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On Galois Groups, Resolvents and Applications

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Abstract
This paper examined Galois groups of polynomials with resolvents. Galois theory addresses the solvability of polynomial equations by radicals, specifically to determine when the polynomial can be solved using a sequence of operations and roots extractions for higher degree polynomials. This paper investigated polynomials roots, inherent difficulties and complexities associated with computing Galois groups of polynomials with a zero in on the incorporation of resolvents with specific emphasis to polynomials of order 3, 4, 5 and 6 . Applications are also presented.

Keywords: Galois Groups, Galois Theory, Polynomials, Radicals, Resolvent

Introduction

Galois theory instituted by $\hat{E}$variste Galois a French mathematician, came up with a bridge between group and field theory, also essential in number theory, differential equations, algebraic geometry, and classic problems like squaring the circle and equally used to authenticate if a polynomial is solvable. Galois is conceded as the creator of modern algebra because his hypothesis was among the hypotheses that started the contemporary knowledge of groups theory in the history of mathematics (Leistar, 2023). One of the most essential applications of Galois theory is to give the yardstick for deciding when a polynomial is solvable through the use of rational operations and root extractions which is executed by exploiting the comparability between fields and their specific automorphism groups. In essence, there exists a sequence of sub-fields between the splitting field and the coefficient field which relates to a sequence of subgroups (Bewersdorff, 2006).

In the principality of Galois theory, the computation of Galois groups for polynomials stands intricate. While the theory itself gives a distinguished framework for comprehending the symmetries of polynomial roots, the practical computation of Galois groups, particularly when resolvents are involved, presents a layer of experience that demands careful investigation (Geissler & Klüners, 2000). Galois's introduction allowed him to characterize the polynomial equations that are solvable by radicals in terms of properties of the permutation group of their roots, an equation is solvable by radicals if it's root may be presented by formula involving only integers. This is a generalization of Abel Ruffini's theorem (also known as Abel's impossible theorem) , which states that " No formula exists for the solution of degree five or more , using only the operations of $+,\ \times.\ \div, -$ and $\sqrt{}$ (Ramond, 2020). The fundamental theorem of Galois theory gives a link between algebraic field extension, effective criterion for the solvability of polynomial equations in terms of the solvability of the corresponding Galois group. As an example, S_5 the symmetric group of order 5 which has 120 elements is not solvable and this implies that the general quintic equation cannot be solved by radicals in a manner that equations of lower degrees can relate to. The theory being one of the historical roots of group theory is still fruitfully applied to bring on new results in specialty such as classical field theory (Orlitzky, 2022).

In Galois theory, a resolvent for a permutation group is a polynomial whose coefficients depend on the coefficients of a given polynomial. Galois theory studies the symmetries inherent in polynomial equations, has transformed the study of algebraic extensions and presented a profound interrelationship between algebra and number theory. If a polynomial is separable and irreducible then the corresponding Galois group will be a transitive subgroup. Nonetheless, a group can be a subgroup of several groups. A resolvent can provide a direction if the Galois group of polynomial is necessarily a proper subgroup or not. Galois theory stands at the framework of modern algebra and interchanges with many areas

of mathematics. The problem of investigating Galois groups has been detected to be of profound interest not just in algebra, but also from the area of number theory inspiring many questions in other areas of mathematics.

Preliminaries

In this section, preliminaries are presented on key concepts and theorems , further details can be obtained from : Weintraub (2000), Bewersdoff (2006), Bright (2013),Ramond (2020), Mertens (2021) and Laszlo (2024) .

i.A field extension is a larger field that contains the original field. Field extensions helps in the comprehension of the relationships between roots of polynomials.

ii.A cyclotomic extension is a field extension obtained by adjoining roots of unity to the base field.

iii. Automorphism: - A bijective map from a mathematical object to itself, preserving the underlying structure. Field automorphisms play a key role in understanding the symmetries of field extensions.

iv.The symmetric group on n elements, denoted as S_n, is the group of all permutations of n distinct objects.

v. Polynomial:-A polynomial over a field k in the indeterminate x is defined as $F[X] = a_o x^o + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n, n = 0, 1, 2, \cdots, n \in W$, where W represents the set of whole numbers. The scalar a_n is called the leading coefficient of F[X], $a_{i's} \in \mathbb{R}$, the set of real numbers and n is the degree of the polynomial F[X]. F[X] is also regarded as polynomial ring where F[X] is defined over a ring (R, +, ·), "+" is the operation of addition and "·" is the operation of multiplication.

Types of polynomials

Degree	Polynomial	Name
n = 1	$a_0 + a_1 x$	Monic (Linear) Polynomial
n = 2	$a_o + a_1 x + a_2 x^2$	Quadratic Polynomial
n = 3	$a_o + a_1 x + a_2 x^2 + a_3 x^3$	Cubic Polynomial
n = 4	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$,	Quadratic
n = 5	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$,	Quintic Polynomial
n = 6	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6$,	Sextic Polynomial
n = 7	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7$	Septic Polynomial
n = 8	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8$	Octic Polynomial
n = 9	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9$	Nonic Polynomial
n = 10	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10}$	Decic Polynomial
n = 11	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10} + a_{11} x^{11}$	Undecic Polynomial
n = 12	$a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10} + a_{11} x^{11} + a_{12} x^{12}$,	Dodecic Polynomial

There is no general name for polynomials of degree 13th to 20th, such polynomials are routinely called with respect to their degree's, for n=13, polynomial is called 13th degree polynomial etc

vi.An irreducible polynomial is a polynomial that cannot be factored into the product of two non-constant polynomials.

Theorem : Galois group of irreducible polynomial $F \in Q[X]$ act transitively on the set of roots of F[X].

Corolary: If F[X] represents an irreducible polynomial of degree n, then, the order of the Galois group of F[X] is divisible by n, i.e $|Gal| = |\theta(\propto_i)| = |stae\ (\propto)|$

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Hence, F[X] gives an irreducible polynomial of degree n in Q[$\propto$] which means that the Galois group represents a transitive subgroup of S_n.

Galois groups

The Galois group of a polynomial is a mathematical group that expresses the symmetries of the polynomial's roots under field automorphisms. Galois groups of polynomial equations gives a strong framework for understanding the solvability of these equations to figure out the long-standing puzzle of solving polynomial equations of degree five or higher. A polynomial is solvable by radicals if every root of the polynomial can be obtained from rational numbers using formulas together with basic operation of addition, subtraction, multiplication, division and nth roots. The solvability by radicals is revealed through the use of Galois theory together with field theory.

It has been indicated that the problem of defining Galois groups is of great applicability not only in algebra but also from the perspective of number theory, raising several issues in other branches of mathematics (Obi, 2017). Galois first illustrated how the several fundamental bases of a certain polynomial condition are related to one another via stage gatherings. By examining the Galois groups that represents the polynomial equations, it presents a robust foundation for understanding their solvability. The solvability of polynomial equations by radicals is the crux of Galois theory . The long-standing issue of solving polynomial equations of degree five or higher can be resolved by applying a series of arithmetic operations and root extractions to algebraic equations (Stewart, 2009). Brzeziński (2011) demonstrated that the solvability of a radical's Galois group characterizes equations that can be solved by radicals. This reveal that radicals cannot solve general equations with degrees greater than 5. A technique for confirming if the alternating group $A_n$on a solution is included in the Galois group of a Schubert problem on a Grassmannian (a Schubert Galois group) as presented by Vakil (2006) which confirmed that such a Galois group alternates. This permits for identification of an endless family of Schubert issues whose Galois groups were not the entire symmetric group and equally to show that the majority of Schubert problems on small Grassmannians ,which can be a differentiable manifold ,were at least alternating. Transitive permutation groups are typically Galois groups of enumerative problems. There is a distinction between primitive groups, which sustain a nontrivial partition, and primitive groups, which are transitive permutation groups that conserve no nontrivial separation.

The Galois group of any given enumerative issue is a fascinating topic. The Galois group Gal can be presented to be either S_n or to contain its subgroup A_n of alternating permutations using different methods for studying Galois groups in enumerative geometry. Structures in enumerative geometry or polynomial systems provides details about the corresponding Galois groups. Information connecting Galois groups may be employed to identify these structures in an increasing number of algebraic geometry applications, either for comprehension or for solving the application.
Galois groups includes but not limited to the underlisted examples :

- **i.** Cyclic group C_2 as in $x^2 - 2$, the klein four group V_4 for $x^4 - x^2 - 2$, dihedral group D_4 for $x^4 - 2$ over the rationals $\mathbb{Q}$
- **ii.** Polynomials : $x^2 - 2$, splitting field **:** $\mathbb{Q}(\sqrt{2}\,)$, giving the Galois group of C_2 and representing the cyclic group of order 2)
- **iii.** Polynomial : $x^2 + 1$ of splitting field of $\mathbb{Q}(i)$ *c*orresponding to Galois group is the cyclic group of order 2 , C_2
- **iv.** Polynomial : $x^4 - x^2 - 2$ synonymous with the splitting field of $\mathbb{Q}(\sqrt{2}\,, i)$ and corresponding to a Galois group of V_4

Method of Solutions of Polynomials of degree n

Polynomials of degree 1, the method of solution of linear polynomials depends on the type of polynomial, suppose the polynomial is a pair of simultaneous equations then, elimination method, substitution method, graphical method, cramer's rule etc suffices.
For $n = 2$, the method of solution of quadratic polynomials include the followings: factorisation method, completing square method, quadratic formula method, graphical method.
For $n = 3$, the method of solution of cubic polynomials include the following: factorisation method: synthetic division method, substitution (change of variables), Cardano formula, Lagrange cubic resolvent, numerical methods and through radicals.
For $n = 4$, for quartic polynomials, the method of solution follows the same as the cubic polynomial and it includes: factorisation method, synthetic division method, Cardano formula, Lagrange resolvent, through radicals, substitution (change of variable) and numerical method (Newton Raphsons)

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For $n = 5$,the solution of quintic polynomials was a major problem in algebra before the early 19th century when the general solution was ascertained using Abel-Ruffin's theorem which states that "No algebraic expression or formula for the general solution of quintic polynomial over the rational" (Ramond, 2020).This statement is also germane to polynomial of higher degrees, an example is that quintic polynomial cannot be solved in terms of radicals, nonetheless, there are quintic polynomials that can be expressed in terms of radicals but the solution to these polynomials are majorly very complex and cannot be used for practical reasons.

Resolvents of Polynomials

The resolvent is a polynomial whose calculation relies on two other polynomials. The resolvent method of a polynomial is a polynomial of lower degree whose roots relate to the roots of the original polynomial (Alizadeh, 2012). Precisely, the roots of a resolvent polynomial are the outcome of evaluating dissimilarity of a multivariate polynomial at the roots of a second polynomial. The procedure expresses how one can use resolvent polynomials to navigate through the lattice of possible Galois groups and eventually, determine Gal(P). Regrettably, the complexity of such a procedure grows with the intricacy of the additional lattice of possible Galois groups. The methods for resolving higher degree polynomials can be obtained in Computer Algebra Systems (CAS) using Mathematica, Maple and SageMath as well as compuations of Galois groups.

Bartel and de Smit (2013) evolved a machinery of resultants and resolvent polynomials with the ultimate objective of understanding the "resolvent method" for computing Galois groups over the rationals $\mathbb{Q}$. For the determination of invariant polynomials, it is adequate to obtain one V-invariant polynomial which is not U-invariant, in place of bringing about the full invariant ring for V . Kret and Shin (2023) talked through computations of Galois groups. They debated some theoretical fundamentals on numerical methods which are implemented in some computer packages . They illustrated how to calculate and partition Galois groups for low degree polynomials by indicating some numerical invariants, which gives information on the isomorphism type of the Galois group in relation to their values, executed for polynomials of degrees 3 and 4. They further devled into Galois resolvents and used them to ascertain a general theorem by Richard Dedekind, which relates the Galois group of an integer irreducible polynomial to Galois groups of its reductions modulo prime numbers.

Forming a Galois resolvent

Given that the coefficient of *V(F, f)* can be written as integral polynomial in the coefficient of *f*, it will be possible to write down such polynomial for any given *F*. For large n, the polynomial seems to be of high degree and with large coefficients. In practices, different method is applied for the computation of the resolvents. This approach, benefited by the fact that the coefficients of $V(F, f)$ are rational integers. Hence we fix some numerical approximation of the roots of f_n $(a_1\ a_2\ \cdots a_n)$ and calculate the following product S_n acting on $a_i s$ by permuting their indices in the same way as it does on $a_i s$ i.e.

$$\prod_{s \in S_n | S} (X - s\, F(\propto_1 \cdots \propto_n))$$

With the approximation being normal, the coefficient of this product are approximated with an absolute error less than ½. They uniquely evaluated coefficients of *V(F, f)* which we know to be rational integers (Obi, 2017).

Galois Extension

Given a field *K*,which is a field extension collection of G, a group, called the Galois group of the field extension *K* over *F*, usually written as Gal (K/F).

Consider the following examples:

i. $x^2 + 2x + 5 = 0$, we can solve this quadratic equation as follows:

$$x^2 + 2x + 5 = 0 \Rightarrow (x+1)^2 + 4 = 0 \Rightarrow (x+1)^2 = -4$$
$$(x+1) = \pm\sqrt{-4}$$
$$(x+1) = \pm 2i$$
$$x = -1 \pm 2i$$

$x_{1,2} = -1 \pm 2i \implies x_1 = -1 + 2i, x_2 = -1 - 2i$ let E be the splitting field extension, then,

$$E = Q[x_1, x_2] \implies Q\,[-1+2i, -1-2i]$$
$$= Q\,[i]$$

Hence, the Galois group of this polynomial is defined as:

Gal $\left(E/Q\right) = [e, \sigma] = S_2, i.e$

$$\sigma_1(-1+2i) = -1-2i$$

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$$\sigma_2(-1-2i) = -1+2i, \sigma = -i$$

ii. $x^2+2x-5=0 \implies (x+1)^2-6=0$

$$(x+1)^2 = 6$$
$$(x+1) = \pm\sqrt{6}$$
$$x = -1 \pm \sqrt{6}$$
$$x_1 = 1+\sqrt{6}, x_2 = -1-\sqrt{6}$$

Let *k* be the splitting field extension, then $k = Q[x_1, x_2]$

$$= Q\left[-1+\sqrt{6}, -1-\sqrt{6}\right] = Q\left[\sqrt{6}\right]$$

Therefore, the Gal $\left(K/_Q\right) = [e, \Im] = S_2, \Im/_Q = \{e\}$.

Let us consider the cubic polynomial of the form $ax^3+bx^2+cx+d=0$ where a, b, c, d are possible complex or real numbers. Here we need to apply Cardano formular but first we need to construct Lagrange cubic resolvent which will help us prove the Cardano formula before applying.

Let $F \subset E$ be a Galois group extension, Gal $\left(E/_F\right) \cong G \cong \mathbb{Z}_m = \langle\sigma\rangle$. Assume that F contain a primitive m-root of unity, then $\mu = e^{2\pi i/_m}, \mu^m = 1$. Considering the $\mathbb{Z}_m -$ grading of E as

$E = E_0 \oplus E_1 \oplus E_2 \oplus E_{m-1} \cdots and$ dim $E_j = 1$

$$E_j = \{x \in E | \sigma(x) = \mu^k x|\}$$

$E_0 = E^G = F$, since the operator is of order *m*, then;

$$0 = (\sigma - I) = \prod_{k=0}(\sigma - \mu^k I) = (\sigma - I)(I + \sigma + \sigma^2 + \cdots + \sigma^{m-1})$$
$$= I + \sigma + \sigma^2 + \cdots + \sigma^{m-1} = \prod_{k=1}(\sigma - \mu^k I)$$
$$= (I + \sigma + \sigma^2 + \cdots + \sigma^{m-1})x = \begin{cases} 0, x \in E_j, J \neq 1 \\ mx, \ x \in E_0 \end{cases}$$
$$\prod_{k=1}^{m-1}(\mu - \mu^k I) = I + \mu^1\sigma + \mu^2\sigma^2 + \cdots + \mu^{(m-1)}\sigma^{(m-1)} = (\mu^j\sigma),$$

This expression is known as Lagrange cubic resolvent, therefore, the projection of $E \to E_j$ is given as $\frac{1}{m}(\mu^j\sigma)$. Now consider the general cubic equation in a depressed form i.e. cubic equation without the quadratic term.

$$x^3 + ax + b = 0$$
$$x^3 + ax + b = (x - \propto_1)(x - \propto_2)(x - \propto_3)$$

From Vieta's theorem, we know that

$$\propto_1 + \propto_2 + \propto_3 = 0$$
$$\propto_1\propto_2 + \propto_1\propto_3 + \propto_2\propto_3 = a$$
$$\propto_1\propto_2\propto_3 = -b$$

The system of equation presented then represents the values of $\propto_1, \propto_2$ and $\propto_3$, However, the general result about symmetric polynomial in $\propto_1, \propto_2, \propto_3$ is expressible in terms of the coefficient of the depressed cubic polynomial {a, b}. Let *E* be the splitting field extension over rational, the E= Q[μ];

$\mu = e^{2\pi i/_3} = \frac{1}{2} + \frac{\sqrt{3i}}{2}$ be the primitive root of unity, therefore $E = Q[\propto_1\propto_2\propto_3]$, then the Gal $(E/F) = S_3$.

Hence, $S_3 \supset A_3 \supset \{e\}, F = E^G, A_3 =$ Gal $\left(\frac{E}{F}\right)$ which implies that [E:F]=3,[E :K]=$|S_3| = 6$ *and* $[F:K] = 2$.

$$\theta = \prod_{[<]}\left(\propto_i - \propto_j\right) = (\propto_1 - \propto_2)(\propto_1 - \propto_3)(\propto_2 - \propto_3)$$

This is A_3 invariant but not S_3 invariant so if we let $D = \theta^2, then \left(\propto_i - \propto_j\right)^2(\propto_1 - \propto_3)^2(\propto_2 - \propto_1)^2 = S_3$ invariant. After some simplification we have the discriminate $D = -4a^3 - 27b^2$,

$F = K\left[\sqrt{D}\right]$, Gal $\left(\frac{E}{F}\right) = \mathbb{Z}_3 = \langle\sigma\rangle, \sigma\langle(1,2,3)\rangle$, considering the $\mathbb{Z}_3 -$ grading $E = E_0 \oplus E_1 \oplus E_2 \oplus E_3$

$$E_j = \{x \in E \mid \sigma(x) = \mu^j x\} \, but \, E_0 = F,$$

Therefore,

$$\propto_1 = P_o + P_1 + P_2, P_j \in E_j$$
$$P_j = \frac{1}{m}(\mu^k\sigma)(\propto_i)$$

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$$P_o = \frac{1}{3}(I + \sigma + \sigma^2)(\propto_i) = \frac{1}{3}(\propto_1 + \propto_2 + \propto_3) = 0$$

Since $(\propto_1 + \propto_2 + \propto_3) = 0$ from the equation of the system of equation stated above.

$$P_1 = \frac{1}{3}(I + \mu\sigma + \mu^2\sigma^2)(\propto_i) = \frac{1}{3}(\propto_1 + \mu \propto_2 + \mu^2 \propto_3) \in E_1$$

$$P_2 = \frac{1}{3}(\propto_1 + \mu \propto_2 + \mu^2 \propto_3) \in E_2$$

$$\therefore P_o, P_1, P_2 \in E = F$$

$$P_1^3 = -\frac{b}{2} - \frac{\left(\frac{1}{2} - \mu\right)}{9}\sqrt{D}, \qquad P = -\frac{a}{3}$$

Therefore,

$$\propto = \sqrt[3]{-\frac{1}{2} - \frac{\sqrt{3}}{18}\sqrt{D}} - \frac{\frac{a}{3}}{\sqrt[3]{-\frac{b}{2} - \frac{\sqrt{3}}{18}\sqrt{D}}}$$

This prove the Cardano formula with the help of Lagrange resolvent and Galois Theory, but this can be proved by substitution (change of value) as follows. Consider the same depressed cubic polynomial $x^3 + ax + b = 0, Let\ x = p - q,$ then $x^3 + 3pqx - (p^3 - q^3)$. Collecting like terms gives:

$$x^3 + 3pqx - (p^3 - q^3) = x^3 + ax + b$$

$$a = 3pq, -b = (p^3 - q^3)$$

but from a = 3pq, q = $\frac{a}{3p}$, then,

$$-b = \left(p^3 - \left(\frac{a}{3p}\right)^3\right) \Longrightarrow -b = p^3 - \frac{a^3}{27P^3}$$

$$(p^3)^2 + p^3 b - \frac{a^3}{27} = 0, \qquad Let\ y = p^3$$

Then $y^2 + yb - \frac{a^3}{27} = 0$, a quadratic polynomial. With application of the quadratic formula we have:

$$y = \frac{-b \pm \sqrt{b^2 + 4(\frac{a^3}{27})}}{2}$$

But y = p^3, $$P = \sqrt[3]{\frac{-b \pm \sqrt{b^2+4(\frac{a^3}{27})}}{2}}$$

$$P = \sqrt[3]{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 + \frac{a^3}{27}}},$$

Recall that; q = $\frac{a}{3p} \Longrightarrow q = \frac{\frac{a}{3}}{\sqrt[3]{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 + \frac{a^3}{27}}}}$

But $x = p - q$

$$x_{1,2} = \sqrt[3]{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 + \frac{a^3}{27}}} - \frac{\frac{a}{3}}{\sqrt[3]{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 + \frac{a^3}{27}}}}$$

This verified the Cardano formula. Quadratic polynomial in a depressed form $x^4 + ax^2 + bx + c = 0$
$x^4 + ax^2 + bx + c = (x - \propto_1)(x - \propto_2)(x - \propto_3)(x - \propto_4)$, from Vieta's Theorem.

$$\propto_1 + \propto_2 + \propto_3 + \propto_4 = 0$$

$$\propto_1\propto_2 + \propto_1\propto_3 + \propto_1\propto_4 + \propto_2\propto_3 + \propto_2\propto_4 + \propto_3\propto_4 = 0$$

$$\propto_1\propto_2\propto_3 + \propto_1\propto_2\propto_4 + \propto_1\propto_3\propto_4 + \propto_2\propto_3\propto_4 = \propto_1\propto_2\propto_3\propto_4 = C$$

Let E be the splitting field extension over K, $k = Q[\mu], \mu = e^{2\pi i/3}$, here we do not need the 4-root of unity. $E = k\ [\propto_1, \propto_2, \propto_3, \propto_4]$ therefore, the Gal $(E/F) = S_4$, the commutator of this group are;

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$S_4^{(1)} = A_4, S_4^{(2)} = V_4\{(1,2),(3,2),(1,3),(2,4),(1,4),(2,3)\}$ and $\{e\} \cong \mathbb{Z}_2 \times \mathbb{Z}_2, S_4 \supset A_4 \supset V_4 \supset \{e\}\, k \subset k\left[\sqrt{D}\right] \subset F \subset E, if\ \theta = \prod_{i<j\leq 4}(\propto_i - \propto_j), then\ D = \theta^2 = \prod_{i<j\leq 4}(\propto_i \propto_j)^2, therefore\ the\ Gal\left(E/F\right) = V_4 = \langle \sigma_1, \sigma_1 \rangle,\ \sigma_1(1,2),(3,4),\ \sigma_2(1,3),(2,4)$, but let us recall the $\mathbb{Z}_4 -$ grading.

$$E_j \implies E = E_o \oplus E_{1o} \oplus E_{o1} \oplus E_{11}$$

$$E = \left\{x \in E \mid \begin{matrix}\sigma_1(x) = (-1)^j x \\ \sigma_2(x) \quad (-1)^k x\end{matrix}\right\}$$

$E_{o1} = E^{V_4} = F$ and the projection of E_j

$$\frac{1}{4}(I + (-1)^j\ \sigma_1)\left(I + (-1)^k\sqrt{2}\right)$$

The roots of the quadratic polynomial is $x = P_{oo} + P_{1o} + P_{o1} + P_{11}$

$$x = \frac{1}{4}(I + \sigma_1)(I + \sigma_2)(\propto_i)$$

$$= \frac{1}{4}(I + \sigma_1)(\propto_1 + \propto_3)$$

$$= \frac{1}{4}(\propto_1 + \propto_2 + \propto_3 + \propto_4) = 0$$

Hence, $P_{10} = \frac{1}{4}(\propto_1 - \propto_2 + \propto_3 - \propto_4)$, $P_{o1} = \frac{1}{4}(\propto_1 - \propto_2 - \propto_3 - \propto_4)$, $P_{11} = \frac{1}{4}(\propto_1 - \propto_2 - \propto_3 + \propto_4)$

$$K\left[\sqrt{D}\right] \subset F, Gal\left(\frac{E}{K\left[\sqrt{6}\right]}\right) = \frac{A_4}{V_4} \cong \mathbb{Z}_3 = \langle \Im \rangle$$

$x^4 + ax^2 + bx + c = F_0 \oplus F_1 \oplus F_2$ $F_0 = k\sqrt{6}$, $r_{10}^k = \frac{1}{3}\left(\mu^{-k}\Im\ \left(P_{j,i}\right)^2\right)$

$P_{10}, P_{o1}, P_{11} = -\frac{b}{8}$, $r_{11}^{(1)} = r_{10}^2 = \frac{a}{144} + \frac{c}{12}$,

After a complicated simplification we have : $Q = \frac{a^3}{1728} + \frac{b^2}{128} - \frac{ac}{48} - \frac{\left(\frac{1}{2} - \mu\right)}{576}\sqrt{D}\ and$

$D = -4a^2b^2 + 16ac - 27b^4 + 144ab^2c - 128a^2c^2 + 256c^3$, therefore $\propto = P_{1o} + P_{o1} + P_{11}$.

$P_{1o} = \sqrt{-\frac{a}{6} + \sqrt[3]{Q} + \frac{S}{\sqrt[3]{Q}}}$, $P_{o1} = \sqrt{-\frac{a}{2} + \mu\sqrt[3]{Q} + \frac{\mu S}{\sqrt[3]{Q}}}$

$P_{11} = \frac{\left(\frac{-b}{8}\right)}{P_{11}P_{1o}}$, by substituting these resulted to $\propto = P_{1o} + P_{o1} + P_{11}$. We obtain the desired result and It is still possible, if we still apply the change of variable method and obtain this result.

Results

Criterion for Solvability of Polynomial of degree 3

Example $x^3 + 3x + 1 = 0$, recall that the discriminant of the general depressed cubic polynomial is defined as : $x^3 + ax + b = 0, is\ D = -4a^3 - 27b^2$. Let *K* be the splitting field extension over *Q*, then $Gal(k/Q) \leq A_3$ if and only if the discriminant is a perfect square. However, to compute the Galois group of this polynomial, we need to verify whether or not the above polynomial is irreducible.

According to the irreducibility properties stated in the previous chapter, the cubic polynomial is irreducible. Hence, the Galois group of this polynomial is not the trivial group, since $\Delta = -4(-3)^3 - 27(1) = 81 = 9^2$ is a perfect square, therefore, the Galois group of this polynomial is a subgroup of A_3 with 3-cycles and it is a transitive subgroup because the polynomial is irreducible. Therefore,

$$Gal\left(K/Q\right) = A_3 \cong S_3\ with\ \langle(1,2,3)\rangle$$

$H(X) = x^3 - 2$, This polynomial is irreducible using Eisenstein criterion of simple roots, suppose let the roots of H(x) be $\omega_1, \omega_2, \omega_3 \cdots$, forming the Galois resolvent by choosing the coefficients that are integers. Simplifying this process means that the coefficients have been taken with the crux that they give different values of the resolvent for each conjugation.

$V_0 = \omega_1 + 2\omega_2 + 3\omega_3$, $V_1 = \omega_3 + 2\omega_1 + 3\omega_2$

$V_2 = \omega_2 + 2\omega_3 + 3\omega_1$, $V_3 = \omega_2 + 2\omega_1 + 3\omega_3$

Then the polynomial will be values of the resolvent and it is conjugate as roots

H(x) = $(x - y_0)(x - y_1)(x - y_2)(x - y_3)(x - y_4)(x - y_5)$.

$= x^6 - (y_0 + y_1 + y_2 + y_3 + y_4 + y_5)x^5 + [(y_0 + y_1)(y_2 + y_3 + y_4 + y_5) + (y_2 + y_3)(y_4 + y_5)]\, x^4 + [y_0y_1 + y_2y_3 + y_4y_5]x^4 - [y_0y_1(y_2 + y_3 + y_4 + y_5)(y_2y_3(y_4 + y_5) + y_4y_5(y_2 + y_3)]x^3[(y_0 + y_1)(y_2y_3 +$

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$y_4y_5 + (y_2 + y_3)(y_4 + y_5)]x^3 + \left[(y_2y_3y_4y_5 + (y_0 + y_1)(y_2y_3(y_4 + y_5))\right]x^2 + \left[(y_4y_5(y_2 + y_3)) + y_0y_1(y_2 + y_3)\ (y_4 + y_5) + y_2y_3 + y_4y_5\right]x^2 - [(y_0 + y_1)y_2y_3y_4y_5 + y_0y_1(y_2y_3(y_4 + y_5) + y_4 + y_5(y_2 + y_3)]x + y_0y_1y_2y_3y_4y_5$

Expanding the coefficient of H[X] in terms if the roots $\omega_1, \omega_2, \omega_3$, gives the symmetric polynomial in the roots e.g. $x^5\ and\ x^4$.

Hence, $x^6 - y_0y_1y_2y_3y_4y_5$ is irreducible in *Q*.

$$12\ (\omega_1 + \omega_2 + \omega_3)x^5\ and\ 58(\omega_1^2 + \omega_2^2 + \omega_3^2) + 122(\omega_1\omega_2 + \omega_1\omega_3 + \omega_2\omega_3)]x^4$$

Criterion for Solvability of Polynomial of degree 4

$$G[X] = x^4 - 2x^3 - 8x - 3$$

Given that $G[X] = x^4 - 2x^3 - 8x - 3$, for x = 3, $G(3) = 81 - 54 - 24 - 3 = 0$. Hence $G[X] = (x - 3)(x^3 + x^2 + 3x + 1)$.Let $f[X] = x^3 + x^2 + 3x + 1$, there is the need to verify whether or not f[X] is irreducible, if f[X] is irreducible over finite G_p , it means that f[X] is irreducible over $Q[X] and\ \mathbb{Z}\ [X]$. Hence, f[X] is irreducible over G_5, since f[X] is a cubic polynomial, using the intermediate value theorem f[X] has at least one real root in $\mathbb{R}$ and two complex conjugation, $Gal\ (F_5/Q) = G\ \leq\ S_3$ and the Galois group is transitive with (1, 2) and (1, 2, 3) cycles.

Criterion for Solvability of Quintic Polynomial

To apply Galois theorem, there is the need to establish some irreducibility criteria that will allow us to solve Quintic polynomials and higher degree polynomial up to degree *n*. Let F ∈ $\mathbb{Z}[X]$, then there exist a rational scalar C ∈ $\mathbb{Q}$ such that $C\ \neq 0\ \forall\ C\ F[X]\ \in\ \mathbb{Z}[X]$ and the greatest common divisor (gcd) of the coefficients is 1, if $F[X] = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{n-1}x^{n-1} + a_nx^n, a\ \in\ \mathbb{Z}[X]$, then the following properties holds:

a. F[X] is irreducible in $\mathbb{Q}$[X] if $f[X]$ is irreducible in $\mathbb{Z}[X]$.
b. Let p be prime and suppose, the leading coefficient $a_n \neq 0$ and not p, if $f[X]$ is irreducible in$\mathbb{Z}[x], then\ F[X] is\ irreducible\ in\ Q[X]$.

Example: $x^3 - x + 1\ is$ irreducible $in\ \mathbb{Z}_p[x]\ implies\ that\ 7x^3 + 4x^2 + x + 3$ is irreducible in $\mathbb{Z}[X]$.

Eisenstein Criteria

Given P is prime and suppose the leading coefficient a_n is indivisible by P and other coefficients are divisible by p and a_0 is not divisible by $P^2\ then\ F[\propto]$ is irreducible in $\mathbb{Z}[X]$.

Example: $x^3 + 4x + 2 = 0$ is irreducible by Eisenstein's criteria.

Shifting Invariant

Let $a\ \in\ \mathbb{Z}[\propto]$ be irreducible if and only if f(x – a) is irreducible. Let p be prime such that

$$F[\propto] = X^P + X^{P-1} + \cdots + X^{p+1}$$

becomes irreducible because, from the general solution of the above polynomial

$$F[X] = \frac{X^P - 1}{X - 1}, \qquad F(x - 1) = \frac{(X + 1)^P - 1}{(x + 1) - 1} = \frac{1}{X}\left(\sum_{k=0}^{p}\binom{P}{K}X^k - 1\right)$$

$$= X^{P-1} + \binom{P}{1}X^{P-1} + \binom{P}{2}X^{P-2} + \cdots + \binom{P}{P-1}$$

And $a_o = \binom{P}{P-1} = P$ which is not divisible by P^2 .Therefore, using Eisenstein's criteria this polynomial is irreducible. Suppose that F(x) is irreducible polynomial of degree n, i.e.

$F(X) = a_n(w - w_1)(w - w_2)\cdots(w - w_n) \forall\ w_i\ \in\ \bar{Q}\ \subset C$, implying that F[X] is irreducible, i.e.

$E = Q[w_1, w_2, w_3\ \cdots w_m), Q[\propto_i]\ \cong\ Q\ [w_j] \cong Q\ [x]\ |_{\langle F\rangle}, if\ \sigma: Q[w_i]\ \rightarrow\ Q[w_j], \sigma(w) = w,$ From Homomorphism theorem, the field extension is embedding of E → $\bar{Q}$ and for E normal over $Q[\propto]$ then the splitting field E is $\sigma[E] = F$, Hence we conclude that $\sigma\ \in Gal\ \left(E/_Q\right)$ such that $\sigma\ (w_i) = w_j$.

Example

$$F[X] = x^5 - 4x + 2$$

The Quintic polynomial is irreducible going by Eisenstein's criteria for p =2, hence the Galois group of irreducible polynomials is transitive, hence for every roots of the polynomial say $w_1 \overset{\sigma}{\rightarrow} w_2$ hence,

$|G| = |orbit\ of\ w| \times |G: Stab\ (w)|$

Hence, $Gal\ (F/G) = S_5$

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Using Cauchy theorem, the polynomial $\langle(1,2,3,4,5)\rangle \in G$, to get the roots of this polynomial,
$F[X] = x^5 - 4x + 2$, $F'[X] = 5x^4 - 4$, this means that
For $F'[X] = 0$, $5x^4 - 4 = 0 \Rightarrow x^4 = \frac{4}{5} \Rightarrow x = \pm\frac{2}{\sqrt[4]{5}}$
This means that since Quintic polynomial has more turning points, the Quintic equation has exactly 3-real roots and two complex roots. $\therefore (a,b) \in G \subseteq S_5$

Example
Given that $H(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e \in Q(x)$ be the general Quintic polynomial with roots w_1, w_2, w_3, w_4, w_5 and assume that these roots satisfy $w_i^5 - 5w_i + 12 = 0, i = 1,2,3,4,5$. Here we need to construct the cubic resolvent as follows. A general cubic resolvent for quintic polynomial is $G(y) = y^3 + py + q$ where p and q are related to the coefficient of the original equation.
Assume that $G(y) = y^3 - 15y + 12$, then p = −15 and q = 12 by
Cardano formula $D = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$

$$D = \left(\frac{12}{2}\right)^2 + \left(-\frac{15}{3}\right)^3 = 6^2 + (-5)^3 = 36 - 125 = -89,$$

With the Discrimnant D being negative, the cubic resolvent polynomial has three district real roots, applying the trigonometry version of cardano formula by substituting $y = u + v$, i.e. $u^3 + v^3 = -q$,
$uv = -\frac{p}{3}$, $u^3, v^3 = w_1, w_2, w_3$ which are the roots of the quadratic polynomial
Hence, $W_k = \sqrt[3]{-\frac{q}{2} + \sqrt{D}}$ and $W_k = \sqrt[3]{-\frac{q}{2} - \sqrt{D}}$
$u^3 = \sqrt[3]{-6 + i\sqrt{39}}$, $v^3 = \sqrt[3]{-6 - i\sqrt{39}}$,
Hence, $y_k = \sqrt[2]{-\frac{p}{3}\cos\left(\frac{\theta + 2k\pi}{3}\right)}$, $k = 0,1,2$
where $\theta = \cos^{-1}\left(\frac{q}{2\sqrt{\left(-\frac{p}{3}\right)^3}}\right)$, substituting all these values we have
$\sqrt{-\frac{p}{3}} = \sqrt{-\frac{15}{3}} = \sqrt{5}$ and $\cos\left(\frac{\theta}{3}\right) = \cos(\cos^{-1}\left(\frac{12}{\sqrt[2]{5^3}}\right)$,

$$\theta = \cos^{-1}\left(\frac{12}{2\sqrt{125}}\right) = \cos^{-1}\left(\frac{12}{22.36}\right) = \cos^{-1}(0.536) = 57.12^o$$

$$\text{For } k = 0, y_1 = \sqrt[2]{5\cos\left(\frac{57.12^o}{3}\right)} \cong 4.24$$

$$For \text{ k =1, } y_2 = \sqrt[2]{5\cos\left(\frac{57.12^o + 120}{3}\right)} \cong 2.301$$

$$For \text{ k =2, } y_3 = \sqrt[2]{5\cos\left(\frac{57.12^o + 240}{3}\right)} \cong -0.698$$

Therefore, the roots of this resolvent y_1, y_2, y_3 provides a preview to the nature of the quintic polynomial, meaning that the Galois group is likely to be complex, hence, the Quintic polynomial cannot be solved by radical.

Sextic Polynomial

$$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$$

Here we can apply substitution method (change of variable) by Let $y = x + \frac{1}{x}$ but before that, we need to divide through the above polynomial by x^3 i.e.
$\frac{x^6+x^5+x^4+x^3+x^2+x+1}{x^3} = x^3 + x^2 + x + 1 + x^{-1} + x^{-2} + x^{-3}$

$$x^3 + x^{-3} + x^2 + x^{-3} + x + x^{-1} + 1 = 0$$

$$\left(x - \frac{1}{3}\right)^3 - 3\left(x - \frac{1}{x}\right) + \left(x + \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right) + \left(x + \frac{1}{x}\right) + 1 = 0$$

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Assuming that $y = x + \frac{1}{x}$

$$y^3 - 3(y) + 2(y) + y^2 + y + 1 = 0$$
$$y^3 - 4y + y^2 + 1 = 0$$
$$y^3 + y^2 - 4y + 1 = 0$$

which is the minimal polynomial of $x + x^{-1}$ there it's roots are $2\cos\left(\frac{2\pi}{7}\right) = x + x^{-1}, 2\,\mathrm{Cos}\left(\frac{4\pi}{7}\right) = x^2 + x^{-2}$ and $2\,\mathrm{Cos}\left(\frac{6\pi}{7}\right) = x^3 + x^{-3}$, since $\mathrm{Cos}\frac{6\pi}{7} < \cos\frac{4\pi}{7}$.

Solving this by Cardano formula gives $\mathrm{Cos}\left(\frac{2\pi}{7}\right) = \frac{1}{6}\left[-1 + \sqrt[3]{\frac{98}{1+\sqrt[3]{3i}}} + \sqrt[3]{\frac{7}{2}(1+3\sqrt{3i})}\right.$

$$\mathrm{Cos}\left(\frac{4\pi}{7}\right) = \frac{1}{12}\left[2 + (1 - 3\sqrt{3i}) + \sqrt[3]{\frac{98}{1+3\sqrt{3i}}} + \left(1+\sqrt{3i}\right)\left(3\sqrt{\frac{7}{2}\left(1+3\sqrt{3i}\right)}\right)\right.$$

$$\mathrm{Cos}\left(\frac{6\pi}{7}\right) = \frac{1}{12}\left[2 + (1 - \sqrt{3i}) \sqrt[3]{\frac{98}{1+3\sqrt{3i}}} + \left(1-\sqrt{3i}\right)\left(3\sqrt{\frac{7}{2}\left(1+3\sqrt{3i}\right)}\right)\right.$$

But $Sin\,\theta = \sqrt{1 - \mathrm{Cos}^2\,\theta}\ ,\ \forall\ 0 \le \theta \le \pi$

$$\mathrm{Sin}\left(\frac{2\pi}{7}\right) = \frac{1}{29}\sqrt{\frac{1}{3}\sqrt[3]{\frac{7}{4}}\left[2v + (1 - \sqrt{3i} + 2\sqrt[3]{2}\,(-2 + \sqrt{3i})\right.}$$

$$\mathrm{Sin}\left(\frac{4\pi}{7}\right) = \frac{1}{24}\sqrt{\frac{1}{3}\sqrt[3]{\frac{7}{4}}\left[2v - \left(1+\sqrt{3i}\right)w + 3\sqrt{2}\left[5 + \sqrt{3i}\right]\right.}$$

$$= \mathrm{Sin}\left(\frac{6\pi}{7}\right) = \frac{1}{24}\sqrt{\frac{1}{3}3\sqrt{\frac{7}{3}}\left[2v - 2w - 3\sqrt{2}\left(1 + 3\sqrt{3i}\right)\right]}$$

where $u = \sqrt[3]{1+3\sqrt{3i}}$, $v = \sqrt[3]{49\left(-13+3\sqrt{3i}\right)}$, $w = \sqrt[3]{7(1+3\sqrt[3]{3i}}$

Applications

i. In engineering, polynomial systems are usage involves construct mechanisms with a desired range of motion. An example is in the robotic arm movements may need to be able to reach many locations in order to accomplish certain activities. Polynomial systems can be used to represent these movements, and then the discussed procedures can be used to study them. The nine-point synthesis problem for four-bar linkages is one of Alt's problems (Allgower & Georg, 2003).

ii. It also gives a complete answer to ancient questions such as dividing a circle into an equal arc using ruler and compasses. In modern language, Galois theory deals with 'field extensions', and the central study is the 'Galois correspondence' between extensions and groups.

iii.Resolvent polynomials are used to determine Galois groups of polynomials. The computation of the resolvent typically relies on root approximations, requiring a high degree of precision. Leonard Soicher has developed a method to compute absolute linear resolvents symbolically without the need for root approximations (Valibouze, 2020).

iv.One of the most important applications of Galois theory is to provide the criterion for estimating when a polynomial is solvable through rational operations and root extractions, executed by exploiting the interrelationship between fields and their respective automorphism groups. There is a sequence of sub-fields between the splitting field and the coefficient field, and these equates to a sequence of subgroups.

v. In Algebraic number theory , Galois theory forms significant part of number theory particularly in the context of class field theory which relates class groups of number fields to Galois groups of their extension

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vi.Classical mechanics : Galois theory is important in the aspect of integrability of Hamiltonian systems in the study of dynamics system

vii.Application to minimal polynomial : Considering L/K to be a Galois extension and $w \in L$, the Galois group $Gal(L/K)$ provides a systematic way to describe all the roots of the minimal polynomial of w over k. They are the different elements of the Galois orbit $\{\sigma(w): \sigma \in Gal(L/K)\}$. Suppose Gal (L/K) acts on $L[X]$, and not just L,by acting on polynomial coefficients then, we can relate minimal polynomial of the same number over different fields using a Galois group (Conrad, 2020).

viii.Classical Mechanics :Galois theory is essential in the area of integrability of Hamiltonian systems in the study of dynamics systems.

Conclusion

Galois theory as a branch of abstract algebra, explores the connection between field theory and collective theory. Its use in computer algebra to simplify radical formulas is an interesting example (Awtrey, Cesarski & Jakes, 2017). It should come as no surprise that studies in algebraic geometry, group theory, and number theory have taken this task into consideration, creating a lovely connection between group theory and the theory of polynomial equations. Galois's work serves as the basis for several core ideas in group theory. In general, calculation of the Galois group of a given polynomial is numerically complicated when the degree of the polynomial is modestly high. The numerical methods depend on the knowledge of transitive subgroups of the symmetric groups. Most polynomials of degree five or higher does not have closed form solutions because of the computational hurdle of the solutions of these types of polynomials. Hence,it is recommended that Galois groups and resolvents method should be used to solve these polynomials for errors-free solutions. .

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