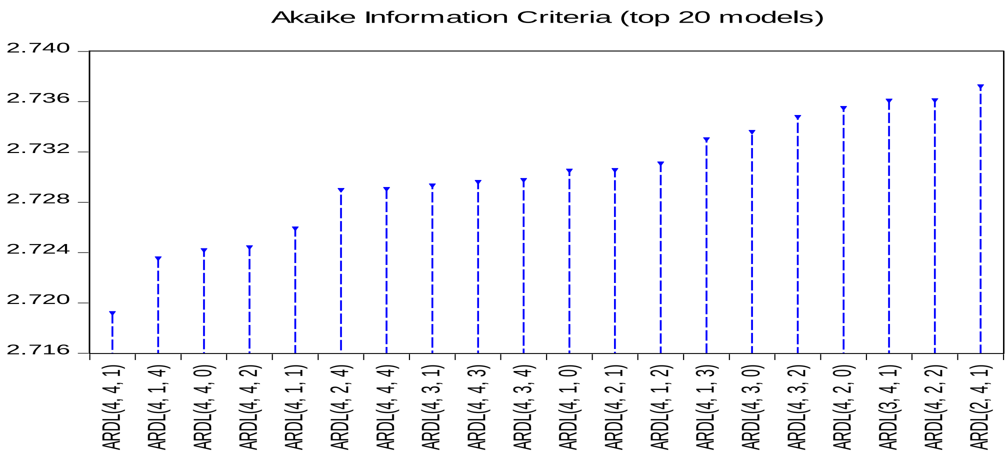

Figure 4: Akaike Information Criteria Graph for COD

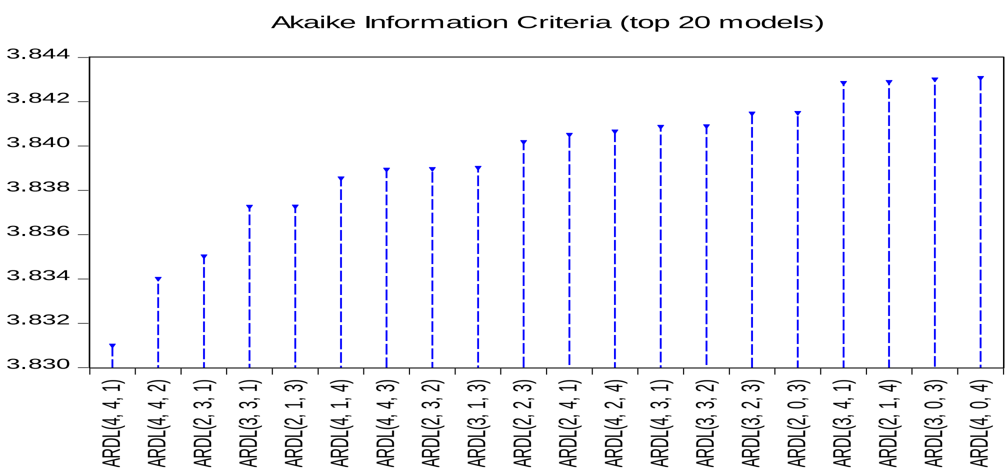

Figure 5: Akaike Information Criteria Graph for COWTI

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Figures 3, 4 and 5 are graphs of the Akaike Information Criteria revealing a visual image of the chosen model.

Table 3: Results of Bounds Test for Cointegration Using ARDL (3,4,1), (4,4,1) and (4,4,1)

Variable	Test Statistic	Value	Signf. level	I(0)	I(1)
COB	F-Statistic	12.68057	5%	4.87	5.85
COD	F-Statistic	7.301270	5%	3.79	4.85
COWTI	F-Statistic	5.446385	5%	3.79	4.85

Table 3 is the results of the Autoregressive Distributed Lag test for cointegration to establish the presence or absence of long term relationship using the F-Stististic of the upper and lower bounds.

Result for Model Estimation

Table 4: Result of Error Correction Coefficient Using ARDL (3,4,1) Model
Dependent Variable: COB

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.040075	0.049111	0.816007	0.4151
D(COB(-1))	-0.006174	0.048465	-0.127397	0.8987
D(COB(-2))	-0.101260	0.048135	-2.103647	0.0361
D(COD)	0.822080	0.027511	29.88167	0.0000
D(COD(-1))	-0.015448	0.051947	-0.297371	0.7664
D(COD(-2))	0.087217	0.051645	1.688776	0.0922
D(COD(-3))	0.016506	0.010783	1.530775	0.1267
D(COWTI)	0.238843	0.026013	9.181582	0.0000
CointEq(-1)*	-0.185396	0.036291	-5.108566	0.0000
R-squared	0.971303	Mean dependent var		0.191859
Adjusted R-squared	0.970640	S.D. dependent var		5.294941
S.E. of regression	0.907279	Akaike info criterion		2.668291
Sum squared resid	284.8116	Schwarz criterion		2.766458
Log likelihood	-464.6217	Hannan-Quinn criter.		2.707345
F-statistic	1463.891	Durbin-Watson stat		1.965147
Prob(F-statistic)	0.000000			

Table 5: Result of Long Run Estimate for ARDL (3,4,1) Model. Dependent Variable: COB

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COD	0.886805	0.049864	17.78439	0.0000
COWTI	0.158931	0.053185	2.988247	0.0030

EC = COB - (0.8868*COD + 0.1589*COWTI)

Table 6: Result of Error Correction Coefficient Using ARDL (4,1,1) Model
Dependent Variable: COD

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.162444	0.062523	-2.598144	0.0098
D(COD(-1))	0.024319	0.055130	0.441123	0.6594
D(COD(-2))	-0.098958	0.053534	-1.848496	0.0654
D(COD(-3))	-0.145554	0.051586	-2.821584	0.0051
D(COB)	0.858567	0.028887	29.72157	0.0000
D(COB(-1))	0.023655	0.052172	0.453397	0.6505
D(COB(-2))	0.096513	0.050091	1.926754	0.0548
D(COB(-3))	0.115318	0.049046	2.351211	0.0193
D(COWTI)	0.051545	0.029333	1.757240	0.0798
CointEq(-1)*	-0.163277	0.034786	-4.693776	0.0000
R-squared	0.966712	Mean dependent var		0.196141
Adjusted R-squared	0.965843	S.D. dependent var		5.000510
S.E. of regression	0.924169	Akaike info criterion		2.707921
Sum squared resid	294.6603	Schwarz criterion		2.816994
Log likelihood	-470.6559	Hannan-Quinn criter.		2.751313
F-statistic	1113.227	Durbin-Watson stat		2.013096
Prob(F-statistic)	0.000000			

Table 7: Result of Long Run Estimate for ARDL (4,4,1) Model. Depedent Variable: COD

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COB	1.015203	0.068402	14.84180	0.0000
COWTI	-0.036010	0.077347	-0.465568	0.6418

EC = COD - (1.0152*COB -0.0360*COWTI)

Table 8: Result of Error Correction Coefficient Using ARDL (4,1,1) Model
Dependent Variable: COWTI

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.472500	0.147590	3.201433	0.0015
D(COWTI(-1))	0.114363	0.047342	2.415698	0.0162
D(COWTI(-2))	-0.044254	0.047062	-0.940338	0.3477
D(COWTI(-3))	-0.117483	0.047543	-2.471106	0.0140
D(COD)	0.182597	0.088420	2.065101	0.0397
D(COD(-1))	-0.127536	0.049761	-2.562967	0.0108
D(COD(-2))	0.106841	0.049860	2.142829	0.0328
D(COD(-3))	0.116177	0.049876	2.329318	0.0204
D(COB)	0.752700	0.081689	9.214252	0.0000
CointEq(-1)*	-0.090484	0.022320	-4.053940	0.0001
R-squared	0.905422	Mean dependent var		0.174479
Adjusted R-squared	0.902955	S.D. dependent var		5.172376
S.E. of regression	1.611299	Akaike info criterion		3.819723
Sum squared resid	895.7182	Schwarz criterion		3.928797
Log likelihood	-668.0009	Hannan-Quinn criter.		3.863116
F-statistic	366.9778	Durbin-Watson stat		1.975455
Prob(F-statistic)	0.000000			

Table 9: Result of Long Run Estimate for ARDL (4,4,1) Model. Dependent Variable: COWTI

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COD	-0.242807	0.623626	-0.389347	0.6973
COB	1.098135	0.610463	1.798856	0.0729

EC = COWTI - (-0.2428*COD + 1.0981*COB)

Tables 4, 6 and 8 are results of the estimated short run (Error correction) coefficients. While tables 5, 7 and 9 are the long run dynamics of ARDL (3,4,1), ARDL (4,1,1) and ARDL (4,1,1) with COB, COD and COWTI as endogenous variables respectively. They allow the understanding of how variables influence each other in both immediate and sustained relationship.

Results for Post Estimation Test

Table 10: Results of Breusch-Godfrey Serial Correlation LM Test for ARDL (3,4,1), (4,1,1) and (4,1,1) Models.

Model	ARDL (3,4,1) Dep. Variable COB	ARDL (4,1,1) Dep. Variable COB	ARDL (4,1.1) Dep. Variable COWTI
F-statistic	0.923303	1.620225	0.647807
Obs*R-square	1.906505	3.341734	1.343702
Prob. F(2,341)	0.3982	0.1994	0.5238
Prob. Chi-Square(2)	0.3855	0.1881	0.5108

The Breusch-Godfrey serial correlation LM test is used to assess the validity of some of the modeling assumptions inherent in applying regression-like models to observed data.

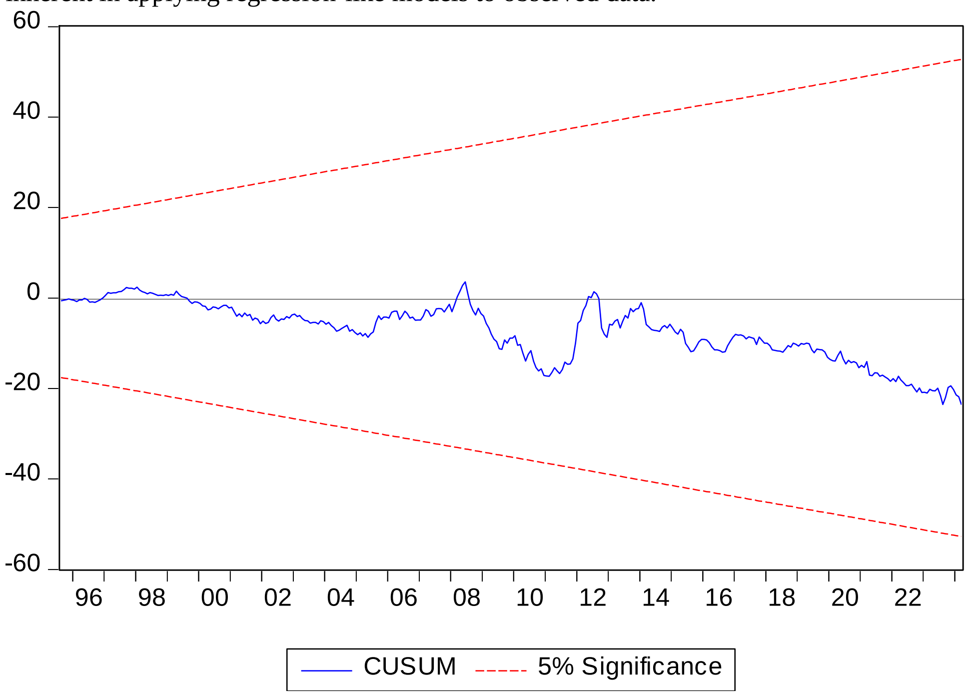

Figure 6: Plot of Cumulative Sum of Residuals, ARDL (3,4,1) Model. Regressand: COB

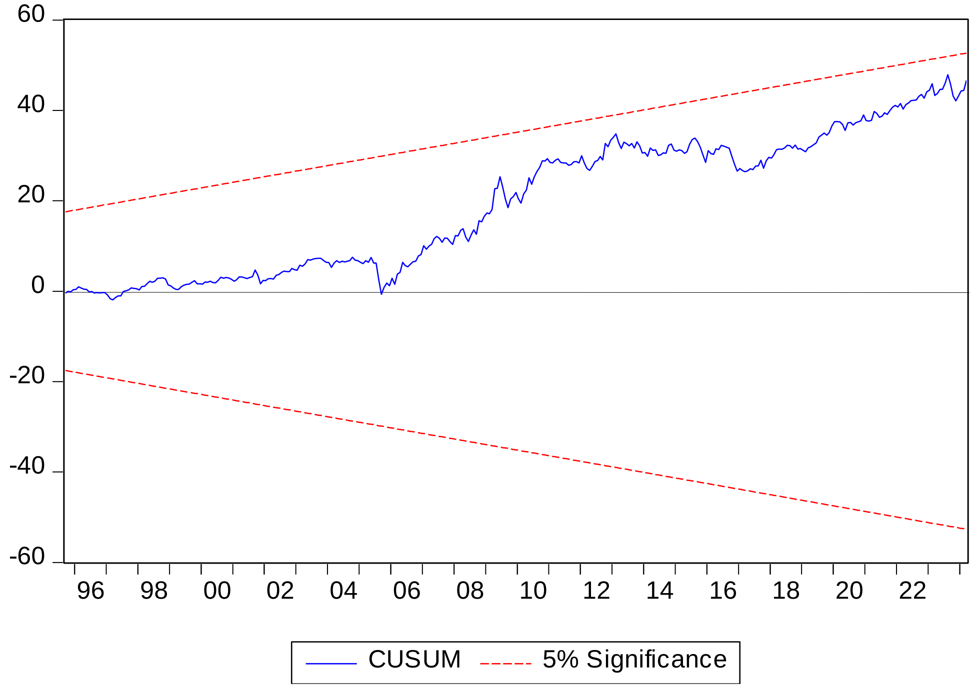

Figure 7: Plot of Cumulative Sum of Rediduals, ARDL (4,4,1) Model. Regressand: COD

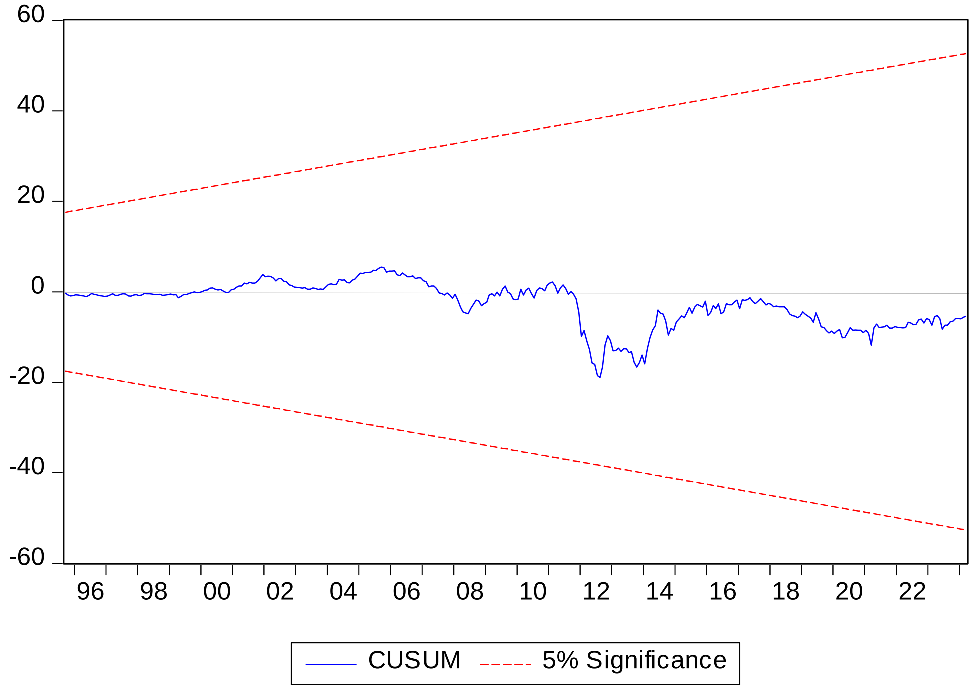

Figure 8: Plot of Cumulative Sum of Residuals, ARDL (4,4,1) Model. Regressand: COWTI

Figures 6, 7 and 8 is to test for stability in the selected models by detecting sudden changes from the constancy of the regression coefficients.

Discussion
This section deals with the discussion of the results estimated in section three.

Summary Statistics for the Raw COB, COD, COWTI
Table 1 shows the summary statistics of Crude Oil Brent, Crude Oil Dubai and Crude Oil West Texas Intermediate. The result show that the average value for all the variables of COB is 55.00482, COD is 52.81838 and COWTI is 52.95797. Also, the maximum value for COB is 133.8700, for COD is 131.2200 and for COWTI is 133.9300, while the minimum values for COB, COD and COWTI are 9.8000, 10.05000 and 11.31000 respectively. The standard deviation simply refers to the deviation from the sample mean and for COB, COD and COWTI the summary statistics records a value of 32.91104, 32.19072 and 29.33917 respectively. For the kurtosis and skewness, COB, COD and COWTI all mirrors positive skewness and platykurtic with a value of 2.070819, 2.035781 and 2.122131 less than 3 respectively.
The Jarque-Bera statistics measures the difference of the skewness and Kurtosis of the series with those from the normal distribution. The null hypothesis for the Jarque–Bera test is that the distribution is normal and from the result, COB assumes a probability value of 0.000002 < 0.05 as a result, we accept the null hypothesis which implies that COB is normally distributed. COD is normally distributed since the probability value of 0.000002 < 0.05 and COWTI is also normally distributed because the Probability value 0.000013 < 0.05.

Time Plot for Trend on Raw COB, COD and COWTI
Figure 1 Shows the time plot for the raw data on crude oil Brent (COB) and the result reveals an upward evolution of COB with huge fluctuations 2010, 2017 and 2021 which is an indication that COB is unstable and characterized by trend.
Figure 2 is the time plot for the raw data on crude oil Dubai (COD) and the result indicates an observable upward trend between 1995 to 2006 for COD and then a steep fluctuation in 2009, 2017 and 2021. This shows that COD price benchmark fluctuates across the period.
Figure 3 is the time plot for the raw data on crude oil West Texas Intermediate (COWTI) and the result largely displayed an upward trend followed by a steady and rapid fluctuation in 2009, 2017 and 2020 indicating COWTI price fluctuation. Also, figures 4, 5 and 6 are all plots of the difference crude oil price Brent, crude oil price Dubai and crude oil price West Texas Intermediate which have been stabilized to picture a series with zero mean and constant variance in the sense of stationarity. The behavior of the crude oil prices in figures 4, 5 and 6 shows stationarity as COB, COD and COWTI all revolve around the origin with evidence of volatility clustering.

Unit Root Test
The Augmented Dickey-Fuller and the Phillips-Perron Unit root tests were conducted on the study variables to test for the presence or absence of unit root to avoid the problem of spurious regression (Ejukwa et al., 2023)1. The result of the Augmented Dickey-Fuller and Phillip-Peron unit root test in table 2 shows that crude oil Brent (COB), crude oil Dubai (COD) and crude oil West Texas Intermediate (COWTI) were not stationary at level because the absolute value of the Augmented Dickey-Fuller (ADF) statistic |-2.236332| for COB, |-2.263152| for COD and |-2.442208| for COWTI are all less than the critical value of |-2.869419| at 5 percent level of significance and the probability value of 0.1939, 0.1847 and 0.1309 > 0.05 respectively confirms the acceptance of the null hypothesis that there is presence of unit root in the three variables. However, at first difference all the absolute value of the ADF statistic for COB (-13.13699), COD (-8.790247) and COWTI (-8.998032) are larger than the critical value of -2.869419, -2.869396 and -2.869534 respectively at 5 percent level of significance and their p-values are less than 0.05. This fulfils the condition for the use of autoregressive distributed lag (ARDL) model.

ARDL Bounds Test for Co-integration
From the result of bounds test shown in table 3, the F-statistic 12.68057 with COB as the endogenous variable is greater than the upper bound I(1) 5.85 , this clearly shows that there exists a long-run relationship between COB, COD and COWTI. There is also co-integration when COD is made the dependent variable because the F-statistics of 7.301270 is greater than the upper bound of 4.85 and lastly, when COWTI is made the dependent variable the F-statistic 5.446385 which is greater than 4.85 upper bound signals the existence of long run co-integrating relationship at the significant level of 5 percent. The Akaike Information criteria graph helps with the visual identification of the chosen model namely, ARDL (3,4,1), ARDL (4,4,1) and ARDL (4,4,1) models. Having established the presence of long-run relationship between COB, COD and COWTI, estimating the autoregressive distributed lag error correction model (ARDL-ECM) is necessary.

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Effect of Crude Oil Dubai and Crude Oil West Texas Intermediate on Crude Oil Brent

The results gotten from table 4 shows that the coefficient of determination R^2 is 0.971303. What this means is that about 97 percent of the deviation associated with crude oil Brent can be explained by crude oil Dubai and crude oil West Texas Intermediate, while the remaining 3 percent is explained by variables not captured in model. This is a good fit with R^2 value close to 1. The error correction term in table 4 is -0.185396 with a negative sign, this imply that the previous period deviation from long rum equilibrium is corrected at a speed of 18.54 percent ceteris-paribus. Considering the short run coefficients, only COD, COWTI and the second lag of COB are statistically significant with absolute value of the t-statistic -2.103647, 29.88167 and 9.181582 respectively larger than 1.96 with corresponding probability of 0.0361, 0.0000 and 0.0000. A percentage change in the second lag of crude oil Brent is associated with a 10.13 percent decrease in crude oil Brent on average all things being equal. A unit change in crude oil Dubai will increase crude oil Brent by 82.21 percent. If there is a percentage change in West Texas Intermediate, crude oil Brent will increase by 23.89 percent assuming all other variables remain constant. However, In the long run as presented in table 5, the coefficients of crude oil Dubai with probability 0.0000 and crude oil West Texas Intermediate with probability 0.0030 were both highly statistically significant. A percentage change in crude oil Dubai and crude oil West Texas Intermediate will increase crude oil Brent by 88.68 percent and 15.89 percent respectively.

Effect of Crude Oil West Texas Intermediate and Crude Oil Brent on Crude Oil Dubai

Table 6 is an expository of the result of the effect of the exogenous variables on crude oil Dubai revealing the R^2 to be 0.966712. This means that approximately 97 percent of the variance in crude oil Dubai (COD) can be described by crude oil West Texas Intermediate (COWTI) and crude oil Brent (COB), the 3 percent remaining is described by variable excluded from the model. The error correction term in table 5 is -0.163277, what this means is that the deviations from the long run equilibrium are corrected gradually by the error correction mechanism at a speed of 16 percent through a series of partial short-run adjustments. The coefficients -0.145554 for the third lag of COD, 0.0051 for COB and 0.115318 for the third lag of COB are all statistically significant as shown in table 6. A unit change in the third lag of COD will decrease COD by 0.145554 and a unit change in COB will increase COD by 0.858567. Again, from table a unit change in the third lag of COB will increase COD by 0.115318. From table 7 however, in the long run a unit change in COB will increase COD by 1.015203 and it is highly significant with a probability value of 0.0000 at 5 percent level of significance. COWTI is not statistically significant hence, does not have a significant effect on COD as they move together.

Effect of Crude Oil Brent and Crude Oil Dubai on Crude Oil West Texas Intermediate

The results obtained from table 8 presents the coefficient of determination R^2 as 0.905422. The implication of this is that about 91 percent of the deviation associated with crude oil West Texas Intermediate can be explained by crude oil Dubai and crude oil Brent, while the remaining 8 percent is explained by variables omitted from the model. This is a good fit with a very high R^2 value close to 1. The error correction term in table 4.8 is -0.090484 with a negative sign, this imply that the previous period deviation from long rum equilibrium is corrected at an adjustment speed of 9 percent ceteris-paribus. The short run coefficients 0.114363, -0.117483, 0.182597, -0.127536, 0.106841, 0.116177 and 0.752700 representing the first lag of COWTI, the third lag of COWTI, COD, first lag of COD, second lag of COD, third lag of COD and COB respectively are all statistically significant at 5 percent level of significance. A percentage change in the first lag of crude oil West Texas Intermediate is associated with a 11.44 percent increase in crude oil West Texas Intermediate on average all things being equal. A percentage change in the third lag of crude oil West Texas Intermediate will decrease crude oil West Texas Intermediate by 11.75 percent. If there is a percentage change in crude oil Dubai, crude oil West Texas Intermediate will increase by 18.26 percent assuming all other variables remain constant. More so, a percentage change in the second and third lags of crude oil Dubai will increase crude oil West Texas Intermediate by 10.68 and 11.62 percent respectively. While crude oil West Texas Intermediate will decrease by 12.75 if the first lag of crude oil Dubai is increased by one percent. However, In the long run as presented in table 9, the coefficients of crude oil Dubai (-0.242807) and crude oil Brent (1.098135) with respective probabilities 0.6973 and 0.0729 were both not statistically significant. This implies that crude oil Dubai and crude oil Brent does not significantly affect the cause of crude oil West Texas Intermediate in the long run.

Post Estimation Test

The results from the Breuch-Godfrey Serial LM test in table 10 show that the residuals from three models were free from serial correlation since their probability of chi-square are all larger than the 5 percent level of significance. This

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result was confirmed in figures 6, 7 and 8 by the plot of cumulative sum of residuals for the three model. The cumulative sum of squares (CUSUM) graph shows that the three models are under stable condition therefore, they are considered stable in long run. The CUSUM line was within the critical bounds at the 5 percent level of significance hence, the reliability of the models

Conclusion

This study tries to examine the interaction between Crude oil Brent (COB), Crude oil Dubai (COD) and Crude oil West Texas Intermediate (COWTI) using autoregressive distributed lag model (ARDL) between the period May, 1994 to March, 2024. The study has reveal that crude oil Dubai has the highest deviation from its mean. Also revealed in the Jaque-Bera statistic is that all the crude oil benchmark is normally distributed with evidence of trend in the movement of the variables under investigation. For non-stationary variables, this amounts to co-integration testing, yet the ARDL approach is flexible and allows for both stationary and non-stationary variables. The error correction mechanism in the models allowed for easy adjustment back to equilibrium as a result of the deviation that occurred in the previous period. When Crude Oil Brent is positioned as endogenous variable, COB is negatively correlated with its own second lag but COD and COWTI are positively correlated to COB in the short run and long run. This means when COD price benchmark increases, COB price benchmark will also increase. In the same way, an increase in the second lag of COB price benchmark will negatively affect the price benchmark of COB only in the short run. In another development, when COD is dependent on the other variables, a unit increase in the third lag of COD will cause COD to decrease. Therefore, there is a negative correlation between COD (-3) and COD, while a positive relationship exist between COB and COD price benchmarks in the short and long run. But COWTI price benchmark is negatively related to COD price benchmark.

The result also confirms that there exists a strong evidence of time-varying interdependence between COB, COD and COWTI. COWTI is negatively related to its second and third lags. This means that past innovations in COB and COD have great influence on present changes in COWTI. The first lag of COD is also negatively correlated with COWTI. However, COB and COD influence COWTI in a positive way. In the long run there is no interconnection between COB, COD and COWTI. More so, the combine lags of COWTI (-1), COWTI (-2) and COWTI (-3) significantly caused own shocks in the short-run. Since own shocks have been identified as determinants of impulse.

Recommendations

Considering the above conclusion, the study recommends the following,

(i) There exists a trend which causes crude oil price benchmark fluctuation, therefore appropriate measures like regulating money supply and exchange rate to prevent the effect of trend from setting into the Nigerian economy. Price stability should be the major focus of monetary policy makers in the short and long term to absorb the shock that may arise in the future.

(ii) There is a need to consider an appropriate past innovation (lags) in estimating the interaction of COB, COD and COWTI and its impact on the economy.

(i) ARDL (3,4,1), ARDL (4,1,1) and ARDL (4,1,1) model are the most suitable in explaining the endogeneity of COB, COD and COWTI within the period under study. Different models answer questions about different aspects.

Contribution to Knowledge

The study has made some contribution to knowledge in the following areas:

(i) The study has provided quantitative evidence that there exists trend in the three crude oil price benchmark of COB, COD and COWTI within the period under investigation.

(ii) The study has also provided numerical evidence that the use of lag innovation is necessary and appropriate in estimating the interaction between COB, COD and COWTI.

(iii) The study has been able to show that ARDL (3,4,1) COB as endogenous variable, ARDL (4,1,1) COD as endogenous variable and ARDL (4,1,1) COWTI as endogenous variable are the most efficient model for modeling the interaction between COB, COD and COWTI within the period under review.

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